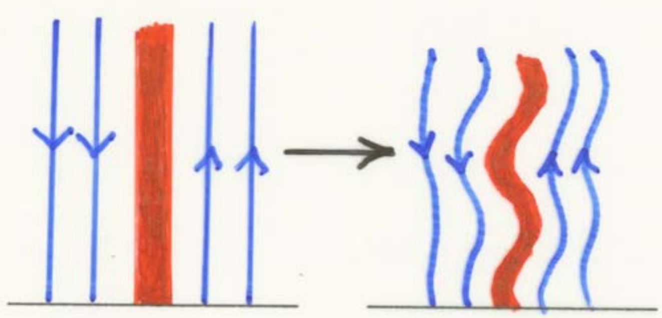
Magnetic Reconnection

Energy Conversion: Two Possibilities

Ideal Process

e.g. kink instability

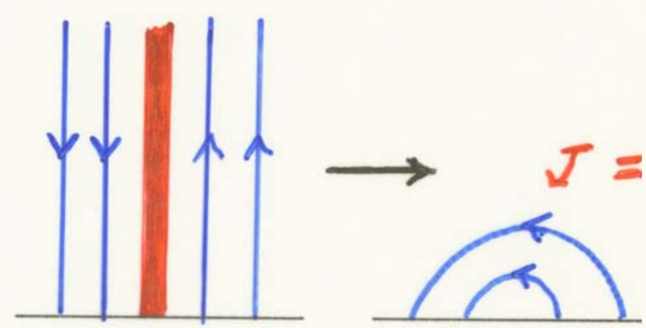


frozen flux constraint

fast but poor efficiency (< 10 %)

Non-Ideal Process

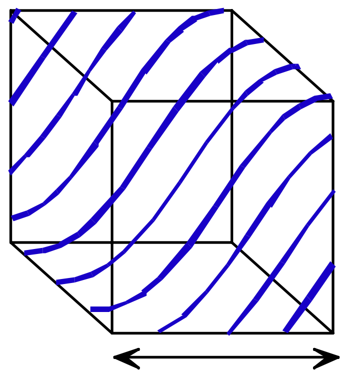
e.g. reconnection



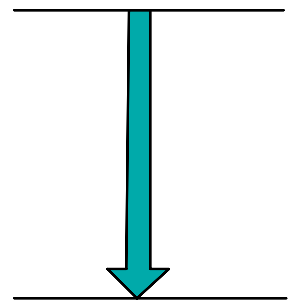
no topological constraints

slow but good efficiency (100 %)

Energy Conversion Rate Based on Simple Diffusion



$L = 6 \times 10^4$ km



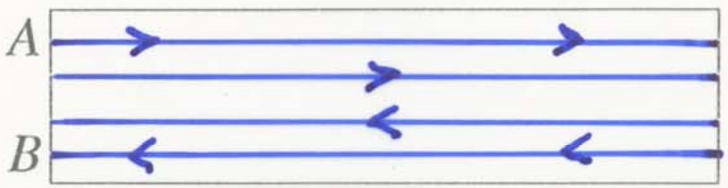
$$W_B = 10^{32} \text{ ergs}$$

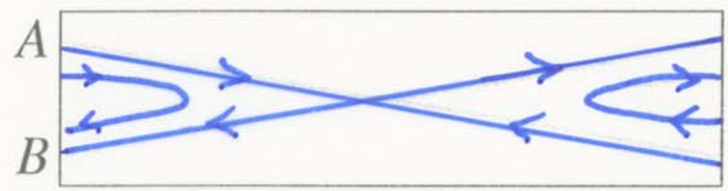
$$\tau_d = L^2 / \eta$$

$\eta = 0.35 \text{ m}^2/\text{s}$ (collisional) → $\tau_d = 3 \times 10^8$ yrs !

Definition of Magnetic Reconnection

Most general definition: Change in connectivity

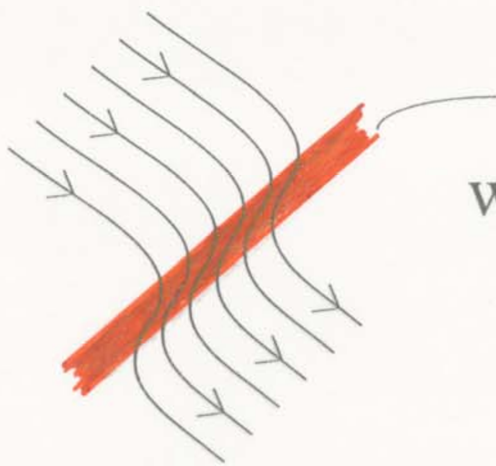




violation of frozen-flux

magnetic diffusion

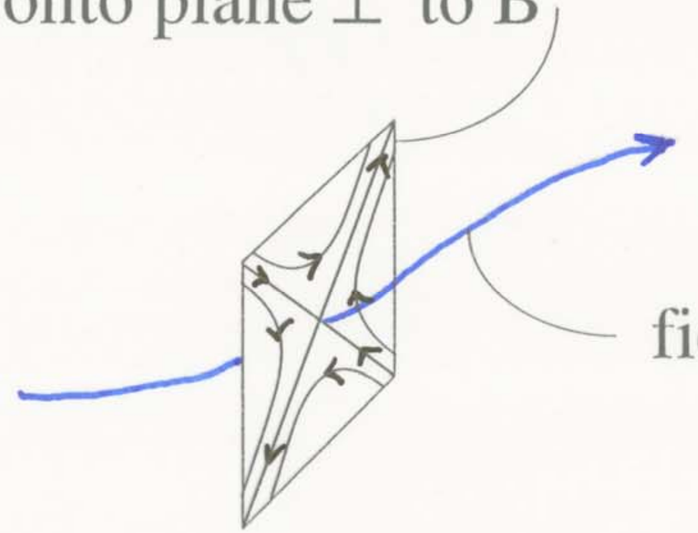
$\eta \mathbf{j} \neq 0$ $\mathbf{E} \cdot \mathbf{B} \neq 0$



3D shock transition with field line slippage

Restricted definition: x-type topology required

projection of adjacent lines
onto plane ⊥ to **B**



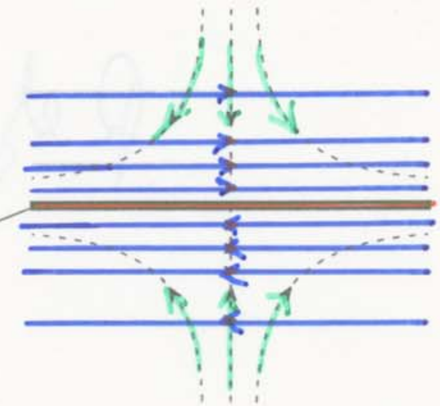
field line with $\mathbf{E}\cdot\mathbf{B} \neq 0$

Changes in Terms with Dimension

1D

merging (annihilation)

$\mathbf{E} \neq 0$ at neutral sheet



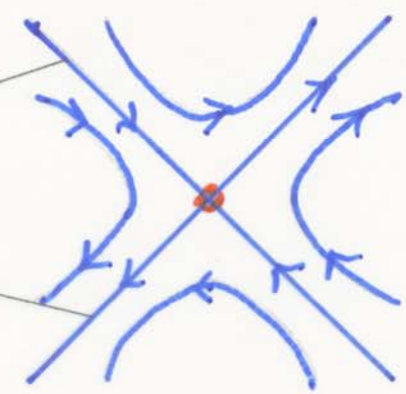
stagnation-point flow

2D

reconnection

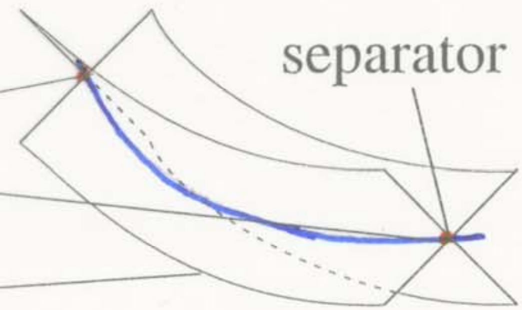
$\mathbf{E} \neq 0$ at x-line (x-point)

separatrix lines



Definition in Three Dimensions

3D with nulls



x-point pairs

separatrix surfaces

$\mathbf{E}_{||} \neq 0$ along separator line

3D without nulls

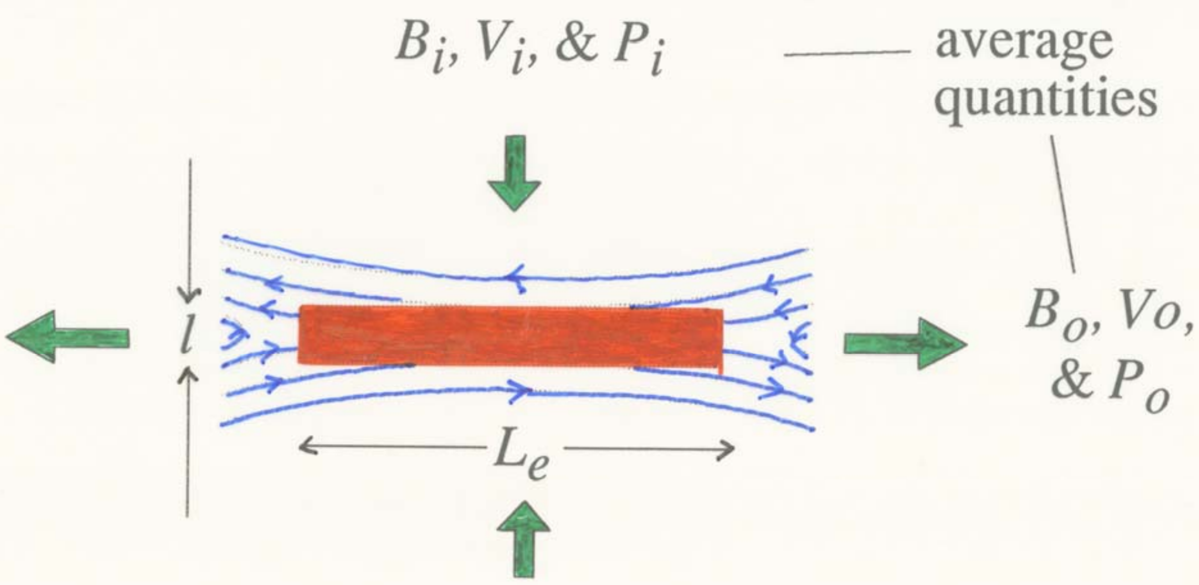
x-type topology

$E_{||} \neq 0$ in volume

separator volume

quasi-separatrix layers

Sweet-Parker Reconnection



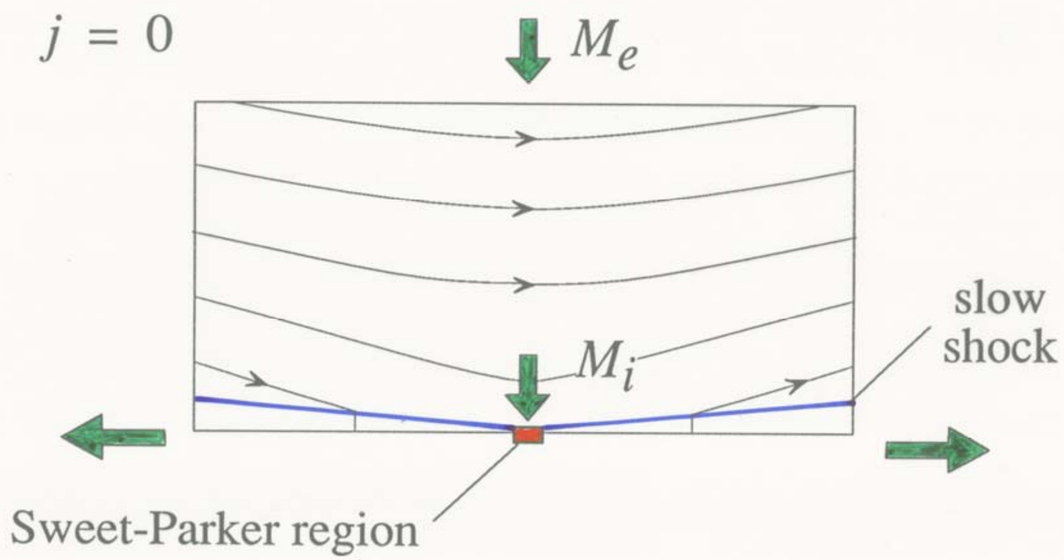
Knowns: L_e, B_i, & P_i

Unknowns: l, B_o, P_o, V_o, & V_i

$$M_e = M_i = R_m^{-1/2}$$

Solar corona: $M_e \approx 10^{-6}$: $t \approx 1$ year !

Petschek's Solution

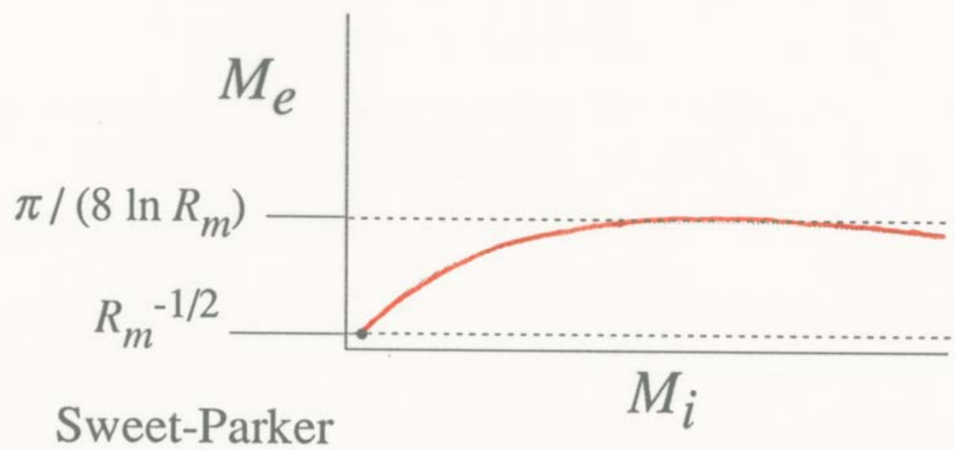


M_e is now a known

solar corona:

$M_e = 0.1$ to 0.01

10 minutes



Petschek's Solution

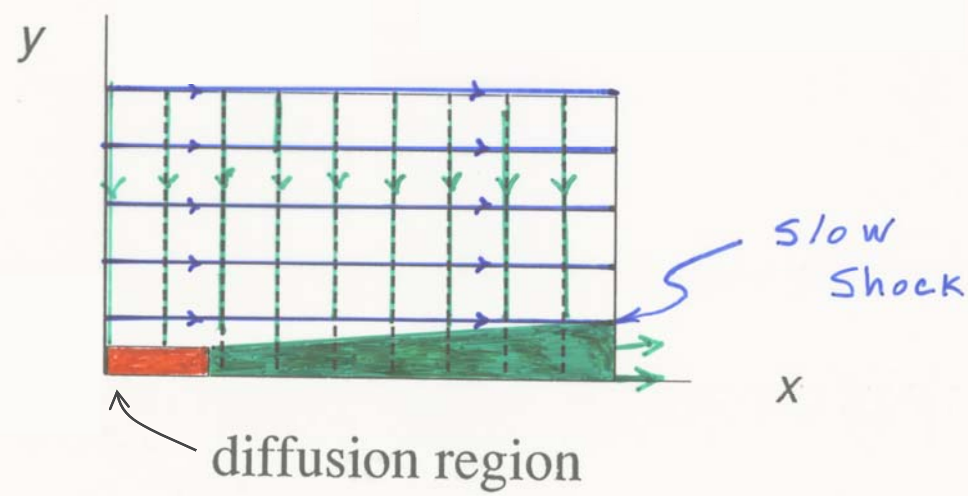
External Region Equations:

$$\rho\,(\mathbf{v}\cdot\nabla)\,\mathbf{v} \;=\; -\nabla p \;-\; \mathbf{j}\times\mathbf{B}\,, \qquad \nabla\cdot\mathbf{v} \;=\; 0$$

Expand:

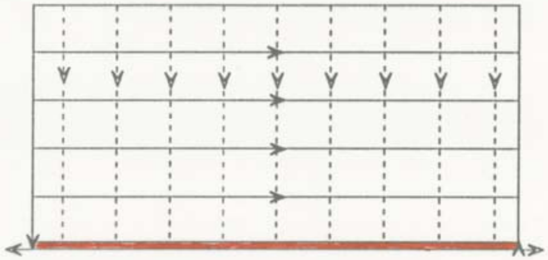
$$\mathbf{B} \;=\; B_o\,\hat{x} \;+\; \mathbf{B}_1 \;+\; \cdots$$
$$\mathbf{v} \;=\; \qquad -\;\mathbf{v}_1 \;+\; \cdots$$

$$\nabla^2 A_1 \;=\; -\mathrm{j}_1 \;=\; 0$$



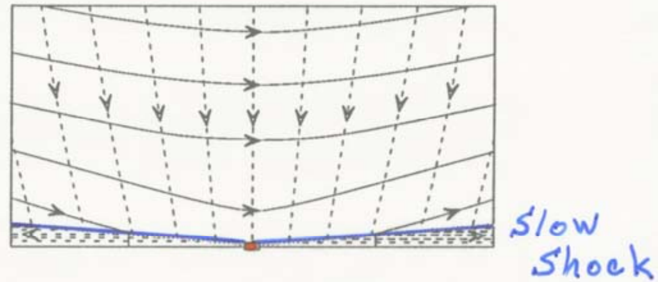
Different Solutions for Different Assumptions

Sweet-Parker



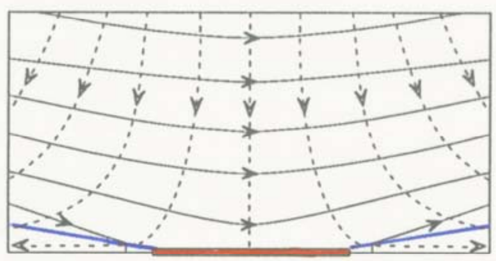
$$V_R = R_m^{-1/2} V_A$$

Petschek



$$V_R \leq \pi / (8 \ln R_m) V_A$$

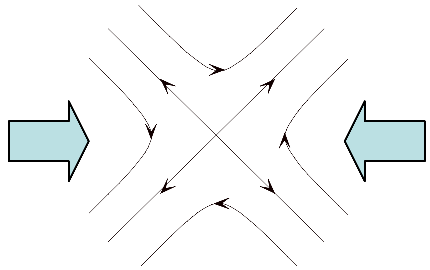
Flux-Pile-Up

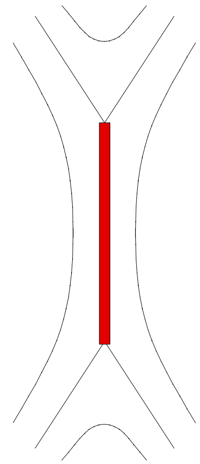


$$V_R \leq (\beta / R_m) V_A$$

↖ plasma beta

Syrovatskii’s Solution





$$V_R \approx R_m^{-1/2} V_A$$

$$V_R = 0$$

$$V_R = 0.5\ V_A$$

QuickTime™ and a
Animation decompressor
are needed to see this picture.

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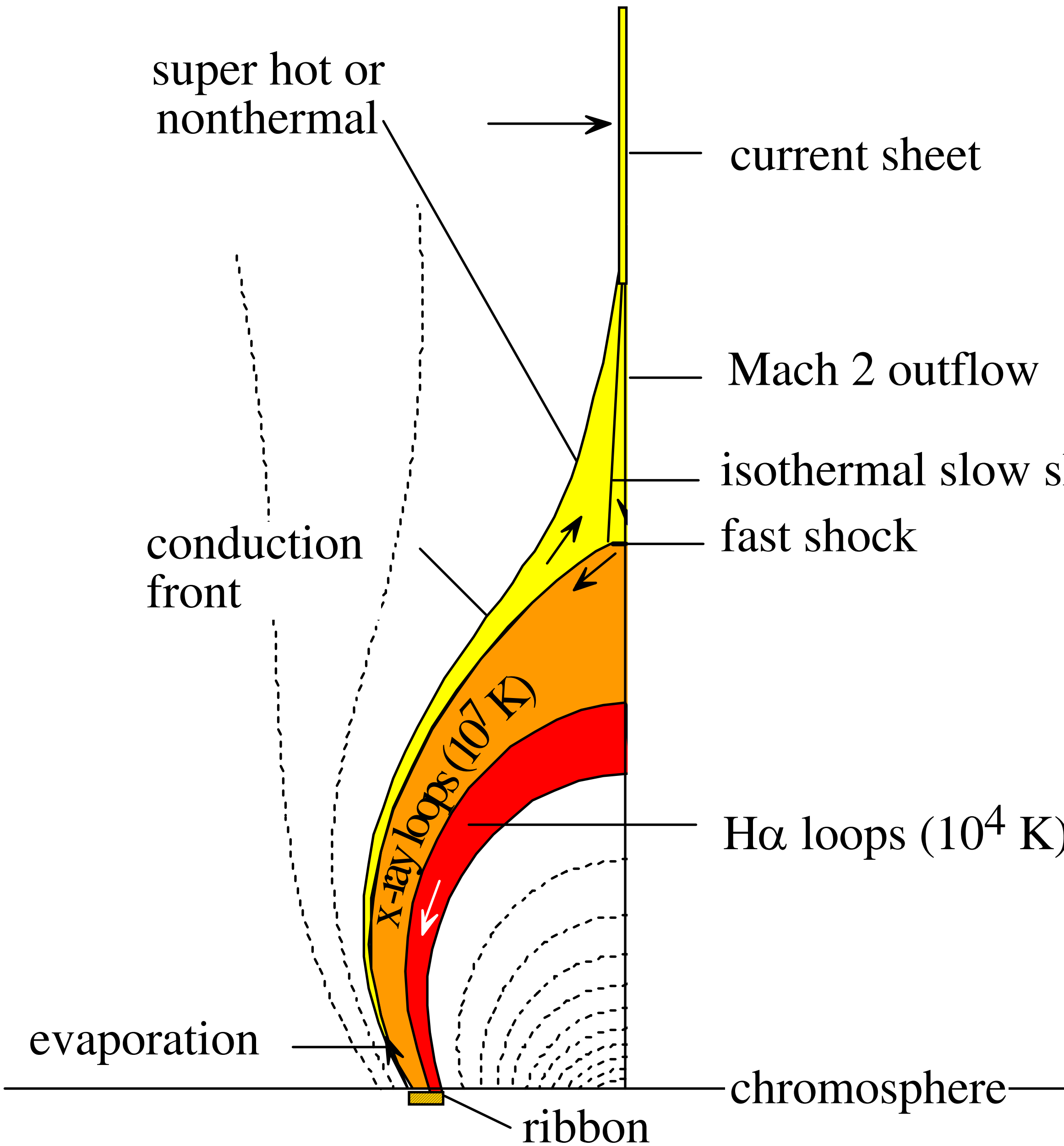
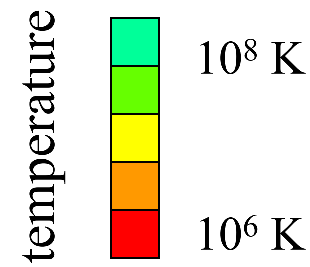
Time Dependent Reconnection

Flare Loop System





QuickTime™ and a
Animation decompressor
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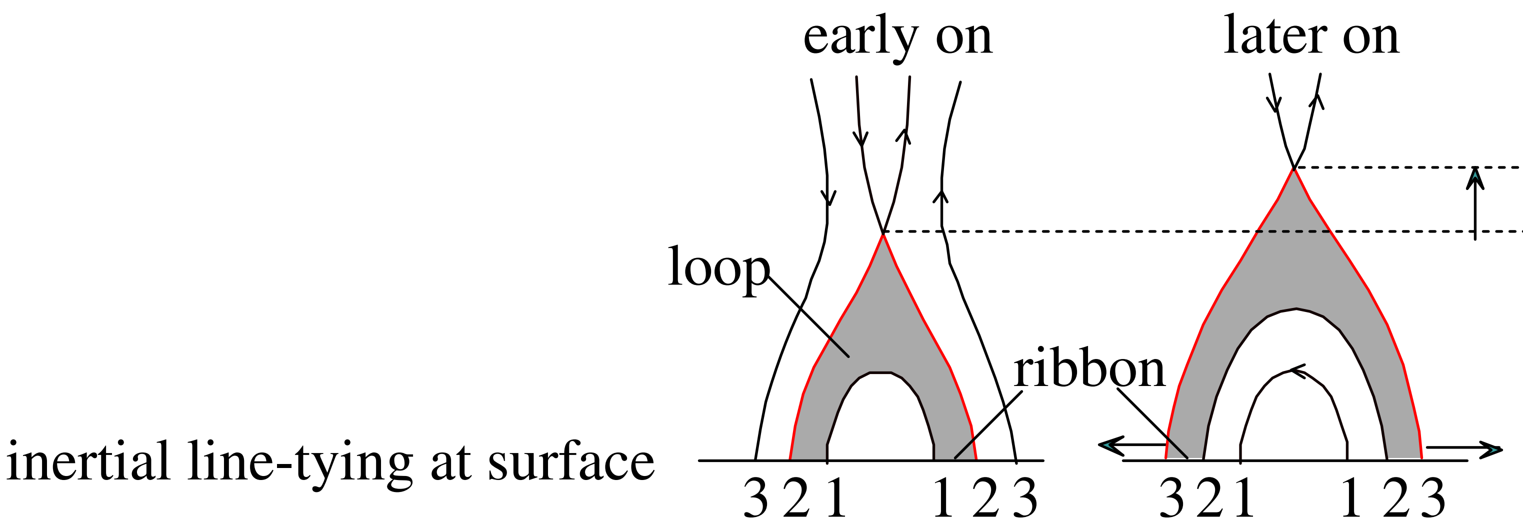
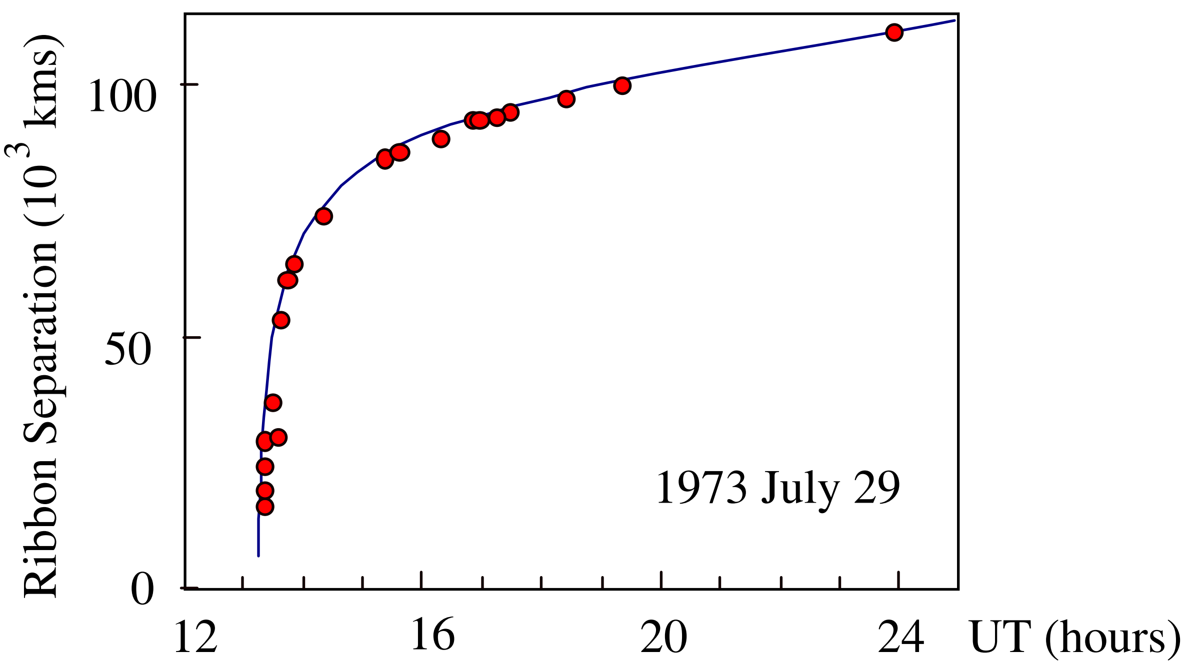
Apparent Motion of Loops & Ribbons

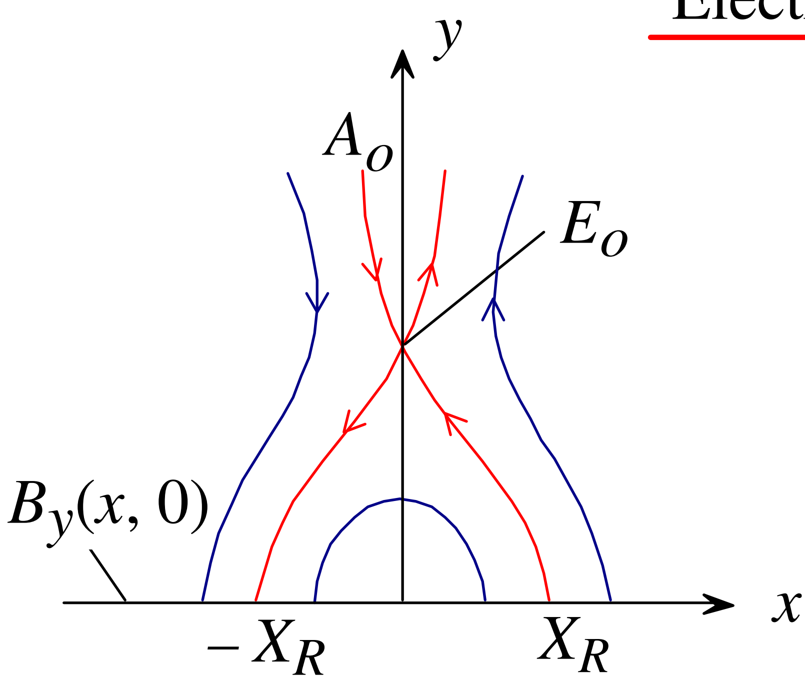




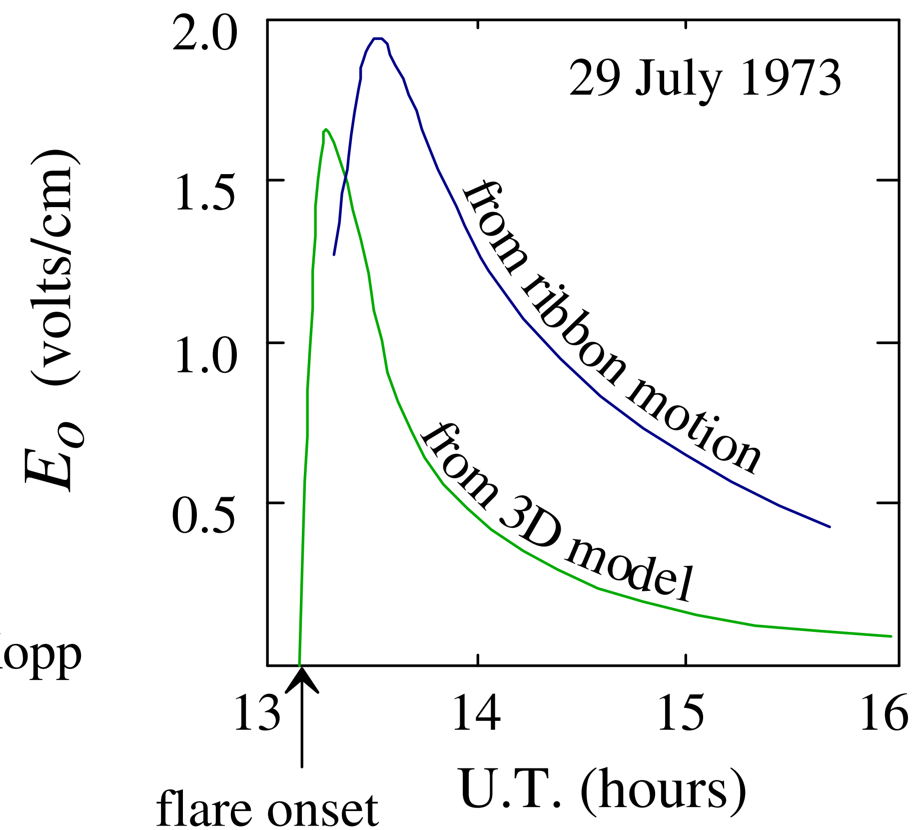
Electric Field at *X*-Line

$$E_o = -\frac{\partial A_o}{\partial t} = \frac{\partial A_o}{\partial x}\frac{\partial x}{\partial t}$$

$$E_o = -B_y\,(X_R)\,V_R$$

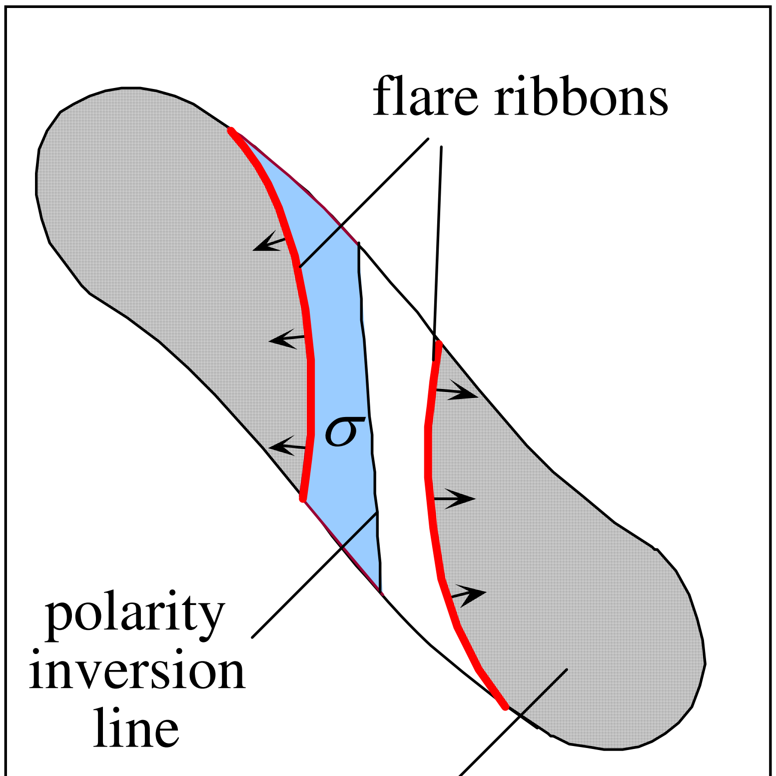


Forbes & Priest
(1984)



Poletto & Kopp
(1986)

Reconnection Electric Fields



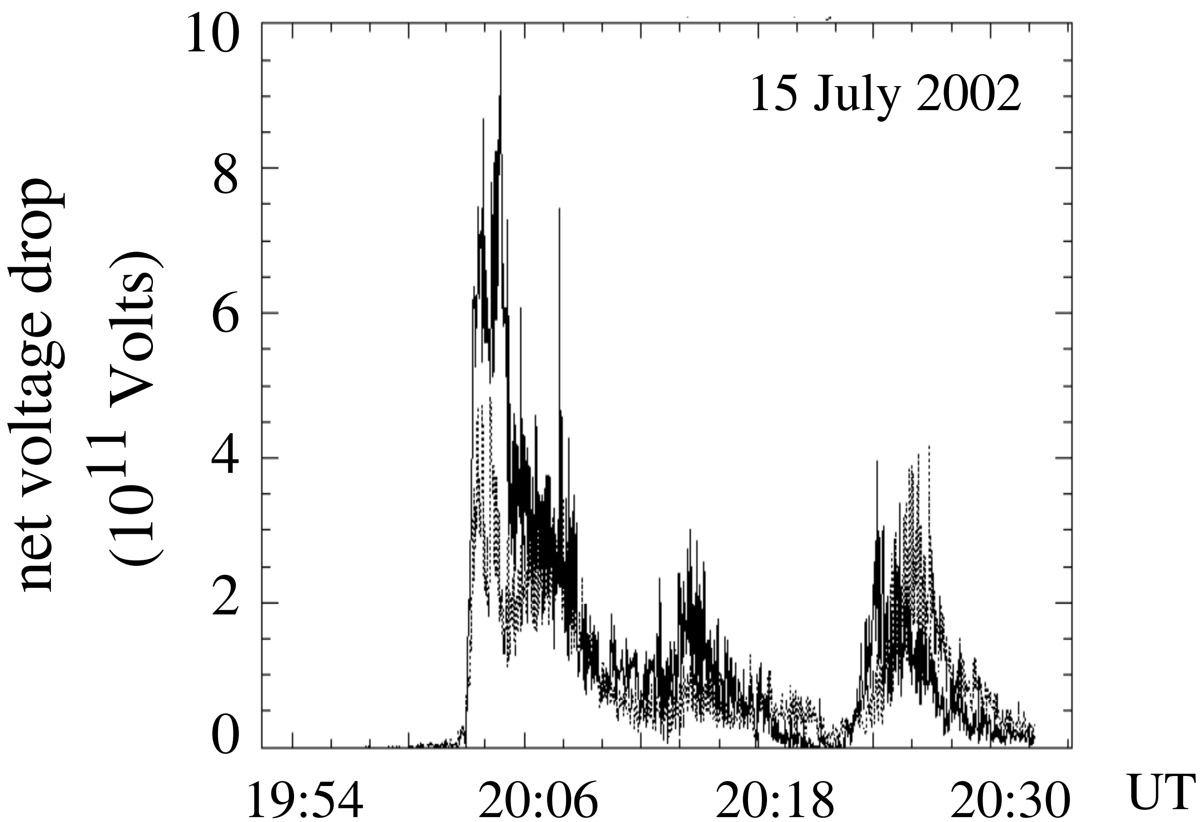
newly reclosed flux:

$$\Phi_B = \iint_{\sigma} B_z \, dx dy$$

global reconnection rate:

$$\int \mathbf{E} \cdot \mathbf{dl} = \frac{d\Phi_b}{dt}$$

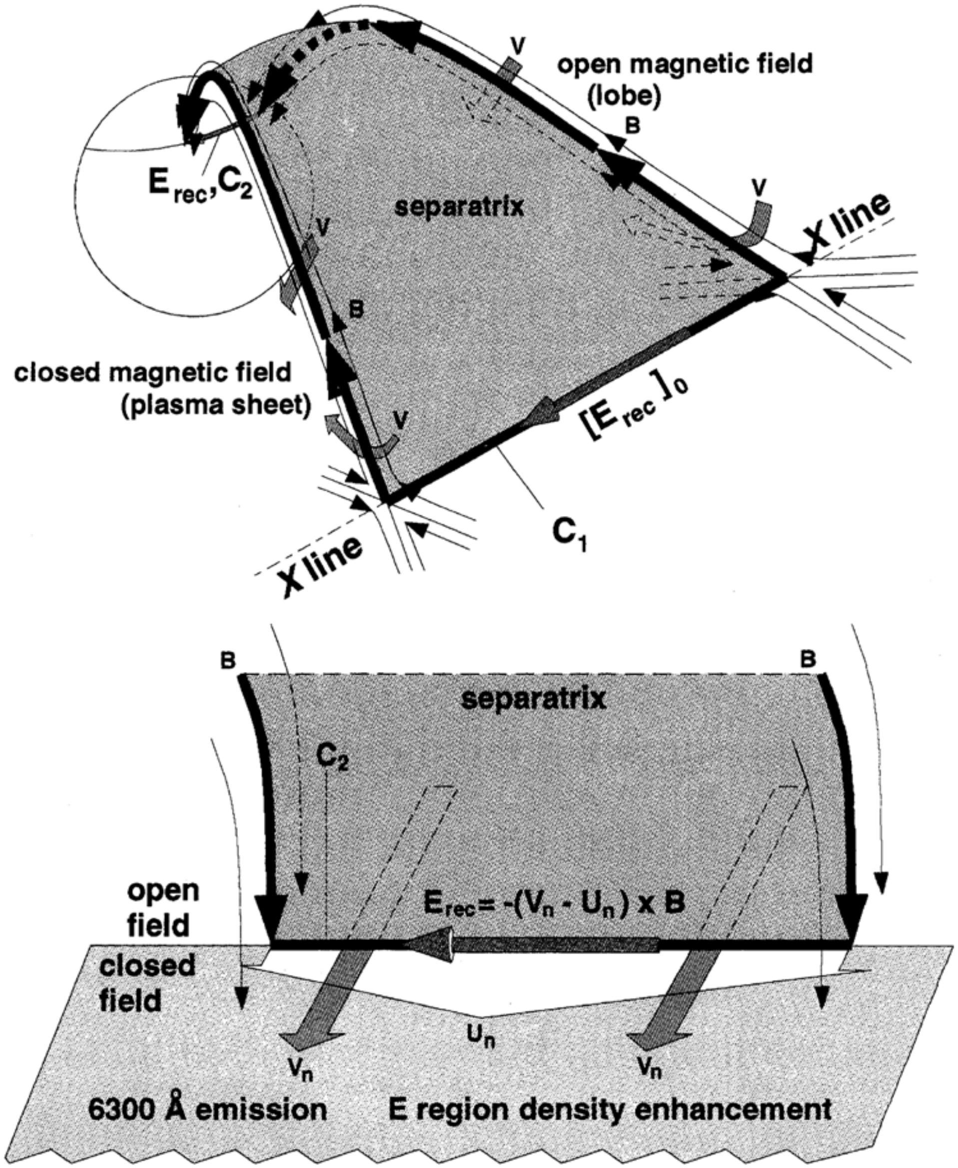
Observed Reconnection Rate for X3 Flare



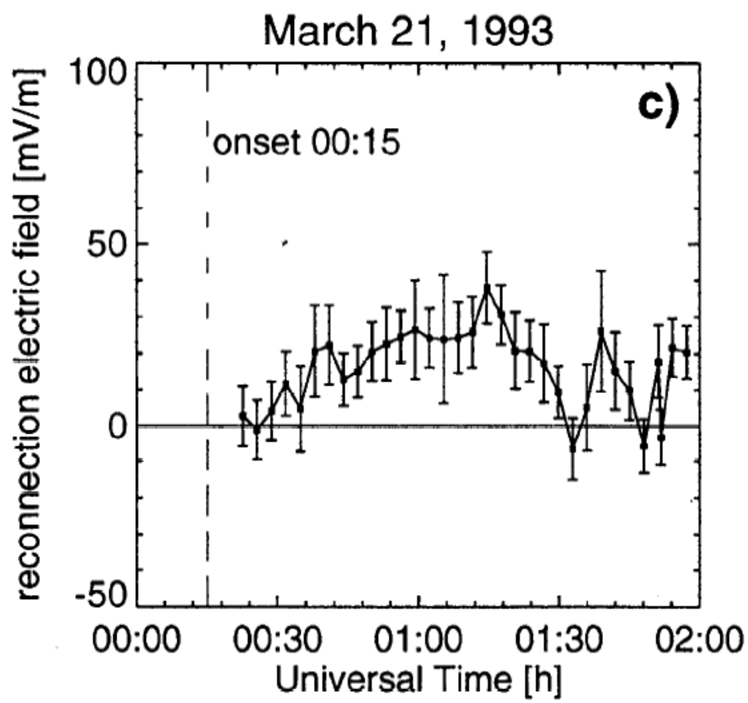
for estimated separator length of 2×10^5 km:

$$E_{ave} \approx 20 \text{ Volts / cm}$$

Magnetospheric Applications

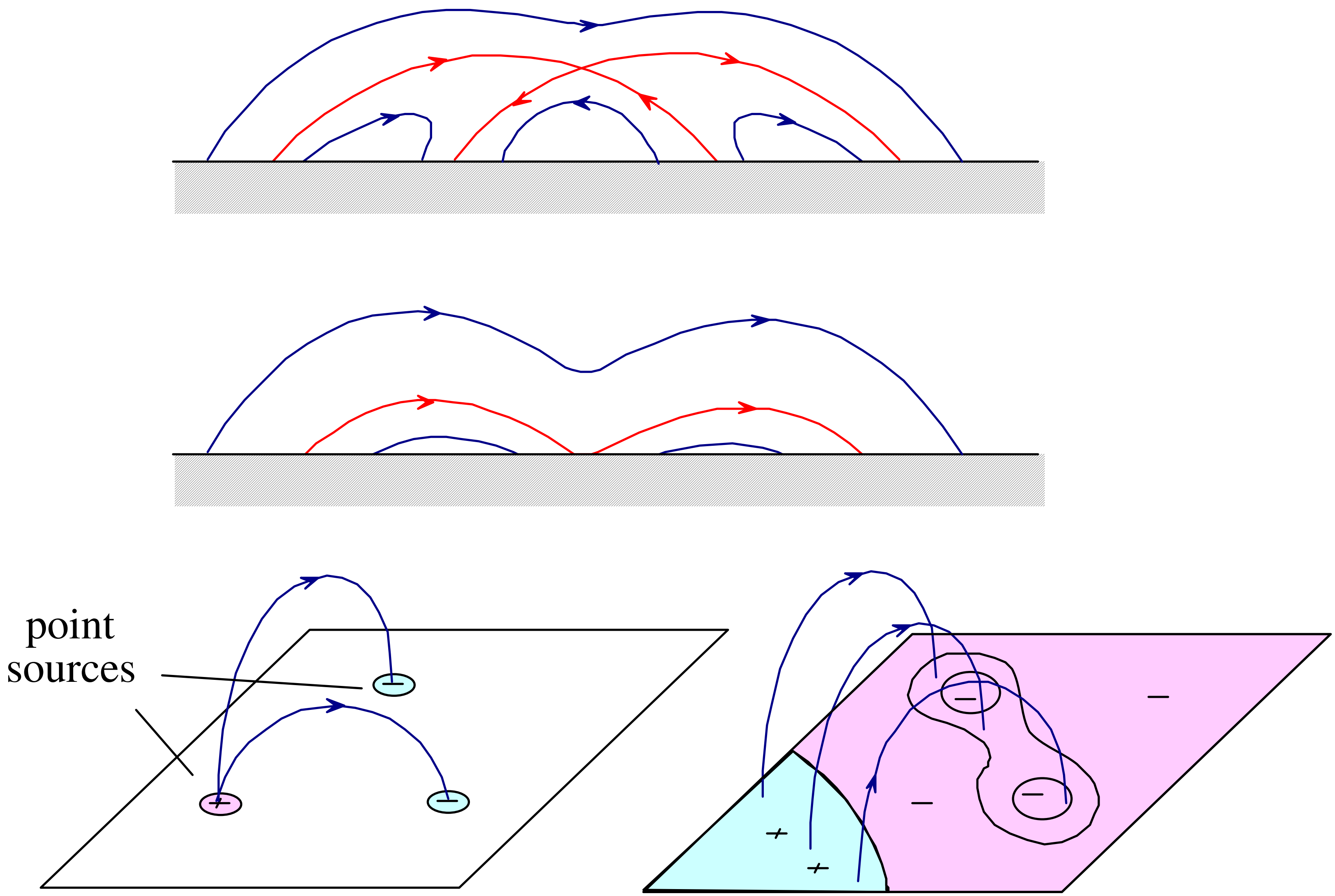


$$\int_{C_1} [\mathbf{E}_{\mathbf{rec}}]_0 \cdot \mathbf{dl} = \int_{C_2} B(V_n - U_n)dl$$

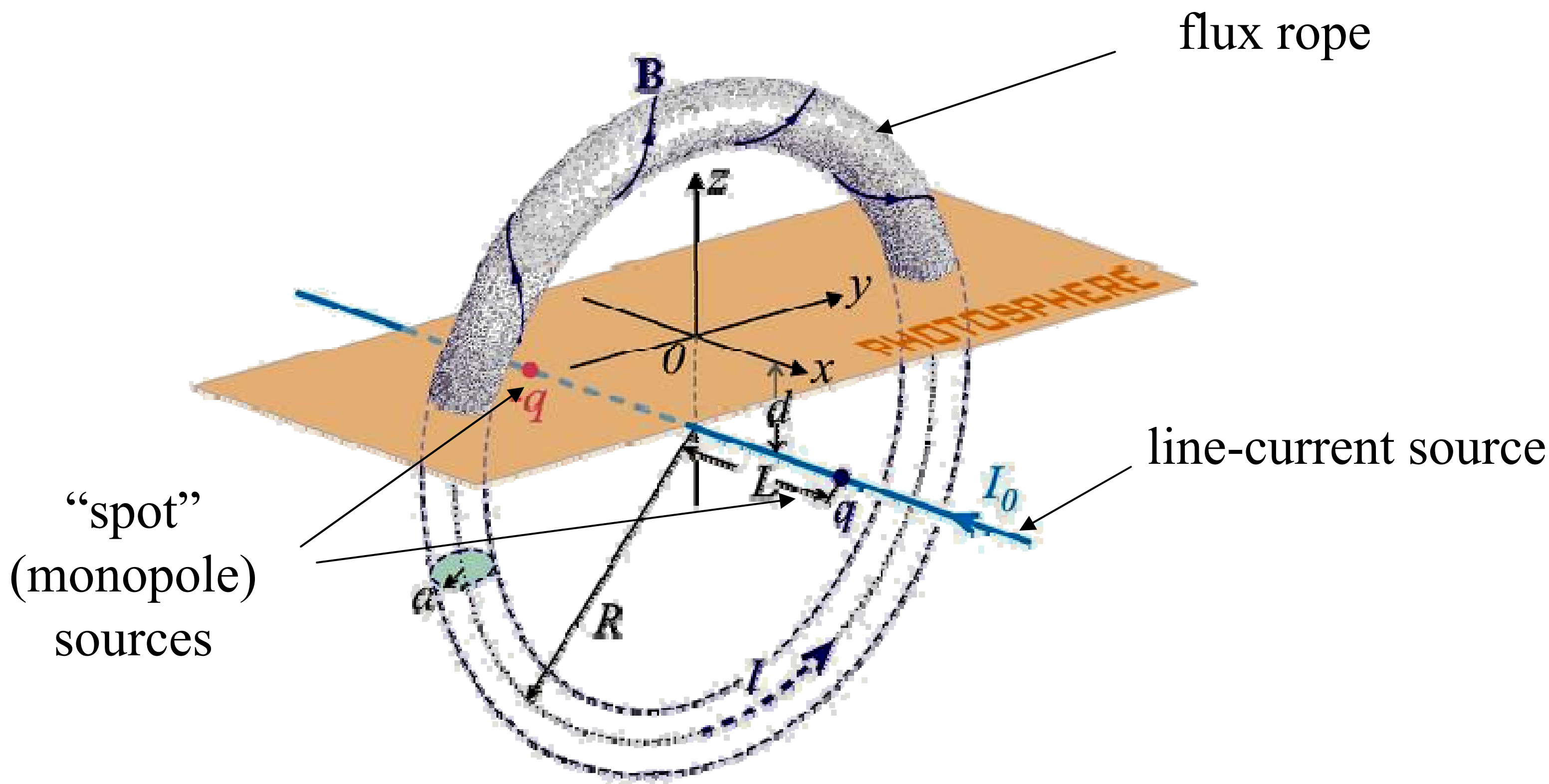


Blanchard et al. (1996)

Different Types of Separatrices



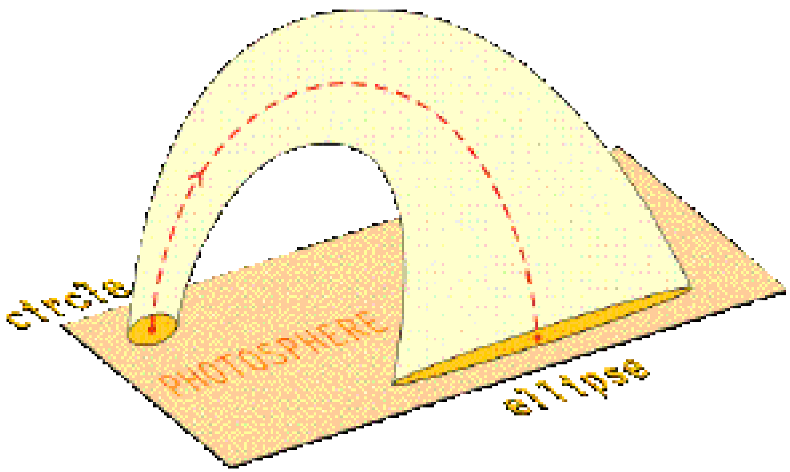
3D Flux Rope Model (Titov & Démoulin 1999)



Squashing Factor Q

- Geometrical definition;

Infinitezimal flux tube such that a cross-section at one foot is curcular, then circle Ź==> Źellipse:



Titov, Hornig & Démoulin (2000)

Q = aspect ratio of the ellipse;

Q is *invariant* to direction of mapping.

Definition of *Q* in coordinates:

$$Q = \frac{(a^2 + b^2 + c^2 + d^2)}{|ad - bc|}$$

a, *b*, *c* and *d* are the elements of the Jacobian matrix

$$D = \begin{pmatrix} \frac{\partial X}{\partial x} & \frac{\partial X}{\partial y} \\ \frac{\partial Y}{\partial x} & \frac{\partial Y}{\partial y} \end{pmatrix} \equiv \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

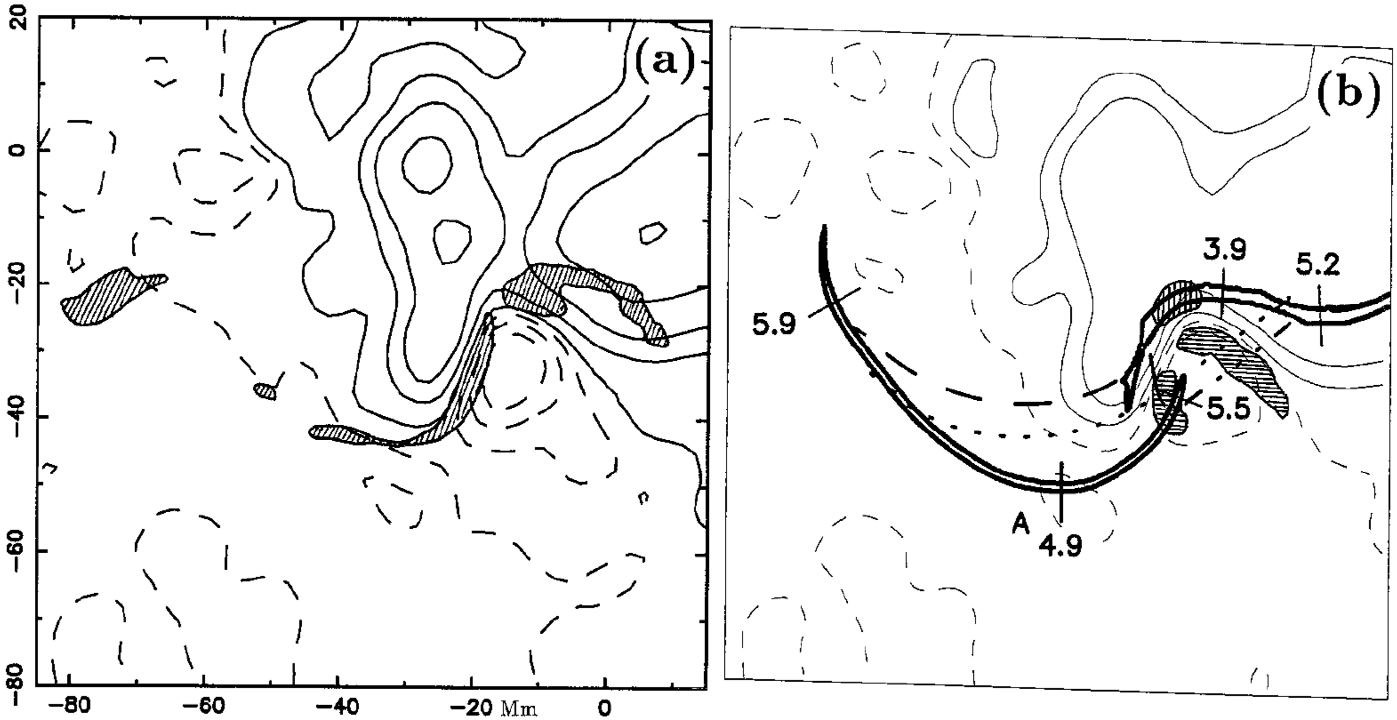
.

Three-Dimensional View of Quasi-Separatrices

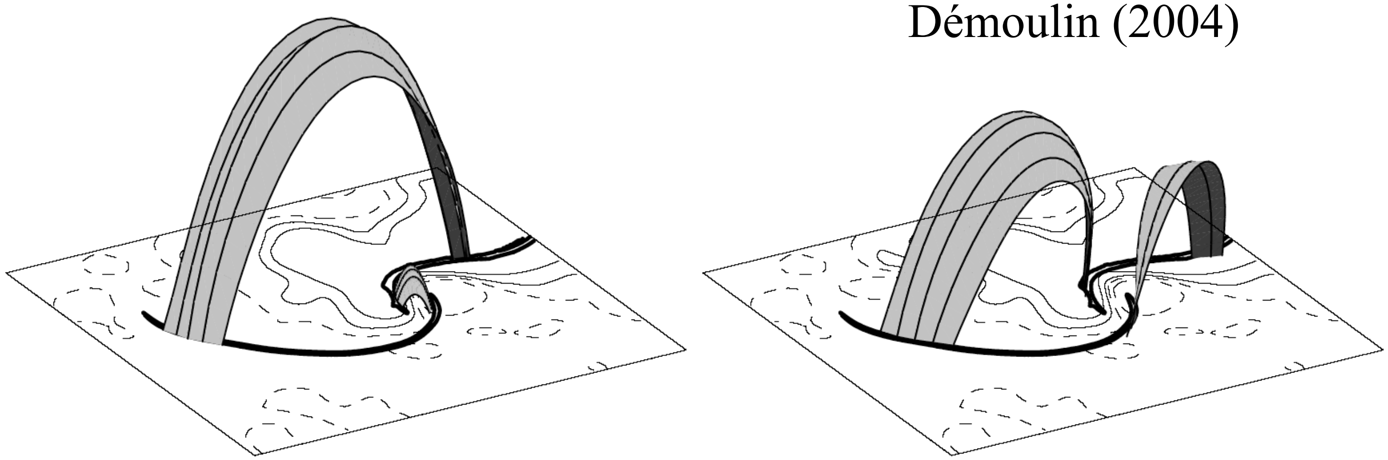
QuickTime™ and a
decompressor
are needed to see this picture.

Titov (2004)

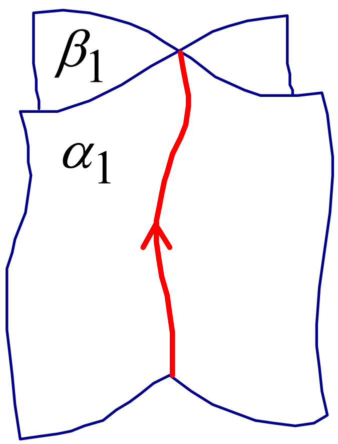
Application of Quasi-Separatrix Theory



Démoulin (2004)

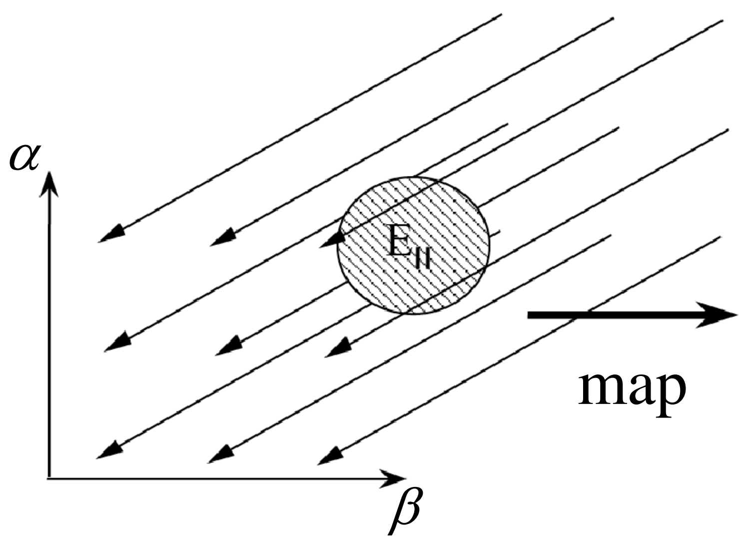


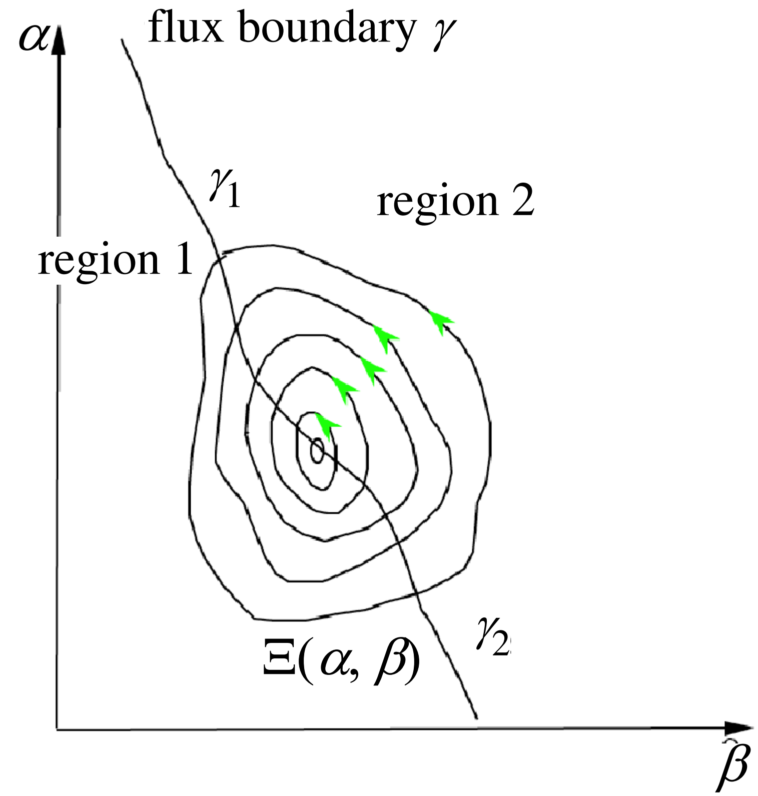
How to Define the Reconnection Rate in the Absence of Topological Distinctions



Euler potentials:

$$\mathbf{B} = \nabla\alpha \times \nabla\beta$$





Hesse et al. (2005)

Reconnection Pseudo-Potential Ξ

$$\mathbf{E} = -\frac{\partial \alpha}{\partial t}\nabla \beta + \frac{\partial \beta}{\partial t}\nabla \alpha - \nabla \psi$$

ψ is related to an electrostatic potential ϕ via

$$\psi = \phi + \alpha \frac{\partial \beta}{\partial t}$$

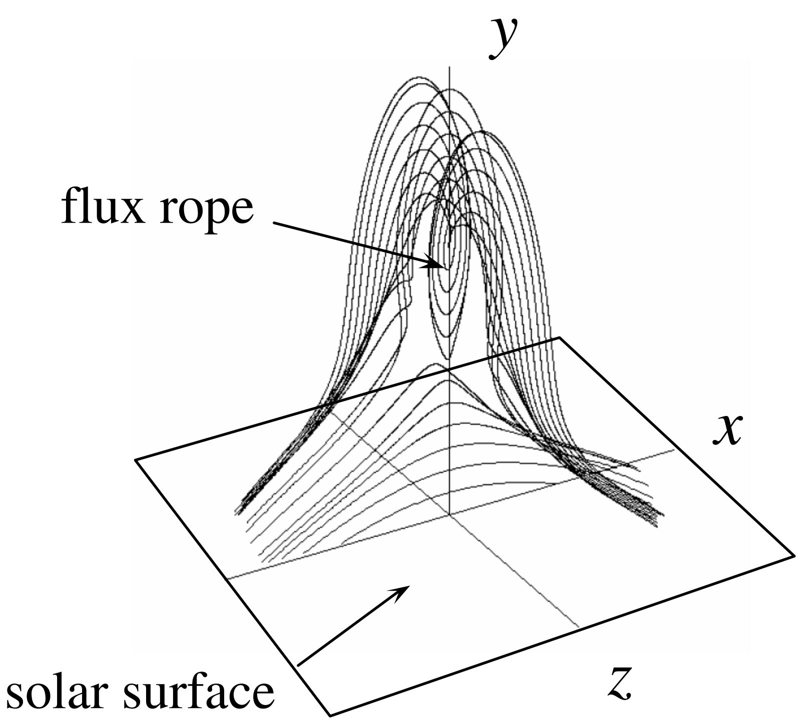
$$\Xi(\alpha, \beta) = -\int_{\alpha, \beta} E_{\|} ds$$

Kinematic Example of 3D Reconnection in an Erupting Field

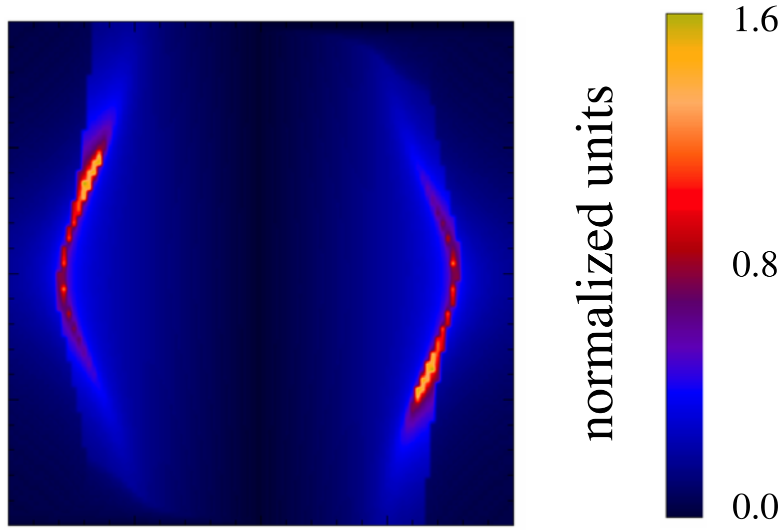
$$B_x = -1 - \varepsilon(t)\,\frac{(1 - y^2/L_y^2)}{(1 + y^2/L_y^2)}\,\frac{1}{(1 + z^2/L_z^2)}$$

$$B_y = x$$

$$B_z = 0.2$$



Net Potential Drop
Mapped onto Surface



Hesse, Forbes, & Birn (2005)

$\varepsilon = 10$

Origin of Non-Idealness

Generalized Ohm's Law: Difference between electron and ion momentum equations

$$\mathbf{E}' = \mathbf{E} + \mathbf{V} \times \mathbf{B}$$

collisions $+ \; \eta \, \mathbf{j}$

inertia $+ \; \frac{m_e}{ne^2} \left[\frac{\partial \mathbf{j}}{\partial t} + \nabla \cdot (\mathbf{vj} + \mathbf{vj}) \right]$

Hall $- \; \frac{\mathbf{j} \times \mathbf{B}}{ne}$

electron stress tensor $- \; \frac{\nabla \underline{\underline{\mathbf{p_e}}}}{ne}$

Origin of Non-Idealness

Comparison of non-ideal terms in Generalized Ohm's Law

Required Length to be Effective	Solar Corona		Terrestrial Magnetosphere	
inertia (λ_e)	10^{-1}	meters	10^{4}	meters
Hall (λ_i)	10^{1}		10^{6}	
e stress	10^{-3}		10^{5}	
collision	10^{-7}		10^{-7}	