Problem set: solar irradiance and solar wind

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1 Stratification of a static atmosphere within a force-free magnetic field

Problem: Write down the general MHD force-balance equation to derive the effect of the curvature and pressure-gradient forces on the stratification of a plasma with a potential magnetic field or with a field with only field-aligned currents. What is the effect of magnetic pressure in a potential or force-free field compared to that in a flux tube surrounded by a field-free atmosphere?

Solution: You could start with:

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla p + \rho \mathbf{g} + \frac{1}{4\pi}(\nabla \times \mathbf{B}) \times \mathbf{B} \tag{1}$$

and then realize that $\mathbf{v} = \mathbf{0}$ while $\nabla \times \mathbf{B} \equiv \mathbf{0}$ in a potential field and that $(\nabla \times \mathbf{B}) \times \mathbf{B} = \mathbf{0}$ in a more general force-free field. For a flux tube with a field free environment, it is the jump in magnetic pressure that needs to be compensated by the jump in gas pressure going from inside to outside the tube.

2 Bright and dark magnetic features and solar irradiance

2.1 When are magnetic concentrations in the solar photosphere seen as bright or dark?

Problem: Strong magnetic fields suppress convective motion. Under which conditions does this occur in the solar photosphere? Estimate the field strength at which convective suppression begins to be effective (note: sound speed $v_s \approx$ 7 km/s, convective velocity far from upflow $v_c \approx 2$ kms). Once convection stops, the gas inside a flux bundle slumps back ("convective collapse") leaving a largely evacuated tube with field of roughly 1 kG. Explain the resulting Wilson depression as a result of the photon mean free path ($\lambda \sim H_p/2$, for pressure scale height H_p), the formation of sunspot umbrae and photospheric faculae. At what flux, roughly, does the transition from bright facula to dark pore occur?

Solution: Equipartition of convective flow dynamic pressure and field pressure occurs at

$$\frac{B_{eq}^2}{4!pi} \approx \frac{1}{2}\rho v_c^2, \tag{2}$$

so that $B_{eq} \approx 30\,\mathrm{G}$.

When the cooling plasma inside strong field adjusts to a much reduced gravitational stratification, the field strength in the tube approaches that which matches the gas pressure:

$$\frac{B_c^2}{4!pi} \approx \frac{1}{2}\rho v_s^2, \tag{3}$$

so that $B_c \approx 100\,\mathrm{G}$. But because of the Wilson depression, the photosphere inside the tube lies lower, and therefore at a higher pressure and related field strength, in fact resulting in about 2 kG.

If the diameter of a flux bundle exceeds several photon scattering lengths λ (about half the photospheric pressure scale height H_p), a dark spot occurs as energy cannot readily diffuse into the tubes interior. The transition between facula and pore thus occurs at fluxes a few times above about $B_c(2H_p)^2 \approx 2\,10^3[\mathrm{G}](150\,10^5[\mathrm{cm}])^2 \approx 5\,10^{17}\,\mathrm{Mx}$.

2.2 Why does solar irradiance peak when the sunspot number reaches its maximum?

Problem: Discuss the relative roles of spots, pores, and faculae in regulating the solar irriadiance, and explain why a sunspot surrounded by magnetic faculae, together forming active regions, result in a dip as the sunspot passes central meridian, even as the overall irradiance near sunspot maximum is higher than at sunspot minimum.

Solution: See Chapter by Lean and Woods in Volume 3 for details. Bottom line: sunspots have their largest contributions as they pass central meridian, with foreshortening reducing their effects towards the limb, while in contrast faculae are (a) much more distributed spatially with (b) their strongest effect in brightening away from disk center as one looks at an angle into the flux tubes to see the relatively bright walls resulting from looking at deeper and thus hotter layers of the solar interior.

3 How much of the Sun's luminosity is available for conversion into phenomena related to its magnetic activity?

Problem: Estimate the maximum mechanical energy flux density available to provide energy into the Sun's atmospheric magnetic field by random horizontal displacements of that field (by the "braiding mechanism" originally proposed by Parker). Use the approximation that the convective enthalpy flux near the photosphere is dominated by the latent heat of hydrogen ionization, that the vertical/depth scale of the convection D equals the sub-photospheric pressure scale height $H_p \approx 400\,\mathrm{km}$ and that the horizontal scale is approximately $L = 700\,\mathrm{km}$. What fraction of the solar luminosity does that amount to? Does solar outer atmospheric activity approach that limit? Why do you think that is?

Solution: The available mechanical energy flux density for the horizontal (braiding) motions can be estimated from

$$F_{mech} = \frac{1}{2}\rho v_r v_h^2, \tag{4}$$

for horizontal (h) and vertical/radial (r) components of the convective flows.

The radial velocity can be estimated by equating the solar bolometric flux density with the energy flux associated with hydrogen ionization:

$$\sigma T_{eff}^4 = \rho v_r f_i N_A \chi_H, \tag{5}$$

where f_I is the ionization fraction (about 0.1), T_{eff} the effective temperature of 5800 K, χ_H th hydrogen ionization energy, N_A Avogadro's number, and $\rho \approx 5\,10^{-7}\,\mathrm{g/cm^2}$ the characteristic density. This yields $v_r \approx 1\,\mathrm{km/s}$. If you add the thermal enthalpy flux ($\rho v_r (5/2) kT$) the number nearly doubles (see Eq. 38 in Nordlund et al., 2009, Living Reviews in Solar Physics 6, 2), .

The horizontal flow velocity can be inferred from the equality of time scales for horizontal and vertical motions:

$$\frac{D}{v_r} \approx \frac{L/2}{v_h} \tag{6}$$

or $v_h \approx 1\,\mathrm{km/s}$.

Thus $F_{mech} = \frac{1}{2}\rho v_r v_h^2 = 2.5\,10^8\,\mathrm{erg/s/cm^2}$, or about $0.004\sigma T_{eff}^4$. The solar outer atmosphere uses considerably less than that, because the field strong enough to couple it into that atmosphere covers only a relatively small fraction of the surface.

4 Which ions carry the dominant radiative losses from the solar corona?

Problem: Why are solar coronal observations commonly made in spectral lines of iron rather than in those of the dominant species, hydrogen and helium? At what temperature do "heavy elements" (heavier than helium) dominate the spectrum of a plasma?

Solution: Look at the ionization energies of the elements. At coronal temperatures, hydrogen and helium are fully ionized, and the optical depths of their bound-free and free-free continua insufficient to lead to substantial emissions. In contrast, iron has residual bound electrons up to in excess of 10 MK, and is abundant enough to result in strong emission.

5 Solar wind and magnetic braking

5.1 Why is there a solar wind?

Problem a: Demonstrate that a hot solar atmosphere must flow outward. Show that a static atmosphere on a star with mass M_* and radius r_* in which

$$\frac{dp}{dr} = -nm_H GM_* \frac{1}{r^2} \tag{7}$$

for fully ionized hydrogen ($n = p/2kT$) and with

$$T(r) = T(r_*) \left(\frac{r_*}{r}\right)^{\gamma} \tag{8}$$

slowly decreasing with distance ($\gamma < 1$) owing to efficient electron heat conduction, the pressure at infinity exceeds the pressure of the interstellar medium.

Solution a: The pressure as function of distance is given by

$$p(r) = p(r_*) \exp\left[-\left(\frac{GM_*}{r_*}\right)\left(\frac{m_H}{2kT_*}\right)\left(\frac{1}{1-\gamma}\right)\left(1-\left(\frac{r_*}{r}\right)^{1-\gamma}\right)\right]. \tag{9}$$

For $v_g = (GM_*/r_*)^{1/2} \approx 450\,\mathrm{km/s}$ and $v_{th} = (2kT_*/m_H)^{1/2} \approx 130\,\mathrm{km/s}$ for a coronal base temperature of $T_* = 1\,\mathrm{MK}$. With a base density of $10^8\,\mathrm{cm}^{-3}$, the base pressure is $p(r_*) = 0.03\,\mathrm{dyne/cm}^2$. Compare that to the pressure of the LISM ($\approx 6000\,\mathrm{K}$, and $0.2\,\mathrm{cm}^{-3}$).

Problem b: Combine conservation of mass with the momentum equation for a steady outflow,

$$\rho v \frac{dv}{dr} + \frac{dp}{dr} = -nm_H GM_* \frac{1}{r^2}, \tag{10}$$

to estimate at what distance from the Sun an isothermal transsonic wind becomes supersonic.

Solution b: Combine the equations for conservation of mass and momentum with the assumption that the accelleration of the wind does not reach zero at the transsonic or critical point r_c. Then $r_c/r_* = \frac{1}{2}v_g^2/v_{th}^2$.

Problem c: Discuss the conditions under which $r_c < r_*$. What kind of stellar and coronal conditions does this require, and what would happen to the stellar wind if they were met? Consider other forces that may be important in driving the wind.

Solution c: This can happen when the surface gravity is low, as for any giant star, particularly if the coronal temperature is high, i.e., in the case of active giant stars. You may explore stellar winds across the HR diagram, noting that evolved giant stars often have a slow, dense breeze in which radiation pressure plays a significant role, effectively countering the stellar gravitational pull.

5.2 What is the time scale of magnetic braking for the present-day Sun of average activity?

Problem a: In a "Weber-Davis" approximation of the solar wind, the angular momentum per unit mass transported by the solar wind (including the specific angular momentum of the plasma and the torque density associated with the magnetic field in the Parker spiral) equals what is carried in a thin, rigidly rotation shell, $L = \frac{2}{3}\Omega r_A^2$, where r_A is the distance at which the wind's radial velocity equals the Alfvén velocity for the radial component of the solar wind, $v_{A,r} = B_r/(4\pi\rho)^{1/2}$. Combine that with conservation of mass and flux of the mostly monopolar-like field to derive the "Skumanich relation": $\Omega \propto t^{-1/2}$, using the approximation that the field strength at the base of the heliosphere scales roughly with angular velocity Ω as $B_{r,0} \propto \Omega$.

Solution a:

$$\frac{dJ}{dt} = 4\pi\rho_A r_A^2 u_{r,A} L, \tag{11}$$

which, with $r_0^2 B_{r,0} = r_A^2 B_{r,A}$ and $B_{r,A} = 4\pi\rho_A v_{r,A}$, yields

$$\frac{d\Omega}{dt} \propto -\Omega^3, \tag{12}$$

resulting in the Skumanich relation for $t \gg t_0$.

Problem b: For a solar moment of inertia of $I \approx 7\,10^{53}\,\mathrm{g\,cm^2}$, and a heliospheric flux of $\Phi =\approx 5\,10^{22}\,\mathrm{Mx}$, and an Alvén velocity close to the sound velocity at 1 MK, what is the time scale of the Sun's magnetic braking?

Solution b: The expression for angular momentum loss under (a) can be rewritten as

$$\frac{1}{\Omega}\frac{d\Omega}{dt} = \frac{2}{3}\frac{\Phi^2}{(4\pi)^2 I v_{r,A}}, \tag{13}$$

which with the numbers given yields a braking time scale

$$\Omega/\frac{d\Omega}{dt} \approx 3\,10^{10}\,\mathrm{yr}, \tag{14}$$

which in this highly simplified approach turns out longer than the value estimated to be of order 10^9 yr based on solar-wind observations and multidimensional modeling.