Problem Set for Chapter 4

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1. In Section 4.2 the brightness temperature was introduced. In this problem we consider a simple example of how the brightness temperature spectrum $T_B(\nu)$ relates to the specific intensity and how the spectrum of either can be used to constrain the properties of an emitting source.

   We consider the case of radio emission from a solar flare on the limb and suppose that it has been observed by a radioheliograph, a telescope that images the Sun at one or more frequencies. We suppose that the resolution of the radioheliograph has been adjusted to be frequency-independent so that the beam size$\Omega_\circ$ is constant.

   The absorption coefficient for thermal bremsstrahlung emission from a hot plasma is given approximately by $\alpha_{ff}(\nu) \approx 0.2\nu^{-2}T^{-3/2}n^2$, where n is the number density of thermal electrons and T is their temperature. The absorption coefficient for nonthermal gyrosynchrotron emission from a power-law distribution of electrons with $\delta = 4$ can be given approximately as $\alpha_{gs}(\nu) \approx 1.85 \times 10^{-10}s^{-5.3}N/B$, where $s = \nu/\nu_B$, N is the number density of nonthermal electrons, and B is the magnetic field strength.

   What is the frequency dependence of the brightness temperature of spectrum for optically thin thermal bremsstrahlung emission? For nonthermal gyrosynchrotron emission? Assuming the source is uniform, how does the spectrum compare in terms of flux density per beam? In reality, both emission mechanisms may contribute to the brightness temperature spectrum. How might one disentangle the two contributions?

   Solution: The optical depth of the thermal emission is given by $\tau_{ff} = \alpha_{ff}L$. With $\tau_{ff} \ll 1$, $T_B \approx \tau_{ff}T \propto \nu^{-2}$. Hence $T_B(\nu) \propto \nu^{-2}$ for optically thin thermal bremsstrahlung emission at radio wavelengths. A proxy for the specific intensity is flux density per beam. Therefore, $F(\nu)/\Omega_\circ \approx 2\nu^2kT_B(\nu)/c^2 \propto \nu^0$. That is, the flux density of optically thin thermal bremsstrahlung emission is independent of frequency - it is flat. Similarly, the brightness temperature spectrum of optically thin gyrosynchrotron emission from a power-law distribution of electrons is $T_B(\nu) \propto \nu^{-5.3}$ for this example and the flux density spectrum $F(\nu) \propto \nu^{-3.3}$. The fact that the optically thin gyrosynchrotron flux density decreases so dramatically with increasing frequency whereas optically thin thermal bremsstrahlung emission remains flat suggests that at high enough frequencies thermal emission completely dominates. Hence, high frequency observations can be used to estimate and remove the thermal contribution.

2. Type II radio bursts are believed to be produced by plasma radiation associated with shocks in the corona or the interplanetary medium. They can therefore be used as probes of these plasma media. Suppose the dynamic spectrum of a type II radio burst is observed by a ground based radio spectrometer over a frequency range of 50-150 MHz. The fundamental band emission is observed to drift from 150 MHz to 50 MHz in 4 min. Assuming a coronal density scale height of 100 Mm, what is the speed of the shock that excites the type II radio burst? As noted in Chapter 8, coronal shocks are believed to have relatively low Alfvén Mach numbers Assuming $M_A \sim 1$, estimate the coronal magnetic field near the shock.

   Solution: The frequency drift rate of the type II radiation can be estimated as $\dot{\nu} = -100/240 \approx -0.4$ *MHz/s. The speed of the shock driver can then be estimated as* $v_{sh} \approx -2\dot{\nu}H_n/\nu_{pe}$. *Taking* $\nu_{pe} = 100$ *MHz one finds that* $v_{sh} = 800$ *km/s. If the Alfv/'en Mach number is such that* $M_A \approx 1$ *we have* $V_{sh} = v_A = B/\sqrt{4\pi n_p m_p}$, *where* n_p *and* m_p *are the proton number density and mass. For a fully ionized hydorgen plasma., the electron number density* $n_e = n_p$. *Since the type II fundamental band emits at the local electron plasma frequency* $\nu_{pe} = \sqrt{n_e e^2/\pi m_e}$ *the Alfvén speed becomes* $v_{sh} \approx eB/2\pi\nu_{pe}\sqrt{m_e m_p}$ *so that* $B \approx 2\pi\nu_{pe}(m_e m_p)^{1/2}v_{sh}/e$. *Substituting for* v_{sh}, ν_{pe}, *and the constants yields an estimate* $B \approx 4$ *G.*

3. A significant amount of the energy released during the course of a solar flare resides in nonthermal electrons. As discussed in section 4.3.2, these electrons produce HXR emission via nonthermal bremsstrahlung radiation. Suppose a HXR photon spectrum of a solar flare is fit to a thick target model yielding a differential electron number density $dN/dE \propto E^{-\delta}$ above some cutoff energy E_c. How does the total energy instantaneously contained in nonthermal electrons depend on the cutoff E_c?

   Solution: The total energy contained in the nonthermal electrons instantaneously is simply

$$E_{tot} \propto \int_{E_c}^{\infty} \left(\frac{dN}{dE}\right) E dE = \frac{E_c^{1-\delta}}{\delta - 1}$$

   Since the spectral index $\delta \sim 4 - 8$, *it is seen that estimates of the contribution of nonthermal electrons to the overall energy budget depend sensitively on* E_c.