

e. The dot product of the normal vector with the magnetic fields gives the angles

$$\cos \theta_1 = \frac{\mathbf{B}_1 \cdot \hat{\mathbf{n}}}{|\mathbf{B}_1|} = \frac{-1.73}{2.00} = -0.866 \quad , \quad \theta_1 = 150.000^\circ$$

$$\cos \theta_2 = \frac{\mathbf{B}_2 \cdot \hat{\mathbf{n}}}{|\mathbf{B}_2|} = \frac{-1.73}{3.42} = -0.507 \quad , \quad \theta_2 = 120.438^\circ$$

f. From the expressions we find

$$M_{A2}^2 = \frac{n_1}{n_2} M_{A1}^2$$

and

$$(M_{A1}^2 - 1) \tan \theta_1 = (M_{A2}^2 - 1) \tan \theta_2 = \left(\frac{n_1}{n_2} M_{A1}^2 - 1 \right) \tan \theta_2$$

Solving this gives

$$M_{A1} = \sqrt{\frac{\tan \theta_1 - \tan \theta_2}{\tan \theta_1 - (n_1/n_2) \tan \theta_2}}$$

$$M_{A1} = 5.000 \quad , \quad M_{A2} = 3.024$$

g. We have several pieces of evidence that this is a **fast shock**. We could have noted early on that $|\mathbf{B}_2| > |\mathbf{B}_1|$. Later it was evident that \mathbf{B}_2 had deflected *away* from the shock normal (although it appears that $\theta_2 < \theta_1$ because the field is directed backwards). Finally, we see that the Mach numbers are ordered

$$1 < M_{A2} < M_{A1} \quad , \quad (41)$$

which is a prerequisite for the *fast shock*.

h. The Alfvén Mach number refers to the normal component of the upstream velocity $v_{1,n}$ in the reference frame of the shock. If we denote the shock speed v_s the inflow speed is therefore

$$v'_{n,1} = v_{n,1} - v_s = -M_{A1} v_{A1,n} \quad , \quad (42)$$

since the inflow is always in the direction opposite $\hat{\mathbf{n}}$. The shock speed is therefore

$$v_s = v_{n,1} + M_{A1} v_{A1,n} \quad , \quad (43)$$

where $v_{1,n}$ is the component of the velocity, in the spacecraft frame,

$$v_{n,1} = \hat{\mathbf{n}} \cdot \mathbf{v}_1 = -2.10 \times 10^7 \text{ cm/s} = -210 \text{ km/s} \quad .$$

The normal component of the Alfvén velocity is

$$v_{An,1} = \frac{|\mathbf{B}_1 \cdot \hat{\mathbf{n}}|}{\sqrt{4\pi n_1 m_p}} = \frac{1.73 \times 10^{-5} G}{9.44 \times 10^{-12}} = 1.83 \times 10^6 \text{ cm/s} = 18.3 \text{ km/s} \quad .$$

This gives the shock velocity in the reference frame of the spacecraft

$$v_s = (-210 + 5.00 \times 18.3) \text{ km/s} = -118 \text{ km/s} \quad , \quad (44)$$

The shock propagates in the direction of the shock normal giving

$$\mathbf{v}_s = \hat{\mathbf{n}} v_s = (103, -32.2, -49.7) \text{ km/s} \quad . \quad (45)$$

The shock is therefore moving *outward* from the Sun, even though it is a *reverse shock*.