Basic Plasma Concepts and Models

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Goal of this lecture

- Review a few basic plasma concepts and models that underlie the lectures later in the week.
- There are several excellent text books in plasma physics: Chen, Nicholson (out of print), Goldston and Rutherford, Boyd and Sanderson, Bellan.
- The book I am most familiar with is by Gurnett and Bhattacharjee, from which most of the material is taken.

What is a Plasma?

Plasma is an ensemble of charged particles, capable of exhibiting collective interactions.

Levels of Description:

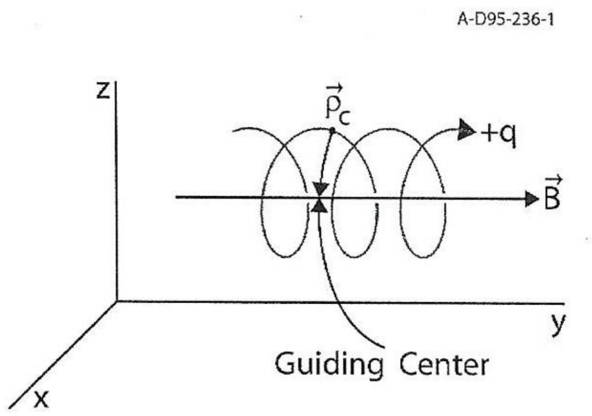
- Single-particle dynamics in prescribed electric and magnetic fields
- Plasmas as fluids in 3D configuration space moving under the influence of self-consistent electric and magnetic fields
- Plasmas as kinetic fluids in 6D μ-space (that is, configuration and velocity space), coupled to self-consistent Maxwell's equations.

Single-Particle Orbit Theory

Newton's law of motion for charged particles

$$m\frac{d\mathbf{v}}{dt} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$$

Guiding-Center: A very useful concept



Single-Particle Orbit Theory

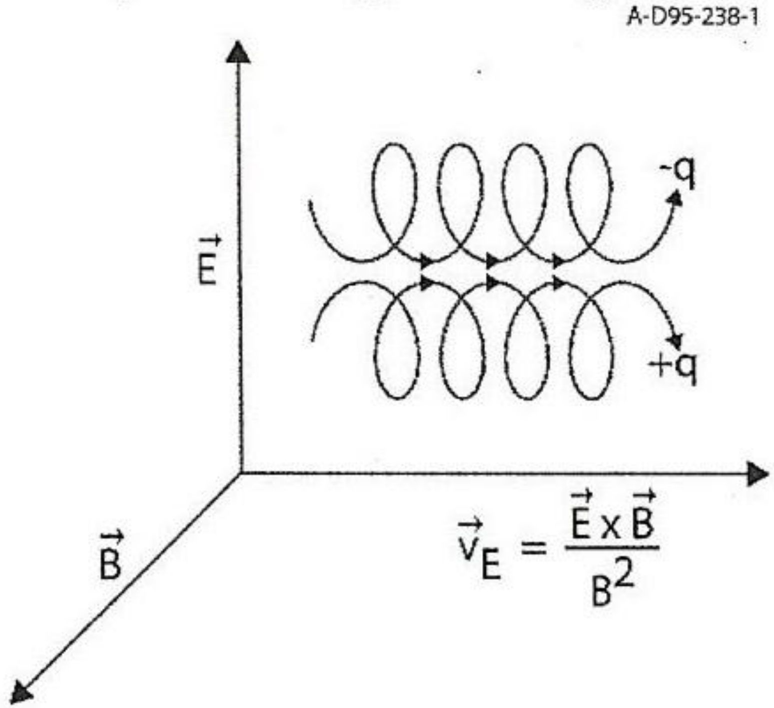
ExB Drift

$$m\frac{d\mathbf{v}}{dt} = q\left(\mathbf{E} + \mathbf{v}\times\mathbf{B}\right)$$

Consider $\mathbf{E} = const., \ \mathbf{B} = const.$

The charged particles experience a drift velocity, perpendicular to both E and B, and independent of their charge and mass.

$$\mathbf{v}_{\mathbf{E}} = \frac{\mathbf{E}\times\mathbf{B}}{B^2}$$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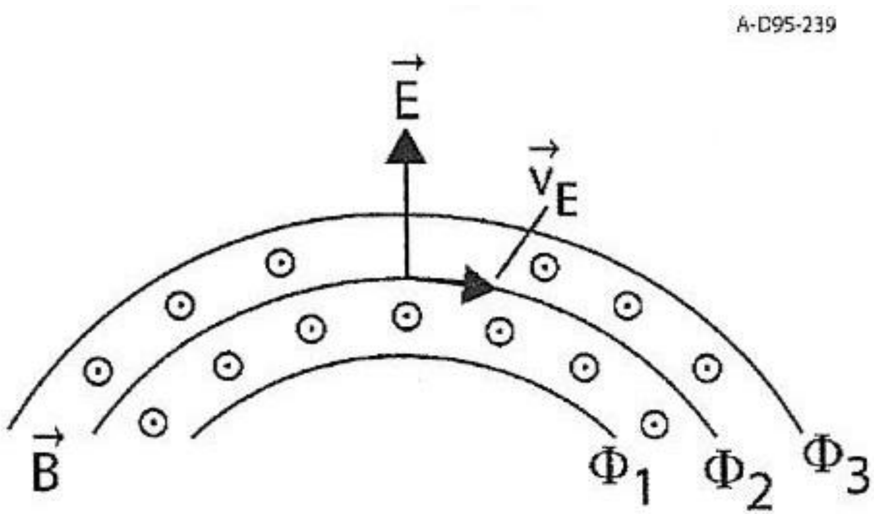
-q
+q
E
B
v_E = E x B / B^2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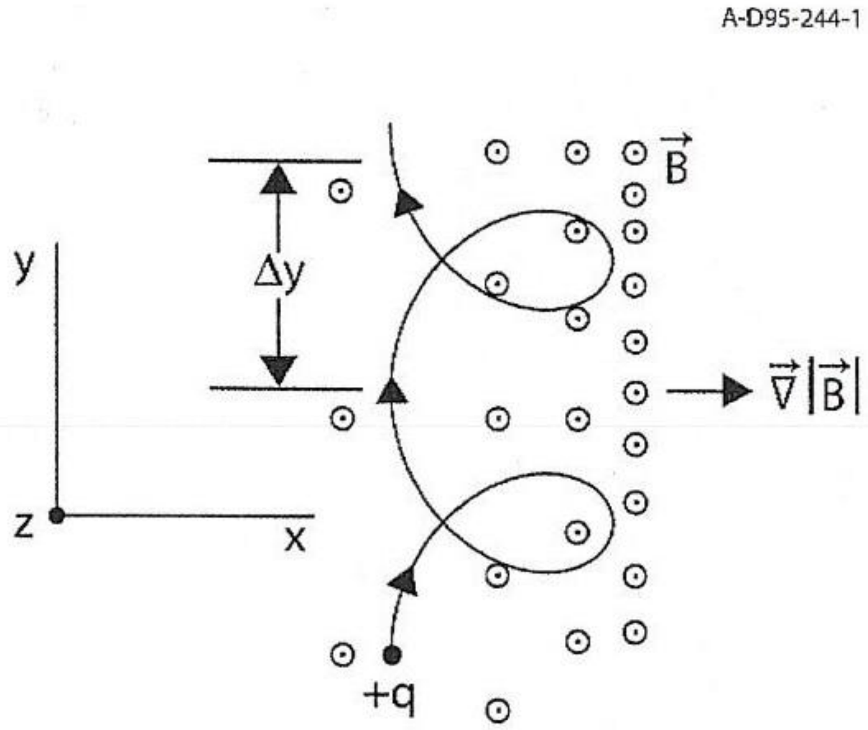
E
v_E
B
Φ1
Φ2
Φ3

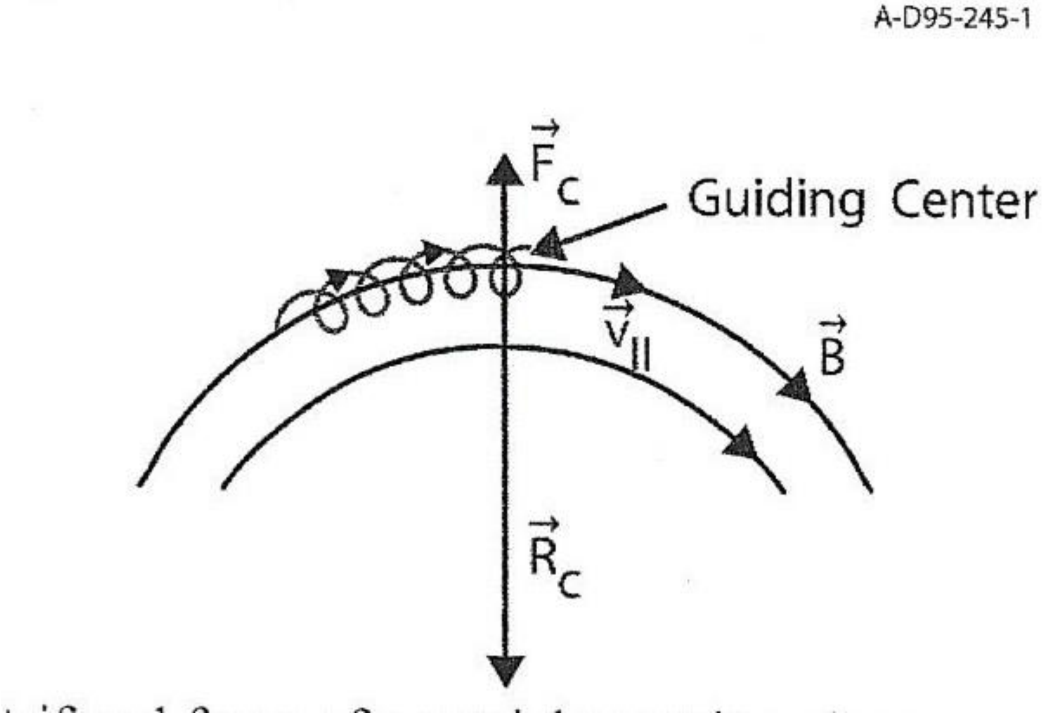
Gradient B drift



$$\mathbf{V_G} = \frac{w_\perp}{qB}\left(\frac{\mathbf{B}\times\nabla\mathbf{B}}{B^2}\right),$$

$$w_\perp = \frac{1}{2}{\omega_c}^2{\rho_c}^2$$

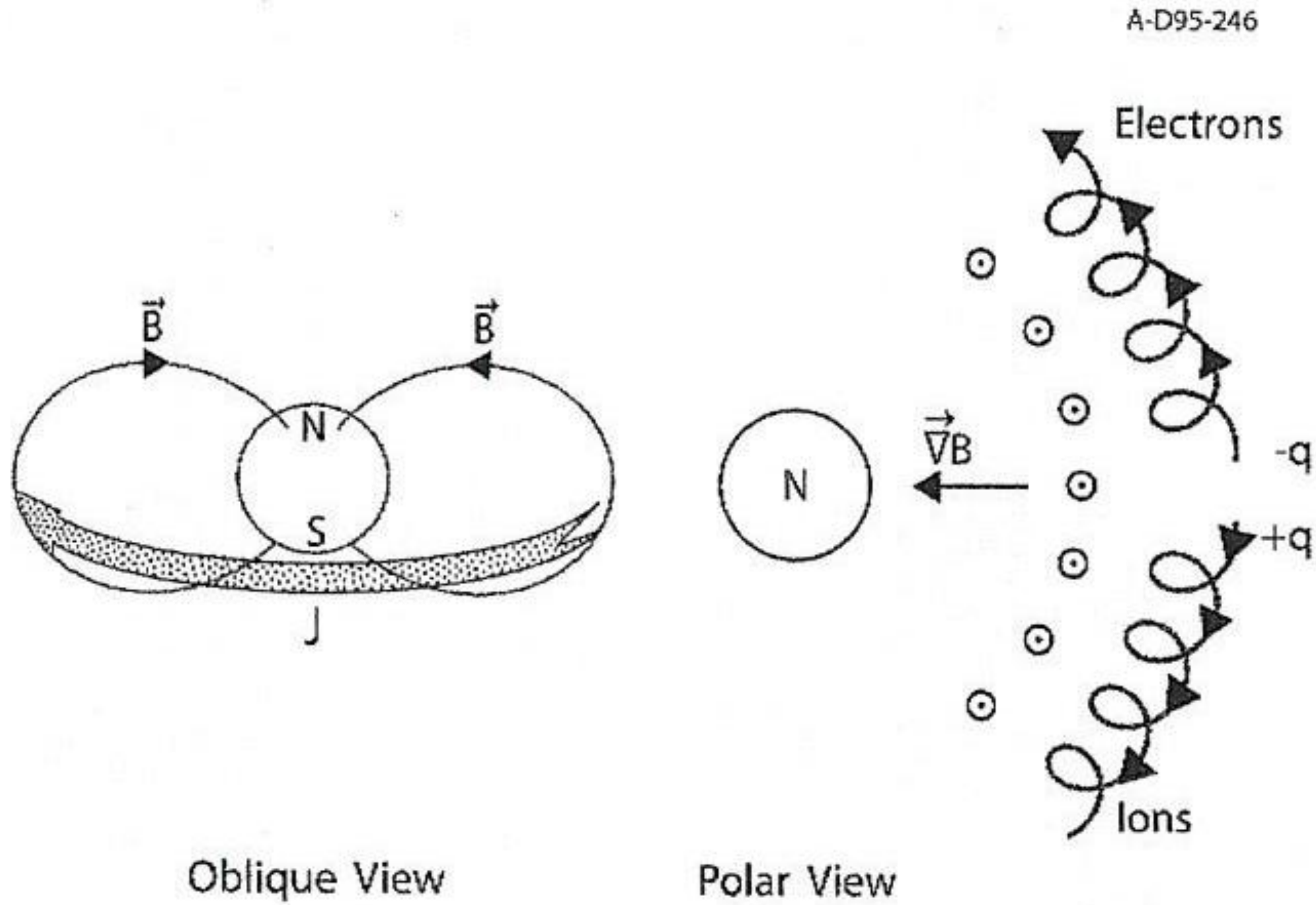
Curvature drift



$$\mathbf{V}_C = \frac{2w_{\|}}{qB^2}\left(\frac{\mathbf{R_C} \times \mathbf{B}}{{R_C}^2}\right),$$

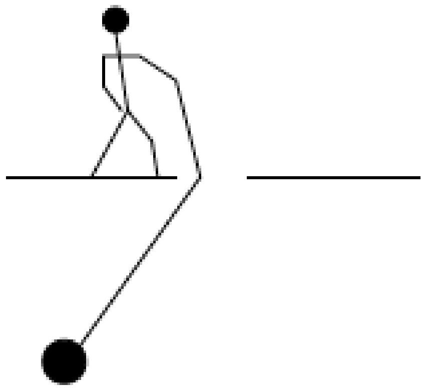
$$w_{\|} = \frac{1}{2}m{v_{\|}}^2$$

The Ring Current in Earth's Magnetosphere: An Example



Adiabatic Invariants

Albert Einstein and the Adiabatic Pendulum (1911)



Einstein suggested that while both the energy E and the frequency ν change, the ratio E/ν remains approximately invariant.

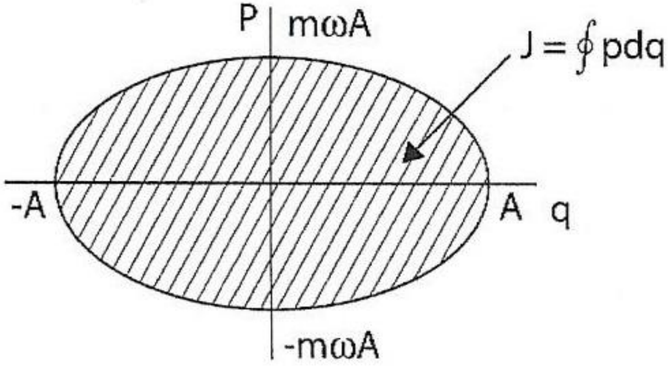
Adiabatic Invariants

Harmonic oscillator

$$\frac{d^2x}{dt^2} + \omega^2(\varepsilon t)x = 0, \ \varepsilon << 1$$

The adiabatic invariant is

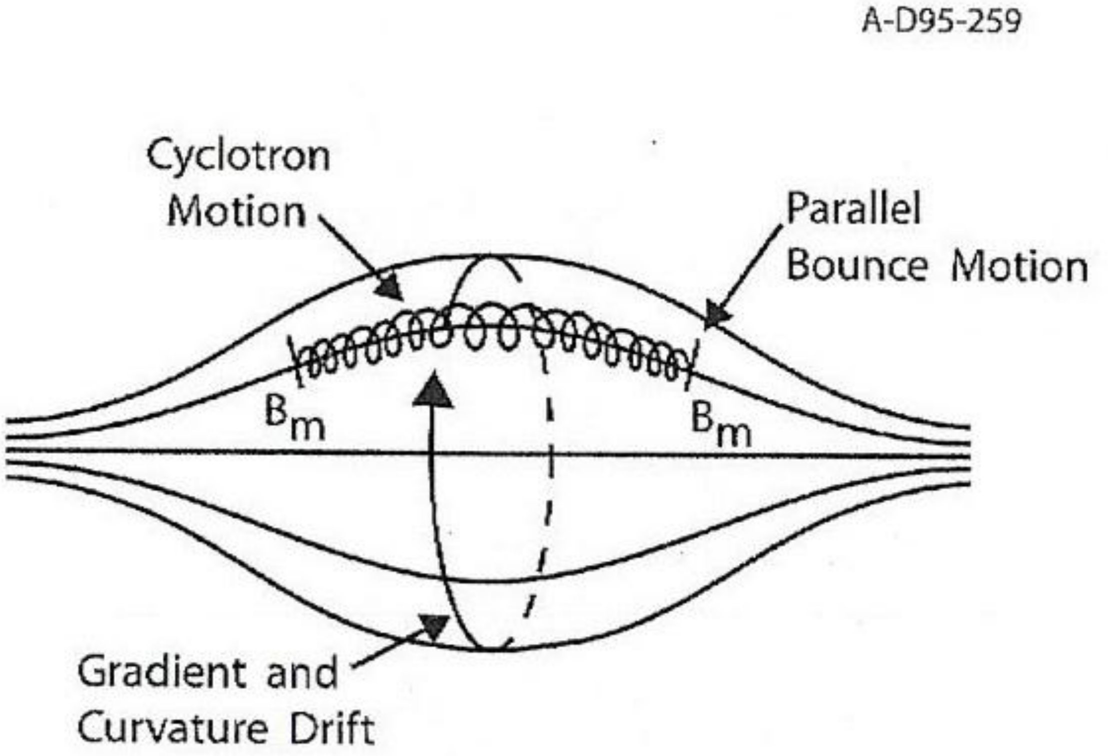
$$J = \oint p dq$$



$$\Delta J / J \sim \exp(-c/\varepsilon)$$

Adiabatic Invariants

Three types of bounce motion



Adiabatic Invariants

Three types of bounce motion

First adiabatic invariant $$\mu = w_{\perp} / B$$

Second adiabatic invariant $$J = m\oint v_{\|} ds$$

Third adiabatic invariant $$\Phi = \pi R^2 B$$

Lectures for which this material is directly pertinent

- *Vasyliunas* : Planetary Magnetospheres
- *Lee* : Particle acceleration in shocks
- *Liemohn*: Energization of trapped particles

Kinetic Description of Plasmas

Distribution function $f(\mathbf{r},\mathbf{v},t)$

Normalization $N = \iint\limits_{phase\ space} d\mathbf{x}\,d\mathbf{v}\,f(\mathbf{r},\mathbf{v},t)$

Example: Maxwell distribution function

$$f = n_0 \exp\left(-\frac{mv^2}{2kT}\right),\ n_0 = N/V$$

Boltzmnn-Vlasov Equation

Motion of an incompressible phase fluid in μ–space

$$\frac{\partial f_s}{\partial t} + \mathbf{v}.\frac{\partial f_s}{\partial t} + \frac{q}{m}\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right).\frac{\partial f_s}{\partial \mathbf{v}} = 0,\ s = e, i$$

In the presence of collisions

$$\frac{\partial f_s}{\partial t} + \mathbf{v}.\frac{\partial f_s}{\partial t} + \frac{q}{m}\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right).\frac{\partial f_s}{\partial \mathbf{v}} = \left(\frac{\delta f_s}{\delta t}\right)_c$$

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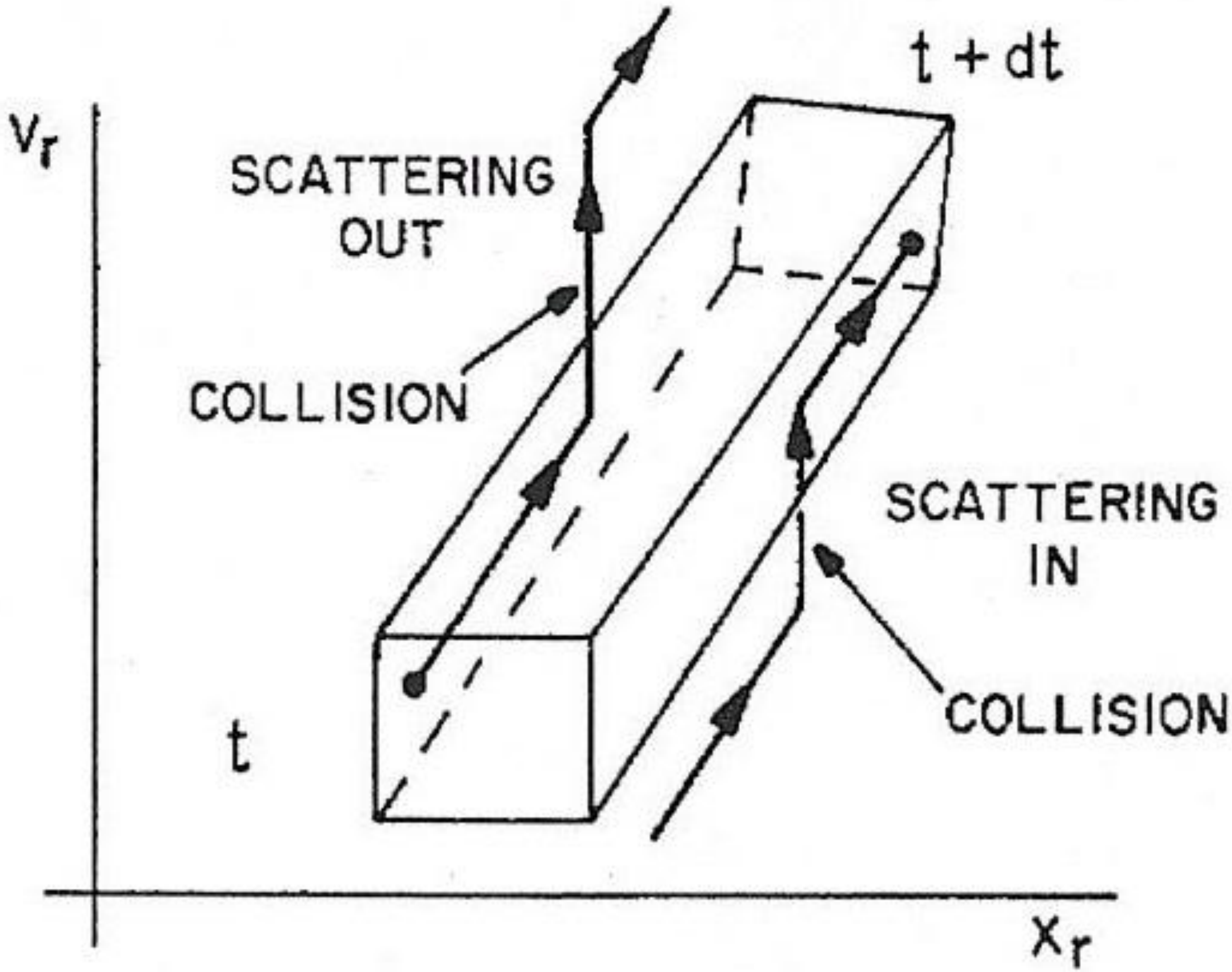

t + dt
V_r
SCATTERING
OUT
COLLISION
SCATTERING
IN
COLLISION
t
X_r

Vlasov-Poisson equations: requirements of self-consistency in an electrostatic plasma

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{r}} - \frac{q_s}{m_s} \nabla \Phi \cdot \frac{\partial f_s}{\partial \mathbf{v}} = 0$$

$$\mathbf{E} = -\nabla \Phi$$

$$\nabla \cdot \mathbf{E} = -\nabla^2 \Phi = 4\pi\rho = 4\pi \sum_s q_s \int d\mathbf{v} f_s$$

Quasilinear theory: application to scattering due to wave-particle interactions

- Consider electrostatic Vlasov equation

$$\frac{\partial f_s}{\partial t} + \mathbf{v}.\frac{\partial f_s}{\partial t} - \frac{q}{m}\nabla\Phi.\frac{\partial f_s}{\partial \mathbf{v}} = 0.$$

Split every dependent variable into a mean and a fluctuation

$$f_s = \langle f_s \rangle + f_{s1},\ \langle f_{s1} \rangle = 0$$

Quasilinear Diffusion

It follows after some algebra that the mean or average distribution function obeys a diffusion equation:

$$\frac{\partial}{\partial t}\langle f_s \rangle = \frac{\partial}{\partial \mathbf{v}} \cdot \left(\mathbf{D} \cdot \frac{\partial}{\partial \mathbf{v}} \langle f_s \rangle \right)$$

Here D is a diffusion tensor, dependent on wave fluctuations (pertinent to Lee, Liemohn, and Opher lectures).

Fluid Models

The primary fluid model of focus in this summer school is Magnetohydrodynamics (MHD)

It treats the plasma as a single fluid, without distinguishing between electrons or protons, moving under the influence of self-consistent electric and magnetic fields.

It can be derived from kinetic theory by taking moments (integrating over velocity space), and making some drastic approximations.

From *Magnetic Reconnection*
by E. Priest and T. Forbes

Frozen Flux/Field Theorem (Alfven's Theorem)

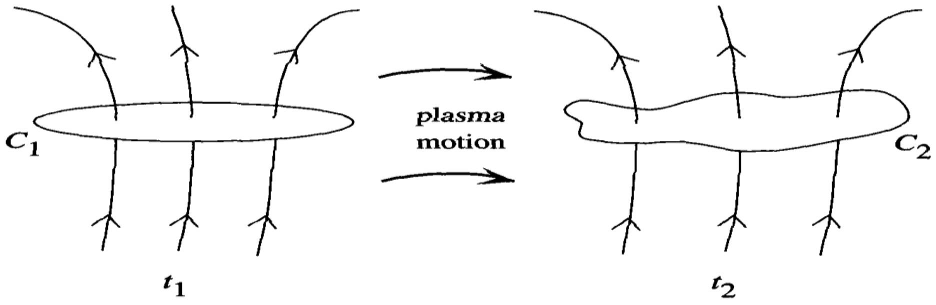

Fig. 1.6. Magnetic flux conservation: if a curve C_1 is distorted into C_2 by plasma motion, the flux through C_1 at t_1 equals the flux through C_2 at t_2.

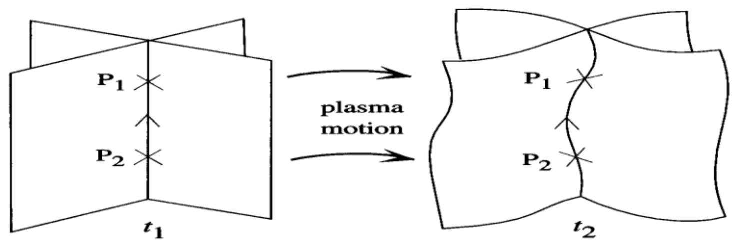

Fig. 1.7. Magnetic field-line conservation: if plasma elements P_1 and P_2 lie on a field line at time t_1, then they will lie on the same line at a later time t_2.

Magnetic Reconnection: Working Definition

If a plasma is perfectly conducting, that is, it obeys the ideal Ohm's law,

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$

B-lines are frozen in the plasma. Departures from ideal behavior, represented by

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \mathbf{R}, \quad \nabla \times \mathbf{R} \neq \mathbf{0}$$

break ideal topological invariants, allowing field lines to break and reconnect.In generalized Ohm's law for collisionless plasmas, **R** contains resistivity, Hall current, electron inertia, and pressure. (More in lecture by *Longcope, Forbes, Vasyliunas,* and *Kozyra*.)