Homework problems for the Heliophysics Summer School 2010
Book 2, Chapter 11: Energization of Trapped Particles
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Show all of your work and explain your answers. No credit without the intermediate steps.

Q-1. Calculate the ExB drift of an ion in the plasma sheet during a magnetic storm.
(a) Derive the general equations of motion (position and velocity vectors), starting from the Lorentz force terms, assuming a northward magnetic field and a dawn-to-dusk electric field.
(b) Find the specific equations of motion for a proton when **B**=+5 nT **z**, **E**=+0.25 mV/m **y**, starting in the plasma sheet 20 R_E downtail (**r**=-20 R_E **x**) with a sunward initial velocity of **v**=+10 km/s **x**.
(c) How long will it take the proton to reach the inner magnetosphere (defined as inside of 7 R_E geocentric distance for this problem), neglecting the gyration motion?
(d) What changes in this solution if the particle is a singly-charged oxygen ion rather than a proton?
(e) Calculate the gyration-averaged energy gain of the two particles (proton and oxygen ion) discussed above (energy of final drift velocity minus energy of initial velocity).

Q-2. Calculate the gyration, bounce, and drift periods for the following particles.
(a) Protons at E = 10 keV, $\alpha_0 = 90^\circ$, at L=4.
(b) Protons at E = 10 keV, $\alpha_0 = \alpha_{0,LC}$, at L=4.
(c) Protons at E = 10 MeV, $\alpha_0 = 90^\circ$, at L=2.
(d) Electrons at E = 10 keV, $\alpha_0 = 90^\circ$, at L=4.

Q-3. Calculate the total mass of the quiet-time plasmasphere.
(a) Calculate the total plasmaspheric mass assuming that the plasmasphere extends out to L=6 where the density is 100 cm^{-3} (assume a composition of only protons). Also assume a density along the magnetic field lines that's proportional to the field strength and a radial dependence of $n \sim L^{-4}$.
(b) Is the content contribution larger from a 1 cm^2 equatorial-plane-area flux tube at L=2 or a 1 cm^2 equatorial-plane-area flux tube at L=6?
(c) How does this change when the comparison is done with a constant magnetic field flux tube comparison? That is, assume an equatorial plane area containing 1 nT at L=2 and L=6.
(d) How much mass is lost to deep space during a large magnetic storm (assume that the plasmapause moves in to L=3).

Q-4. Calculate the total energy of the storm-time ring current.
(a) Calculate the total ring current energy assuming that the ring current extends from L= 4 to 7 with a constant density of 10 cm^{-3}, constant average energy of 40 keV, and uniform density and energy of only protons along the field lines.

(b) Assuming exponential energy content decay with time and τ=20 h, how long with it take to reduce this energy content to a tenth of its peak value?
(c) Compare the storm-time ring current energy content with the quiet-time plasmaspheric energy content. Assume an average energy of 1 eV for the plasmasphere, and use the plasmaspheric number content in the previous question.

Q-5. Calculate the PSD function of relativistic electrons from the radial diffusion equation.
(a) Derive the general solution for the equilibrium state of the PSD function from the radial diffusion equation (that is, assume d/dt terms are zero) with no loss term (that is $\tau \rightarrow \infty$), assuming $D_{LL}=D_0L^4$.
(b) Find the specific solution for the boundary conditions of $f(L=3)=0$ and $f(L=8)=f_0$.
(c) Describe in words the change in your answer if the functional dependence of the diffusion coefficient were $D_{LL}=D_0L^{10}$.