Problem Set on Magnetic Reconnection

Amitava Bhattacharjee

1. Assume that a plasma is evolving slowly in time preserving the magnetostatic equilibrium condition $\mathbf{J} \times \mathbf{B} = \nabla p$ at all times. For simplicity, assume also that the current density, the magnetic field, and the plasma pressure depends only on one spatial dimension (y) and time (t).

(a) For a unidirectional magnetic field $\mathbf{B} = \mathbf{B}(y,t)\hat{\mathbf{x}}$, show that the magnetostatic equilibrium condition yields

$$B^2(y,t)/(2\mu_0) + p(y,t) = const.$$

(b) Assuming that plasma flows are negligible, show that the time-dependent magnetic induction equation is satisfied by the following solution:

$$\mathbf{B} = B_0 erf\left(\frac{1}{2}\left(\frac{\mu_0}{\eta t}\right)^{1/2} y\right)\hat{\mathbf{x}}, \text{ where } \quad erf(y) = \frac{2}{\sqrt{\pi}}\int_0^y \exp(-u^2)\,du.$$

(c) Make plots of the magnetic field, current density, and pressure for a sequence of times $t_1 < t_2 < t_3$.

2. The goal of this problem is to derive an expression for the reconnection rate in the quasi-steady Sweet-Parker model.

(a) Consider a magnetic field profile

$$\mathbf{B} = B_0 \tanh(z/a)\hat{\mathbf{x}}.$$

Make a plot of the magnetic field and identify the neutral line across which the magnetic field reverses direction.

(b) Consider now the reconnection scenario discussed in the lecture. Show from the one-dimensional momentum equation that the maximum outflow velocity along x is given by

$$v_{out} = V_A = B_0/(\mu_0 \rho)^{1/2},$$

where ρ is the plasma mass density, assumed to be constant.

(c) Show, by using the equation of mass and energy conservation that the reconnection rate is given by $v_{in} = V_A / S^{1/2}$, where S is the Lundquist number of the plasma.

(Hint: By energy conservation, we mean that the energy per unit time flowing into the reconnection layer is equal to the Ohmic dissipation per unit time in the layer. Use the result that the Ohmic dissipation per unit volume per unit time in the resistive MHD model is given by ηJ^2.)

(d) How does the typical time scale for reconnection in this problem compare with the time scale for diffusion in problem 1?