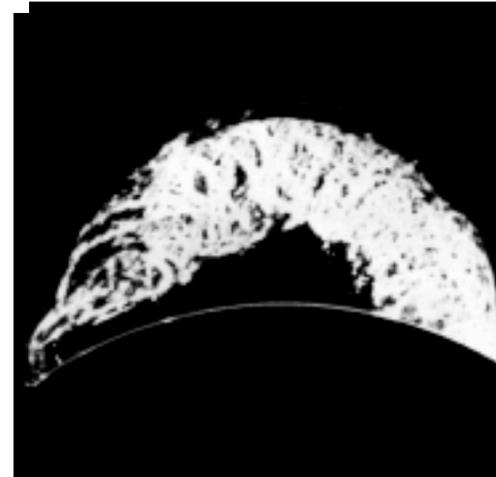
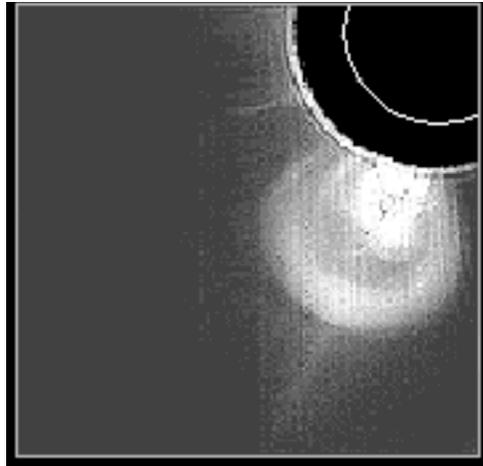
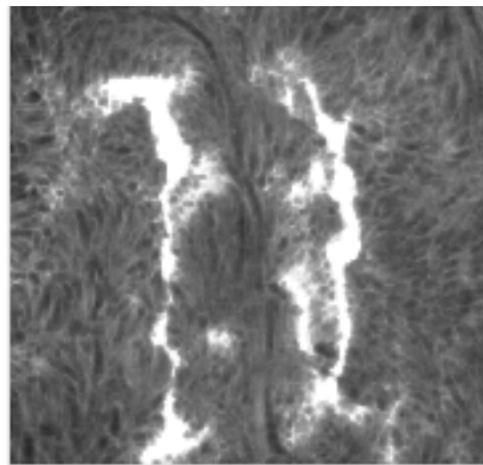


Theory & Modeling of Solar Eruptions

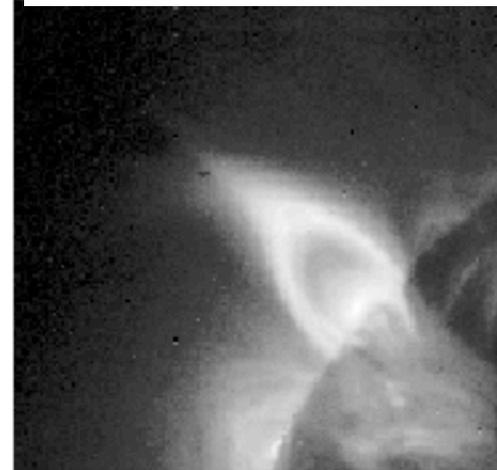
coronagraph (CME) $H\alpha$ limb (prominence)



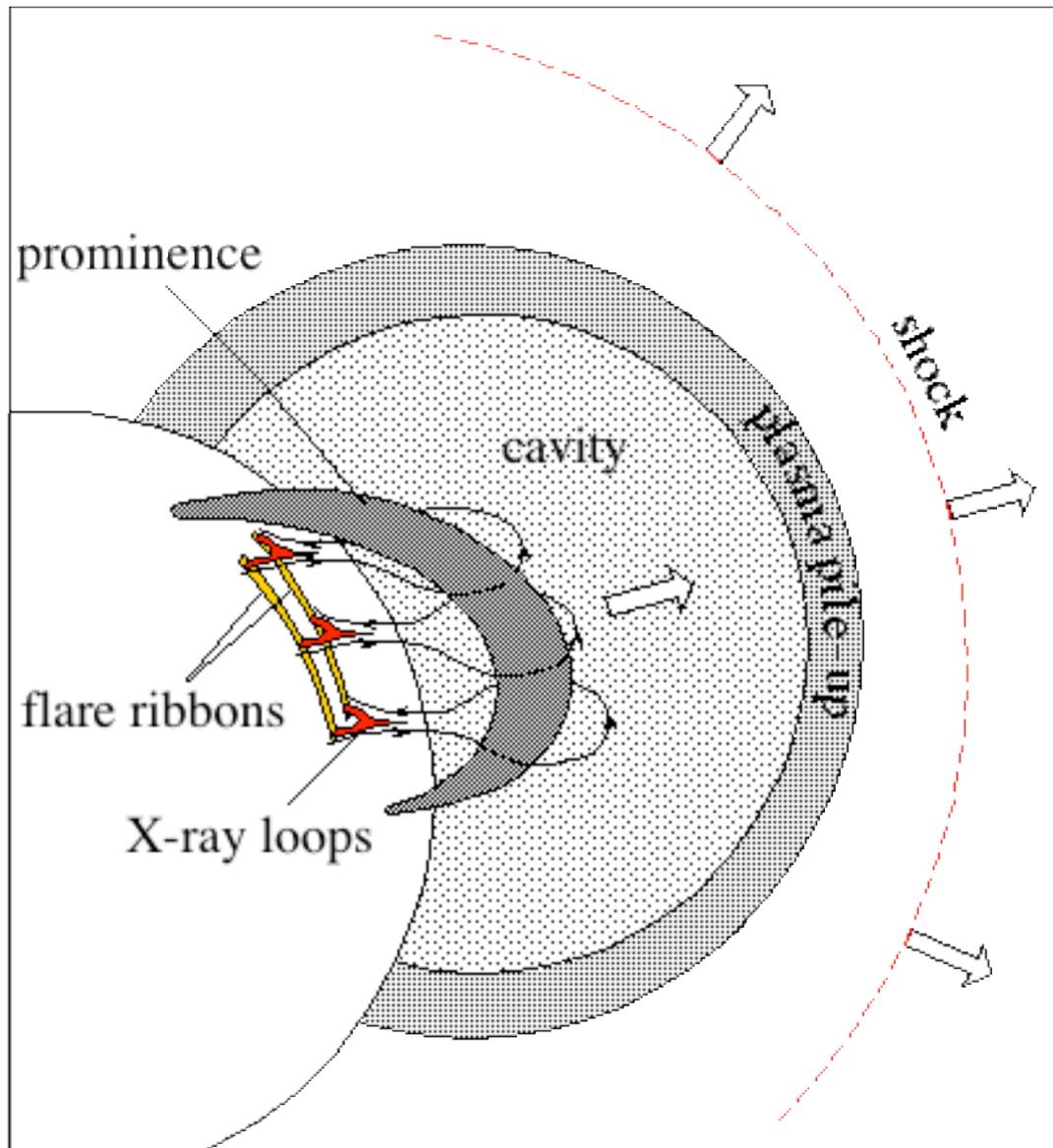
$H\alpha$ disk (flare ribbons)



X-ray (flare loops)

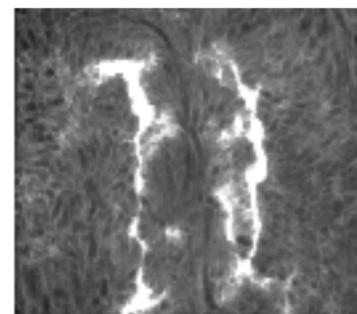
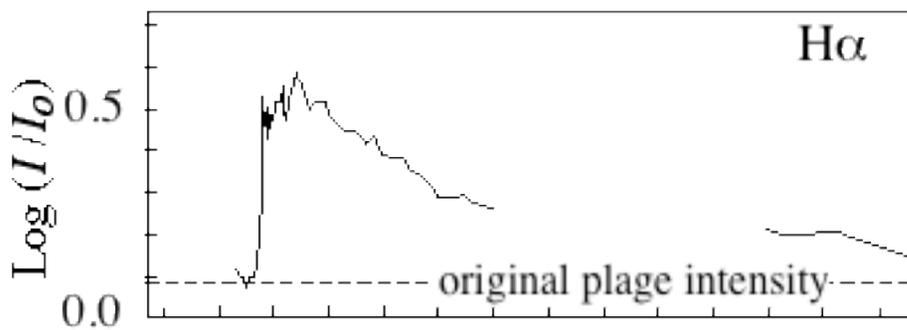


Large Solar Eruptions



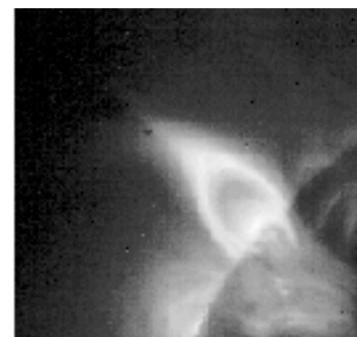
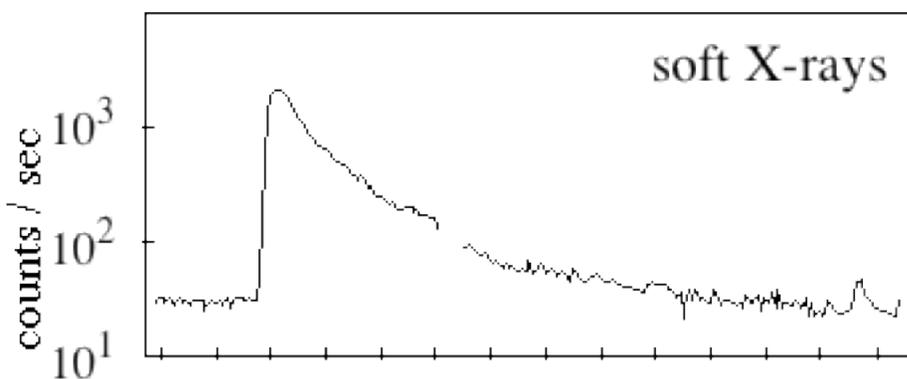
Light Curves

chromosphere



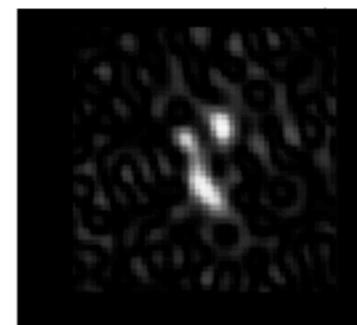
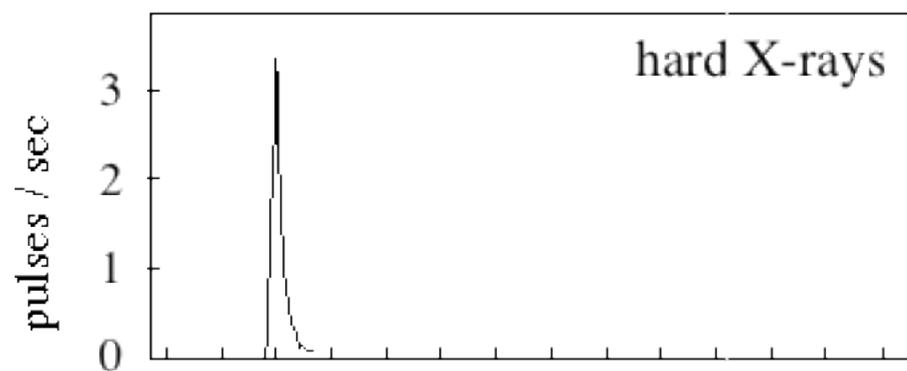
BBSO

corona
(thermal)



Yohkoh

nonthermal



RHESSI

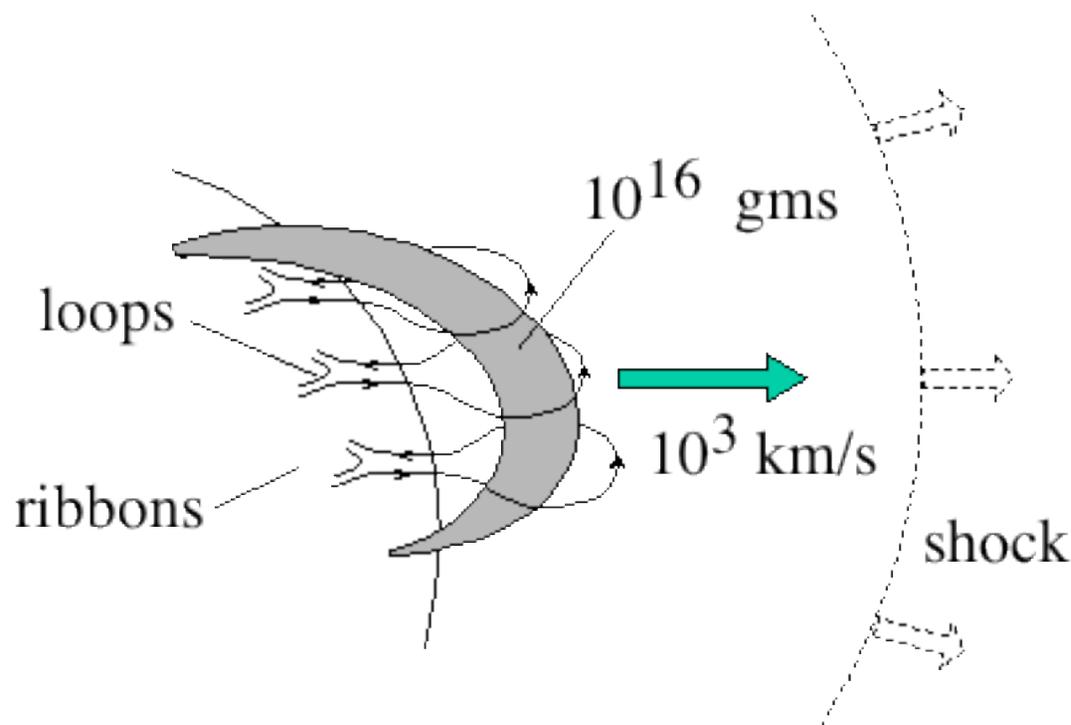
15 16 17 18 19 20 21 UT
(hours)

CME/Flare Energetics

kinetic energy of mass motions: $\approx 10^{32}$ ergs

heating / radiation: $\approx 10^{32}$ ergs

work done against gravity $\approx 10^{31}$ ergs



volume involved:
 $\gtrsim (10^5 \text{ km})^3$

energy density:
 $\lesssim 100 \text{ ergs/cm}^3$

Nature of Energy Source: Required: $\approx 100 \text{ ergs/cm}^3$

Type	Observed Values	Energy Density
kinetic $(m_p n V^2)/2$	$n = 10^9 \text{ cm}^{-3}$ $V = 1 \text{ km/s}$	$10^{-5} \text{ ergs/cm}^3$
thermal nkT	$T = 10^6 \text{ K}$	0.1 ergs/cm^3
gravitational $m_p n g h$	$h = 10^5 \text{ km}$	0.5 ergs/cm^3
magnetic $B^2/8\pi$	$B = 100 \text{ G}$	400 ergs/cm^3

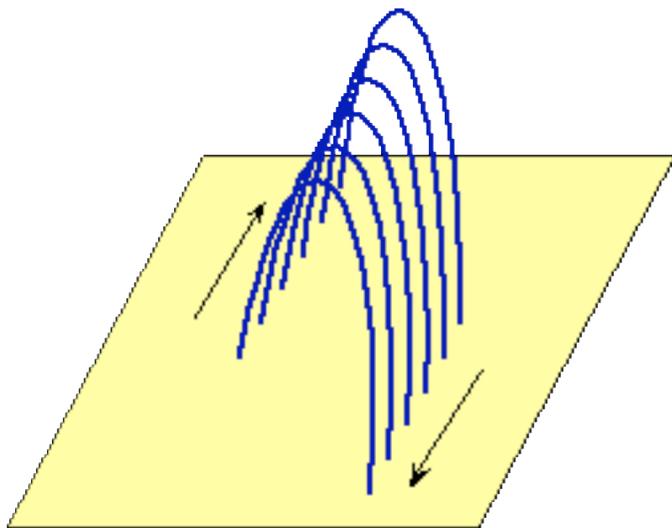
How is Energy Stored?

$$\beta = 10^{-3}$$

$$\nabla p \approx 0$$

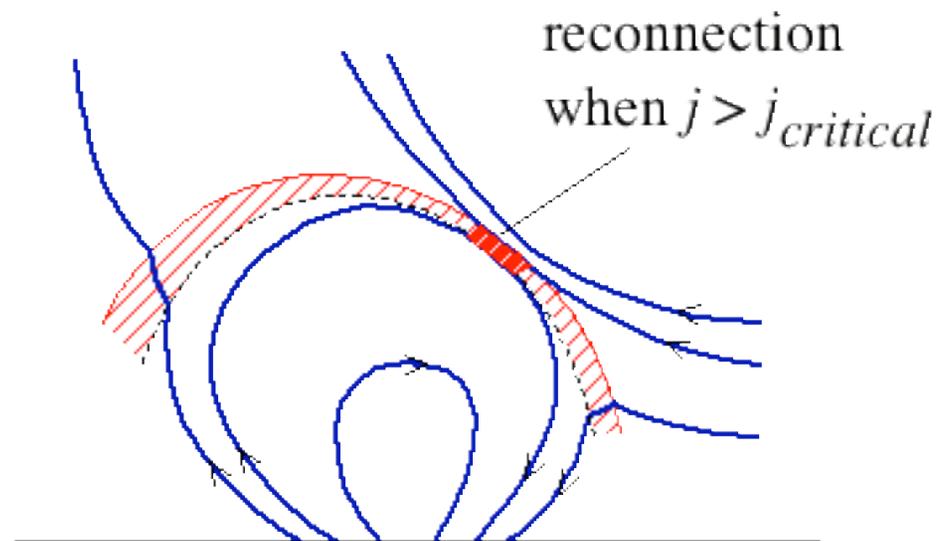
$$\mathbf{j} \times \mathbf{B} \approx 0$$

Force-free fields: $\mathbf{j} \parallel \mathbf{B}$



sheared magnetic fields

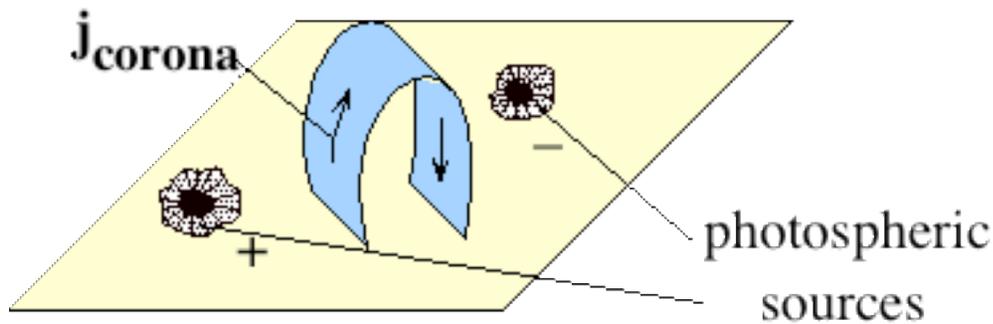
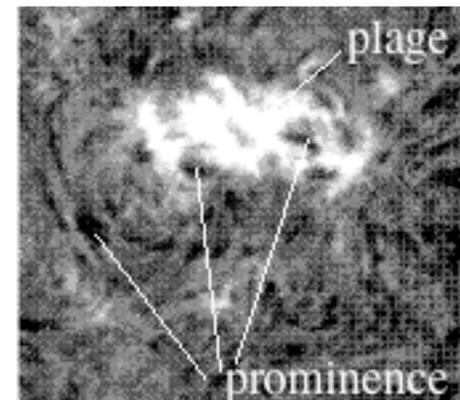
Current sheets:



emerging flux model

How Much Energy is Stored?

H α image



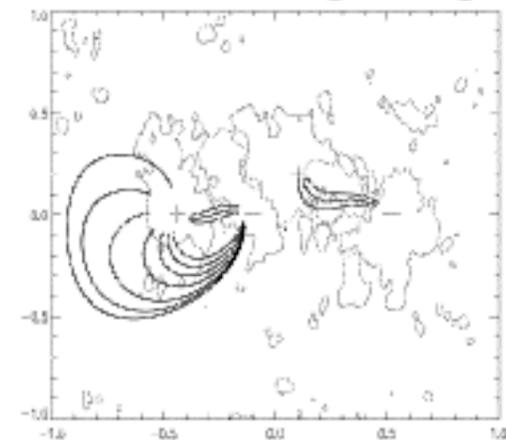
$$\mathbf{B} = \mathbf{B}_{\text{photospheric currents}} + \mathbf{B}_{\text{coronal currents}}$$

invariant
during CME

source of
CME energy

$$B_{\text{from corona}} \approx B_{\text{from photosphere}}$$

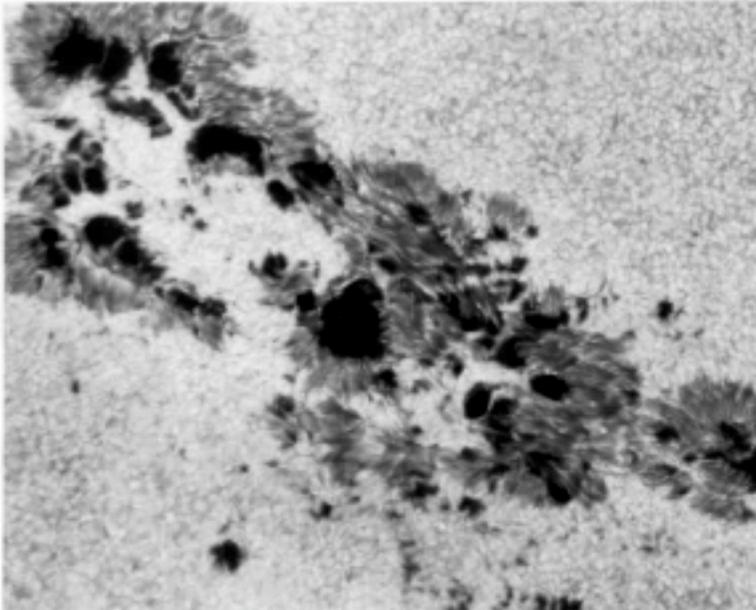
model with magnetogram



from Gaizauskas & Mackay (1997)

free magnetic energy \approx 50% of total magnetic energy

Inertial Line-Tying



Plasma below the photosphere is both massive and a good conductor.

Evolution of the photosphere is slow compared to time scale of eruptions.

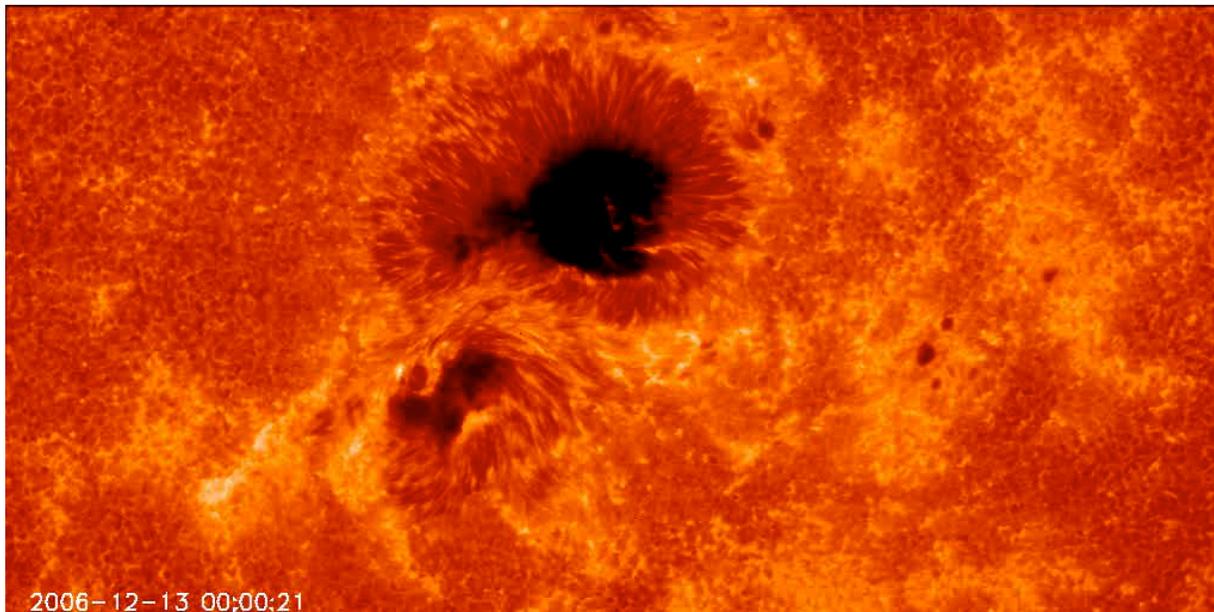
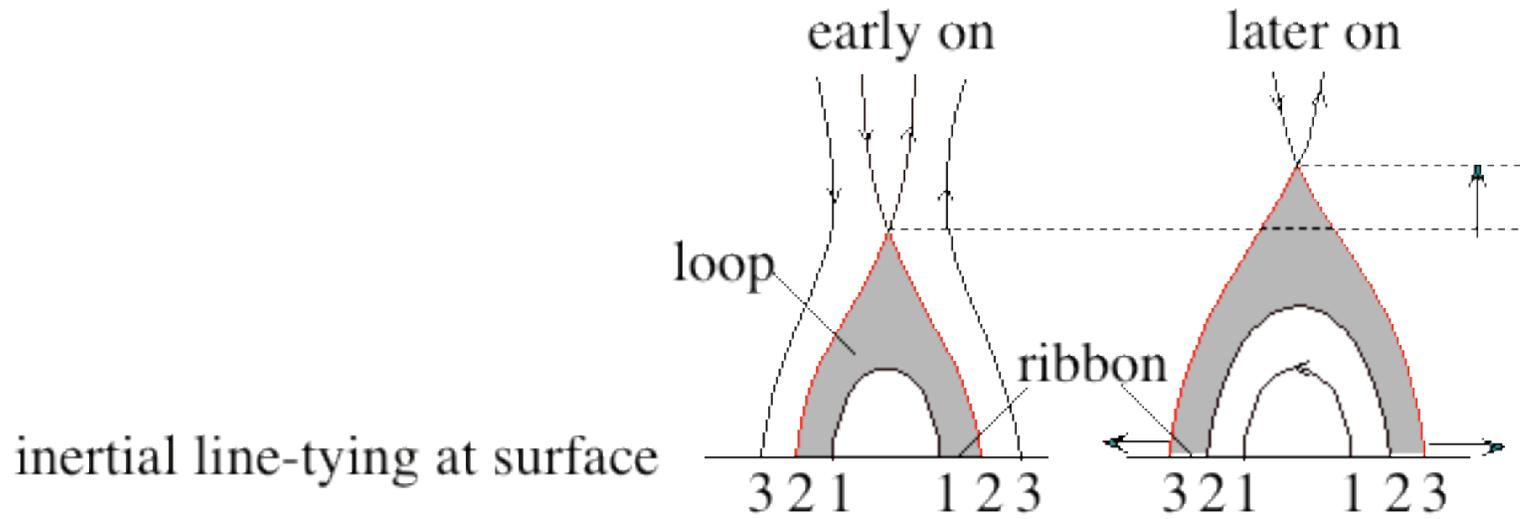
Photospheric boundary condition:

$$\mathbf{E} = -\mathbf{V} \times \mathbf{B} = \mathbf{0} .$$

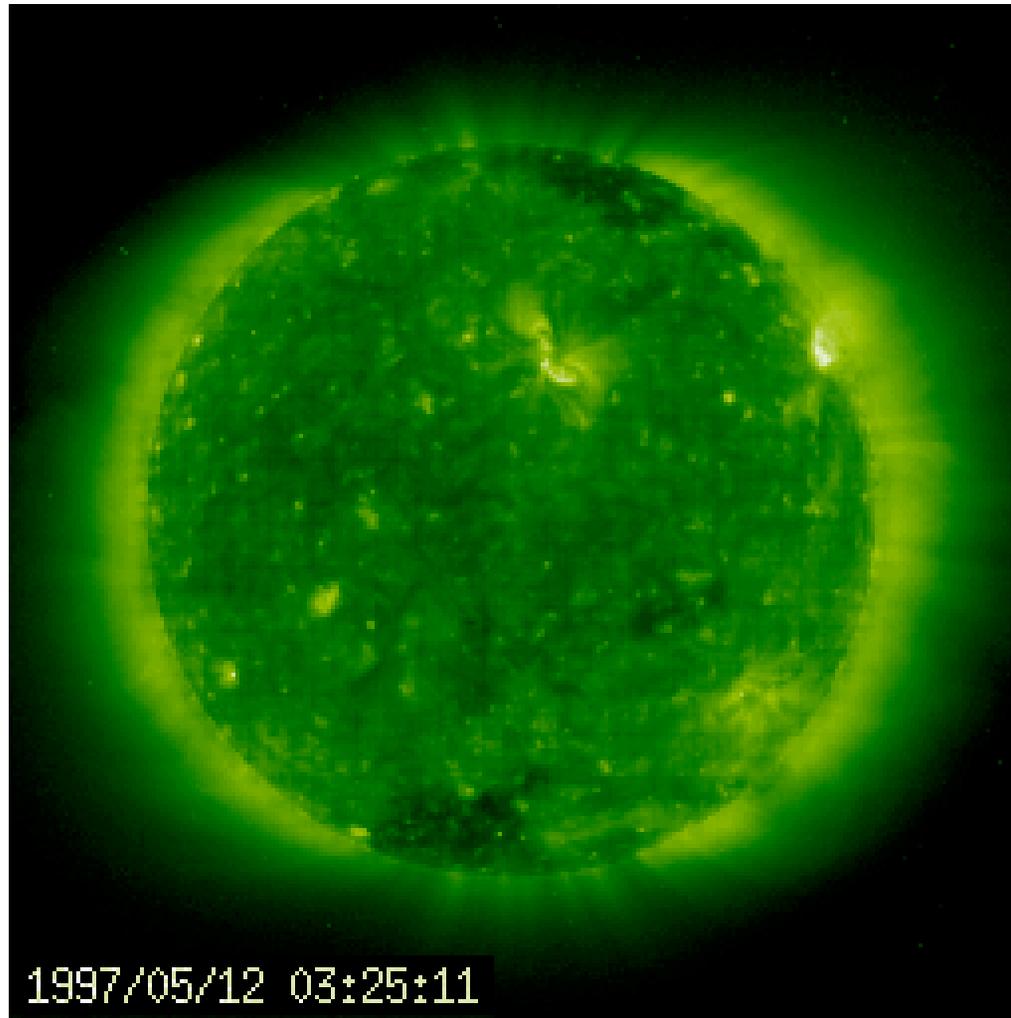
Photospheric convection is negligible

B normal to surface is fixed.

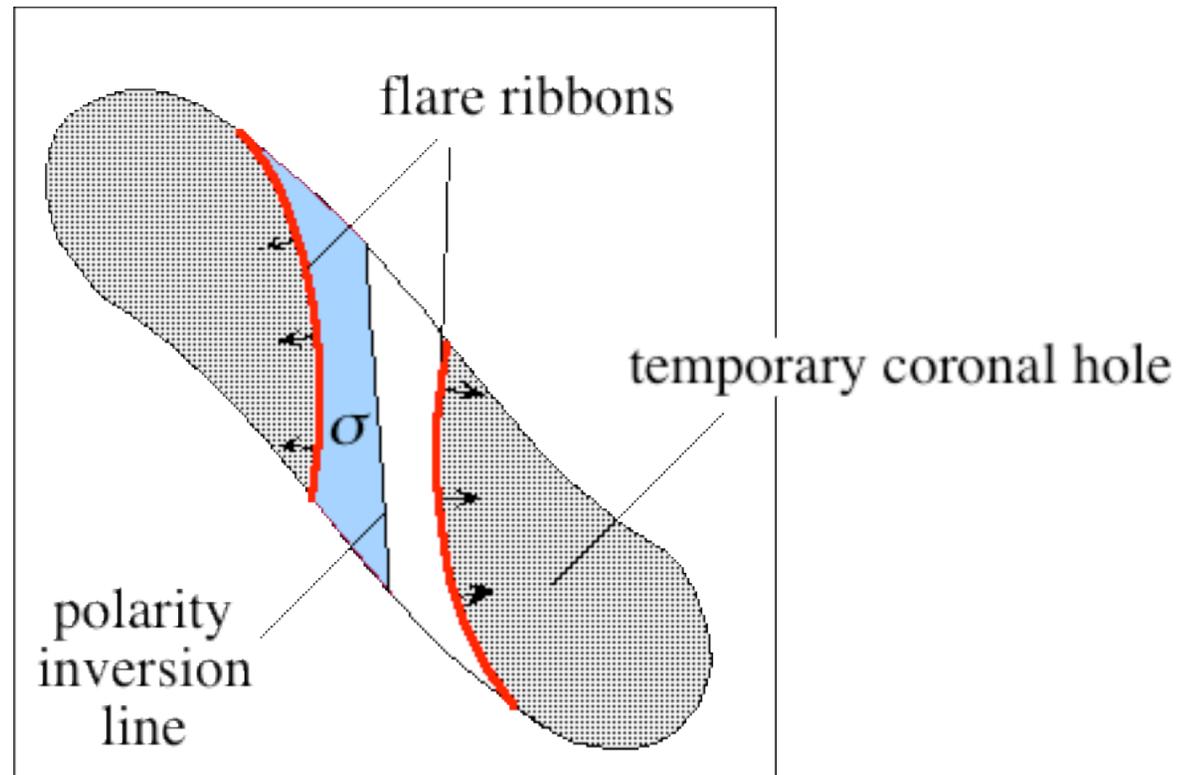
Apparent Motion of Loops & Ribbons



Transient Coronal Holes as Seen by EIT



Reconnection Electric Fields



newly reclosed flux:

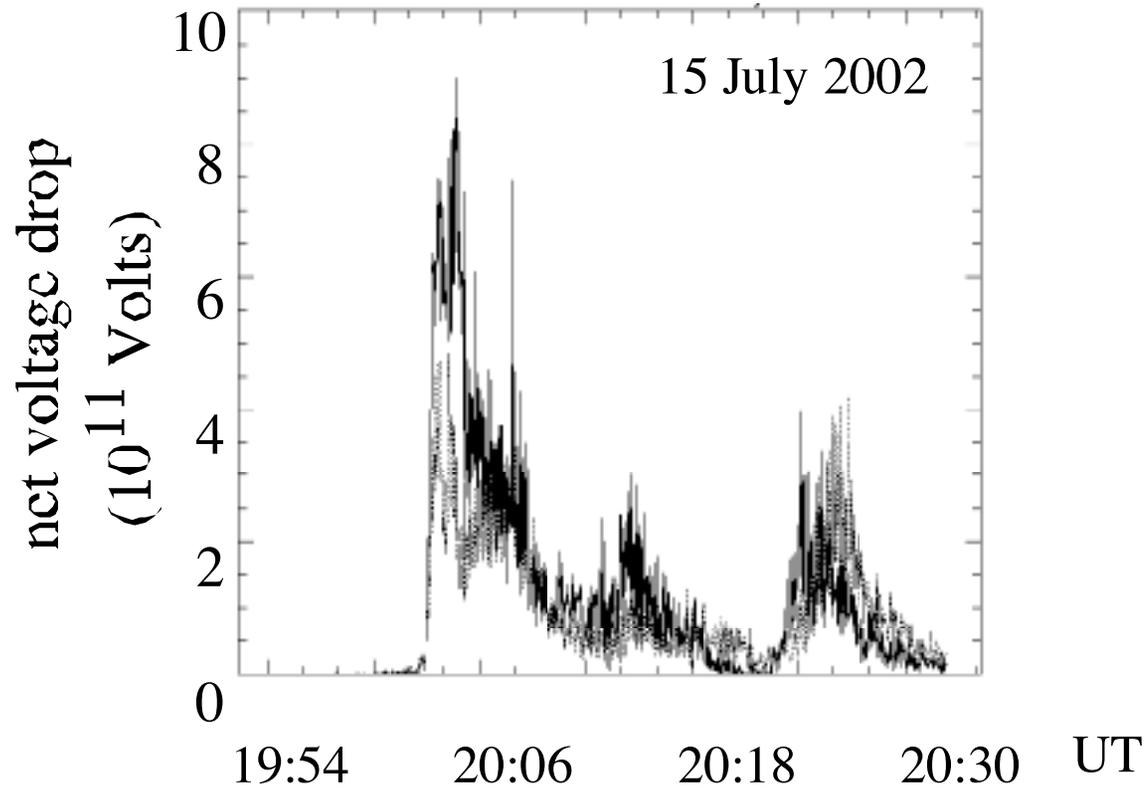
$$\Phi_B = \iint_{\sigma} B_z dx dy$$

global reconnection rate:

$$\int \mathbf{E} \cdot d\mathbf{l} = \frac{d\Phi_b}{dt}$$

CME/Flare Reconnection Rate

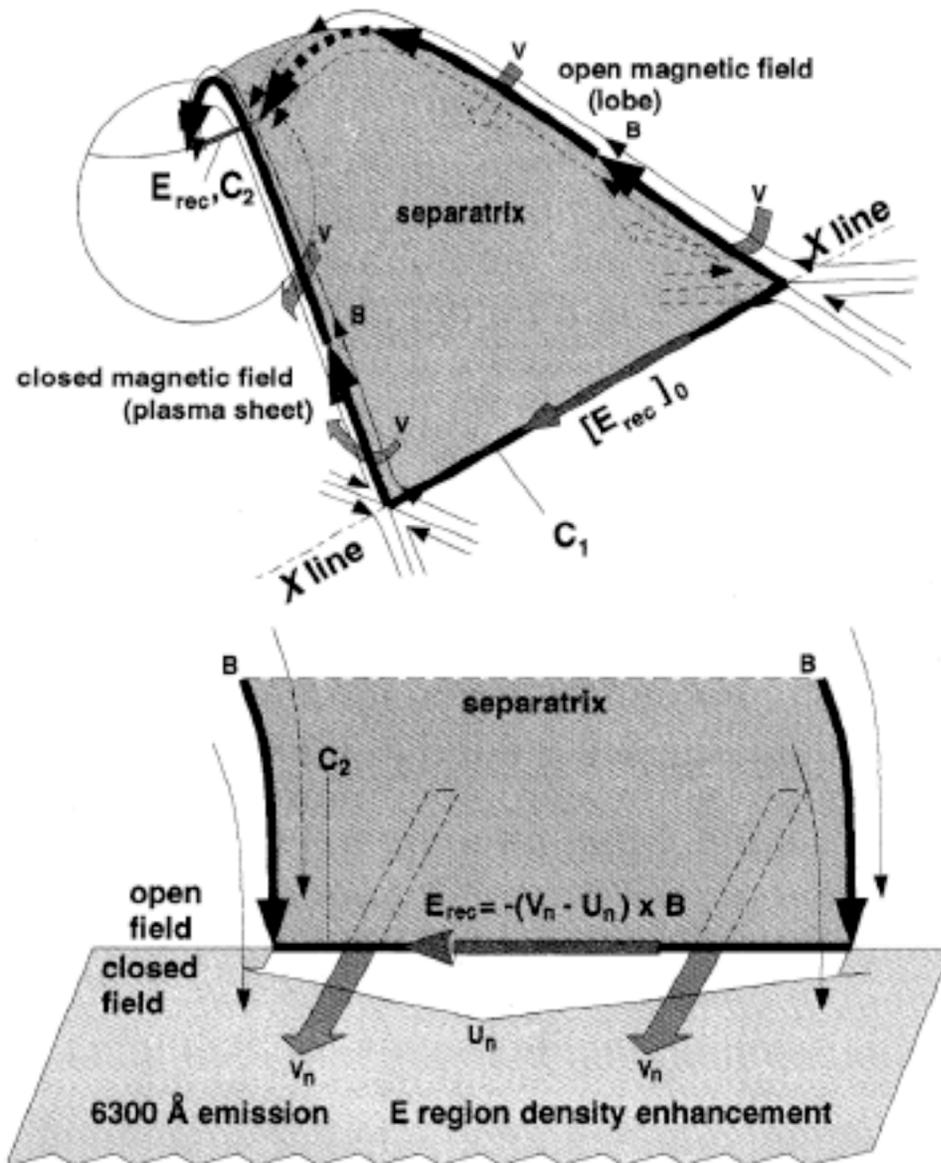
Observed Reconnection Rate for X3 Flare



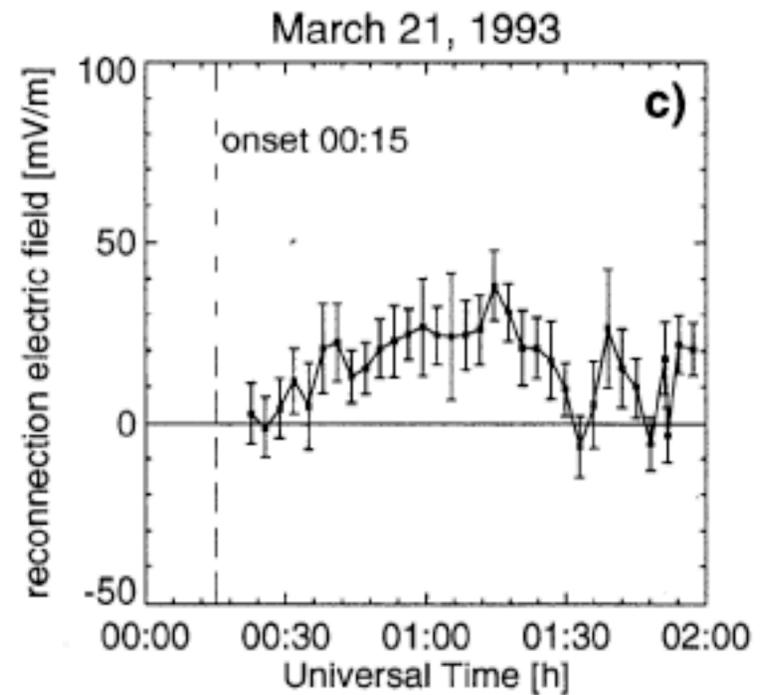
$$E_{\text{reconnect}} / E_{\text{Dreicer}} = 10^6 \gg 1$$

collisionless processes involved

Substorm Reconnection Rate



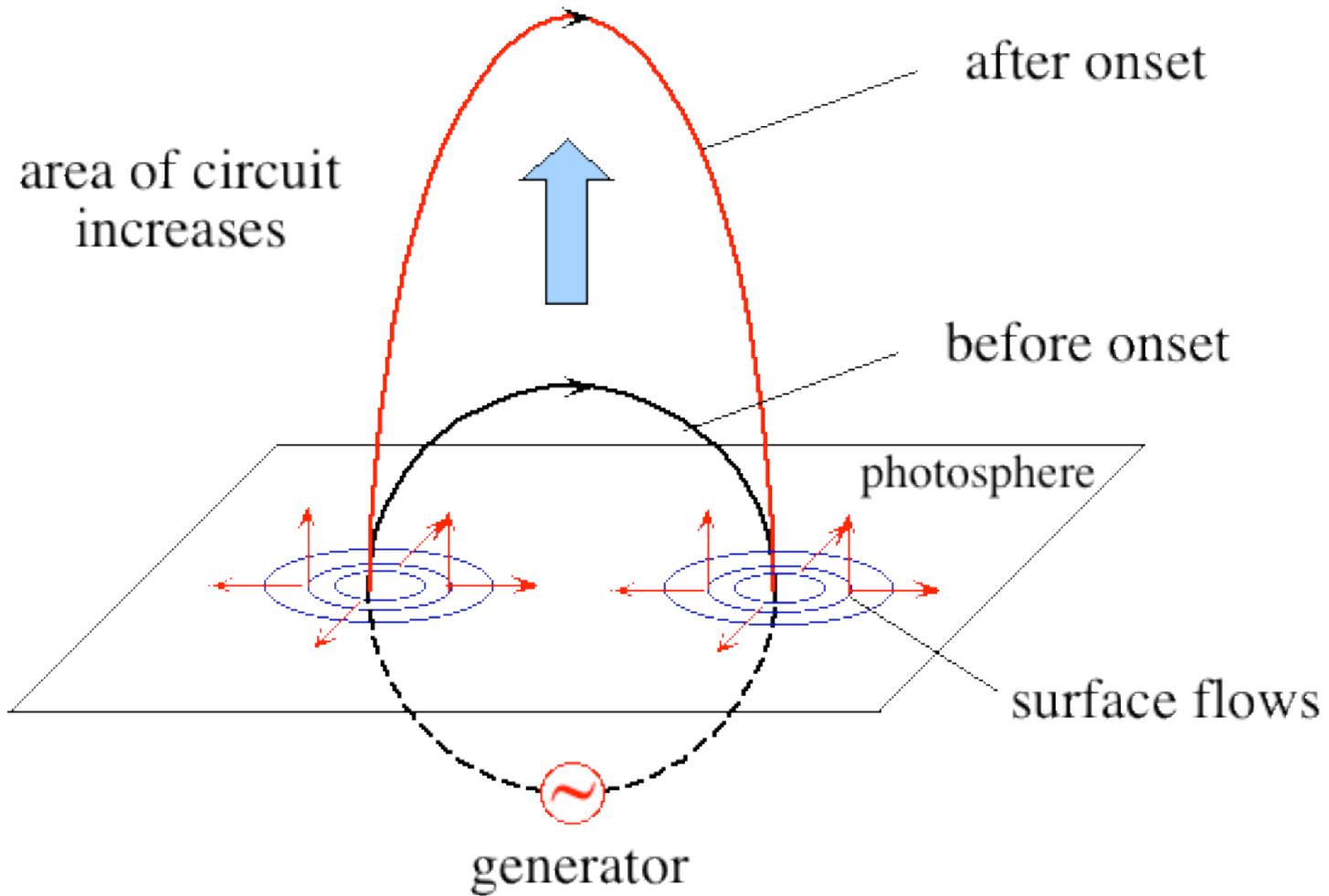
$$\int_{C_1} [E_{rec}]_0 \cdot d\mathbf{l} = \int_{C_2} B(V_n - U_n) dl$$



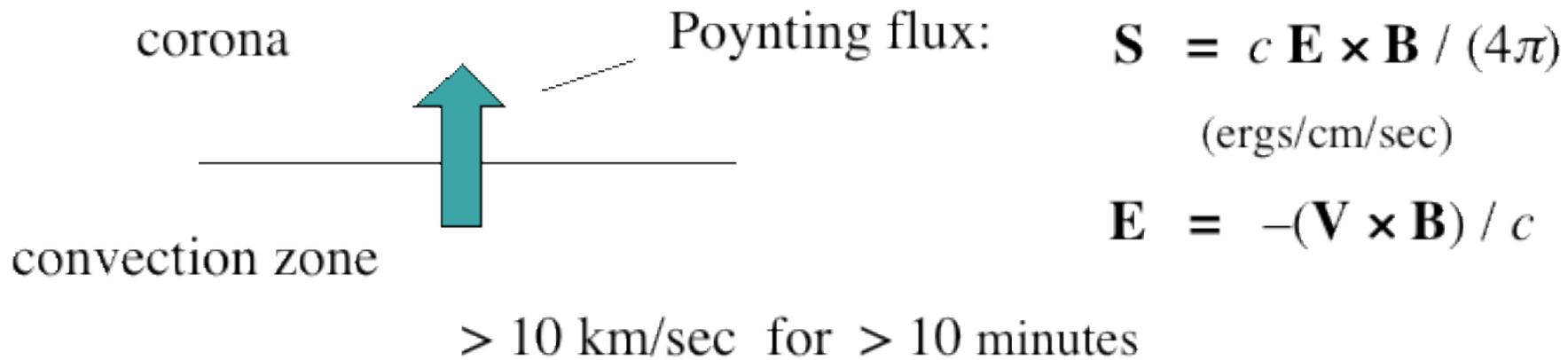
Blanchard et al. (1996)

Flux Injection Models

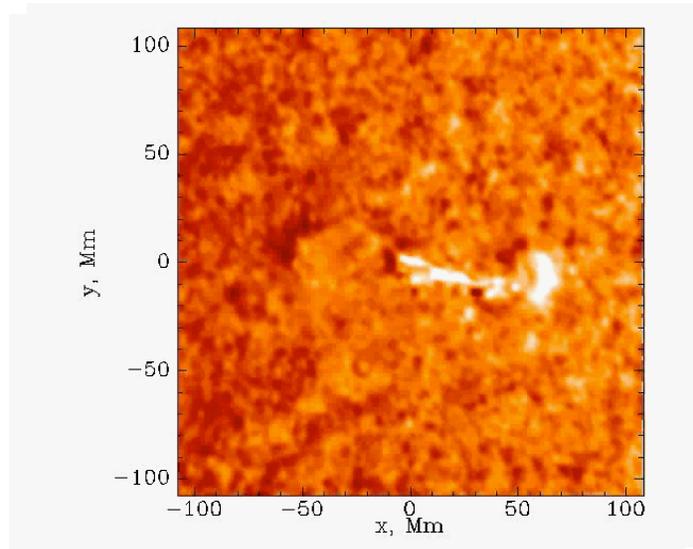
(e.g. Chen 1989)



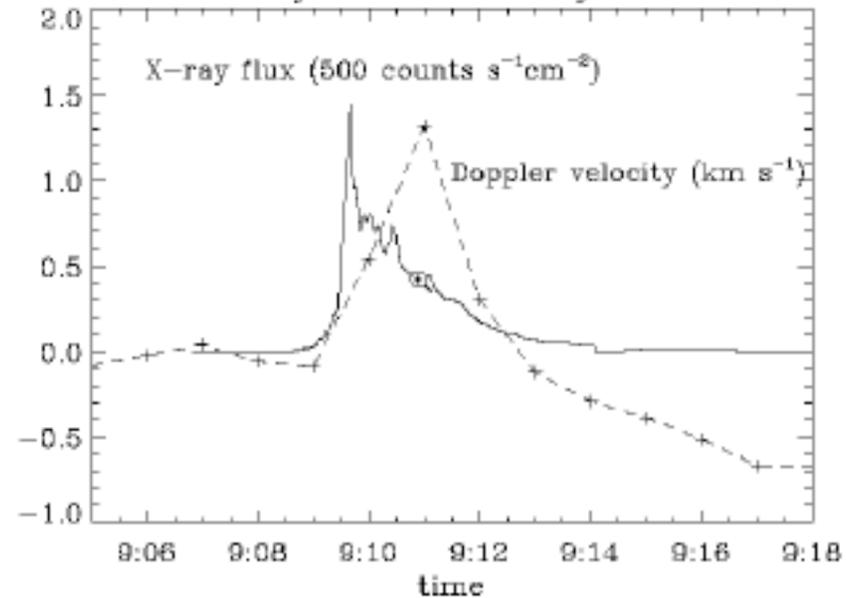
During injection energy flows through photosphere.



Kosovichev et al. 1998

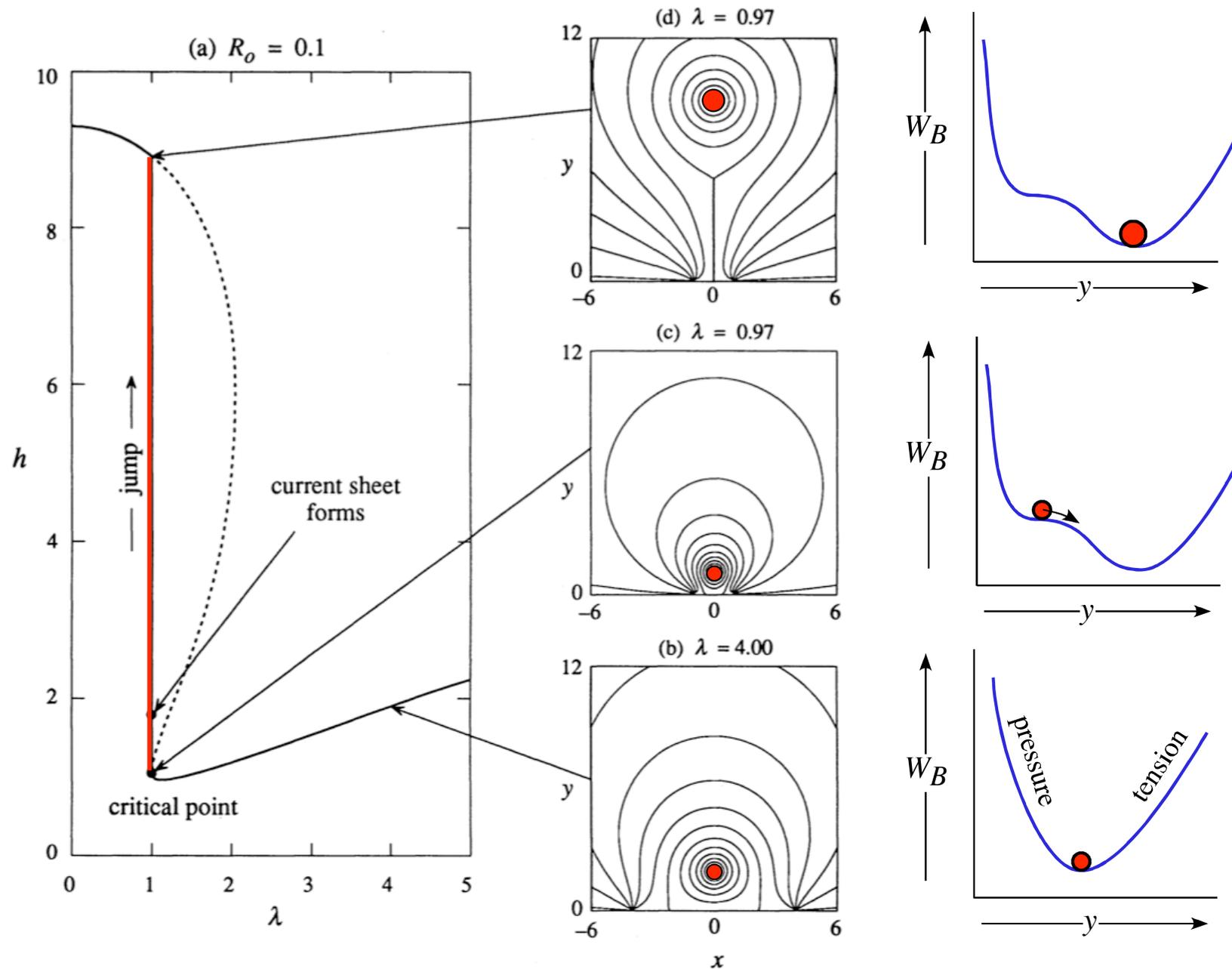


X-ray flare of 9 July 1996

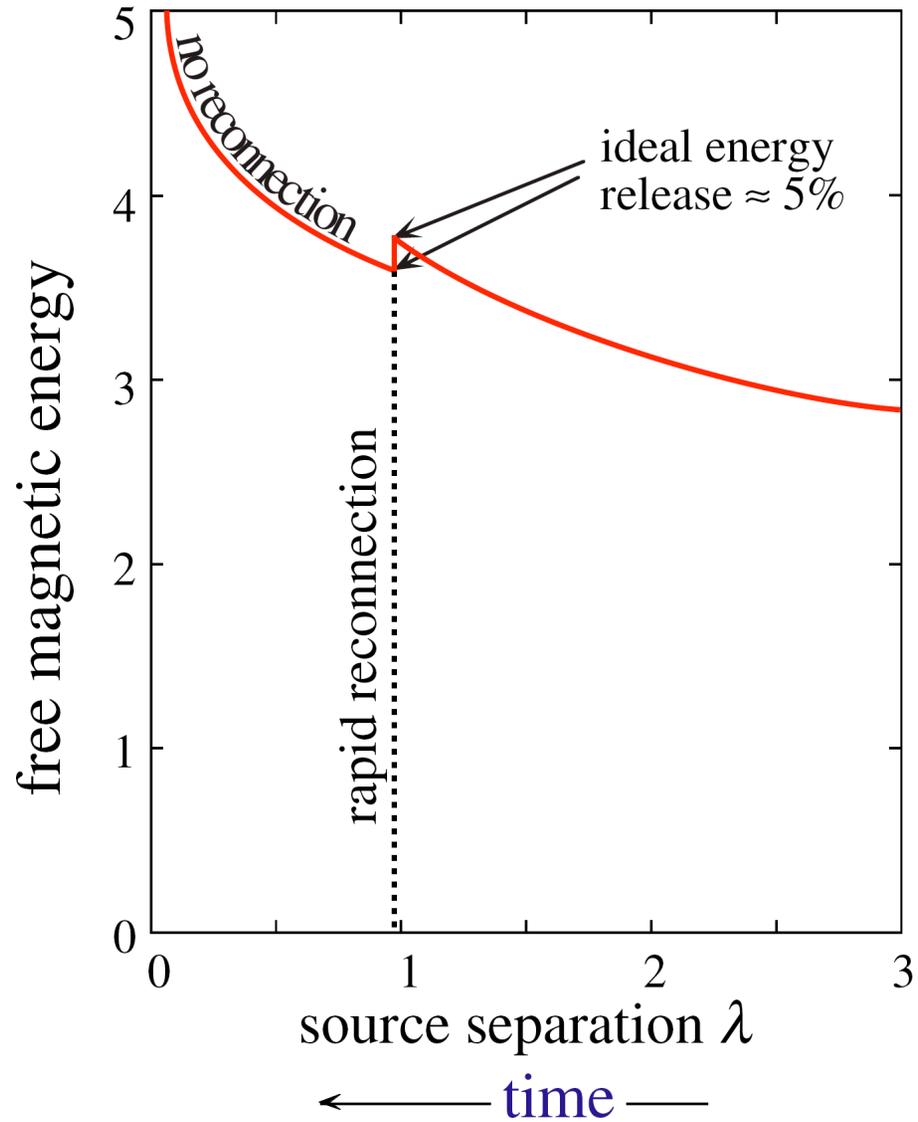


Injection models predict large surface flows which are never observed.

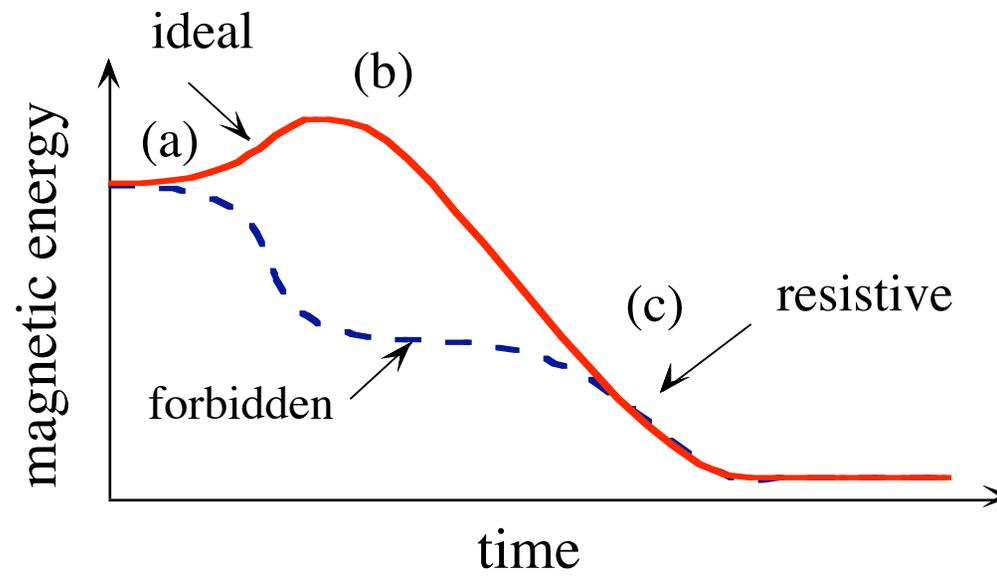
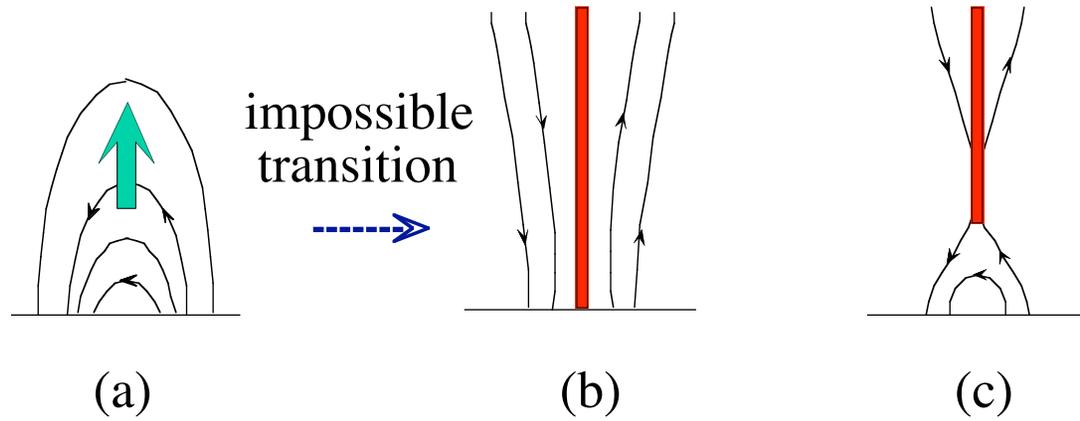
Loss of Equilibrium Model



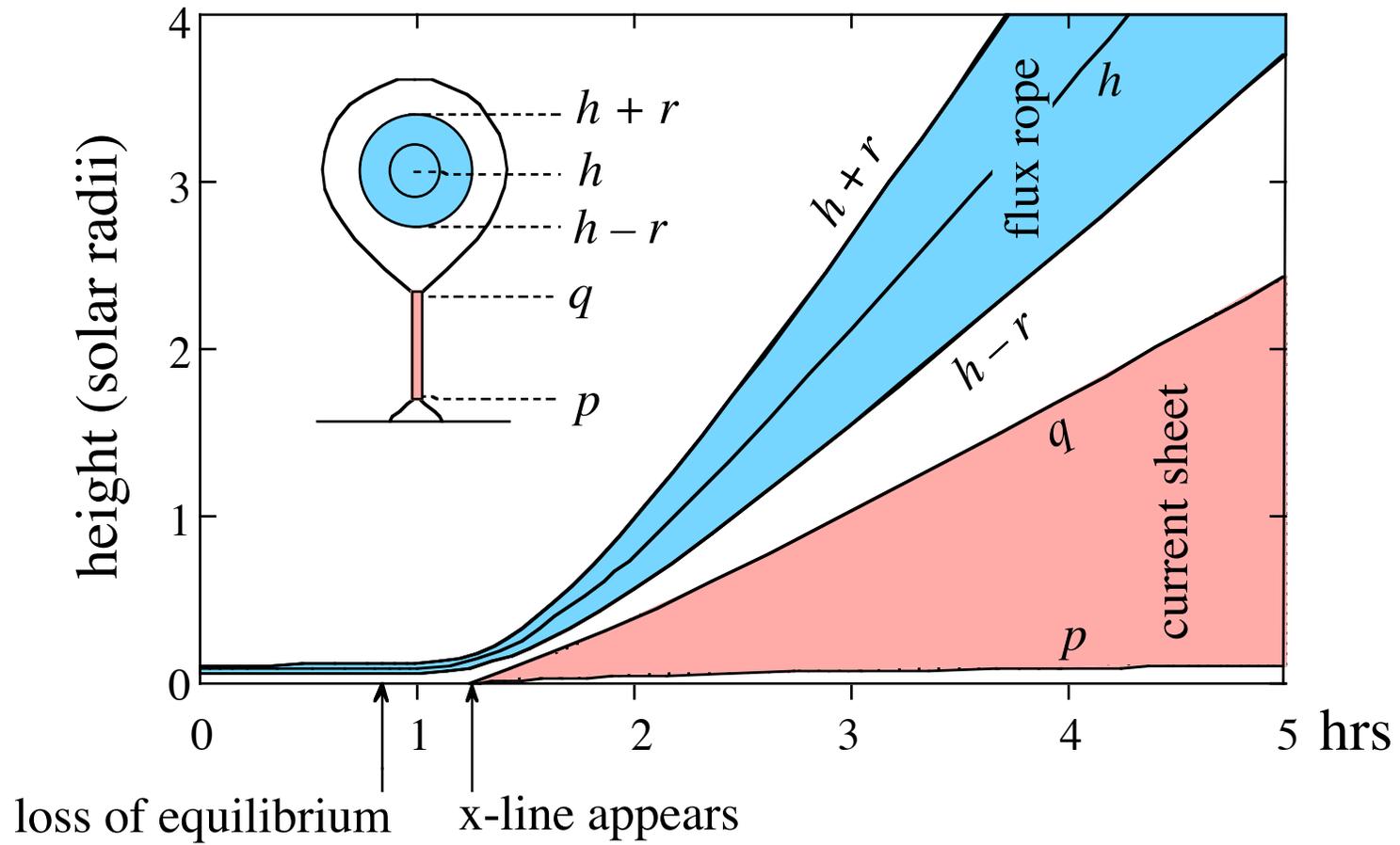
Energy Release in 2D Model



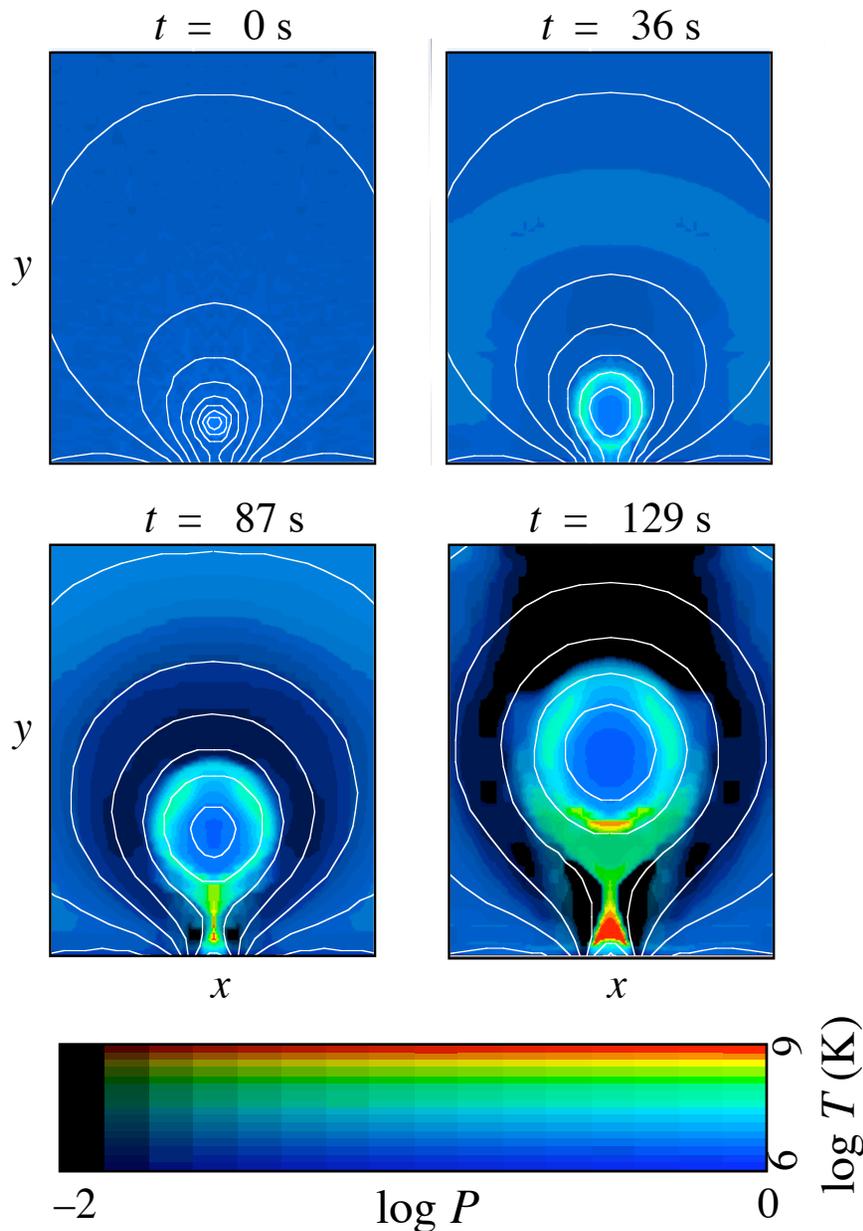
Aly - Sturrock Paradox



Trajectories



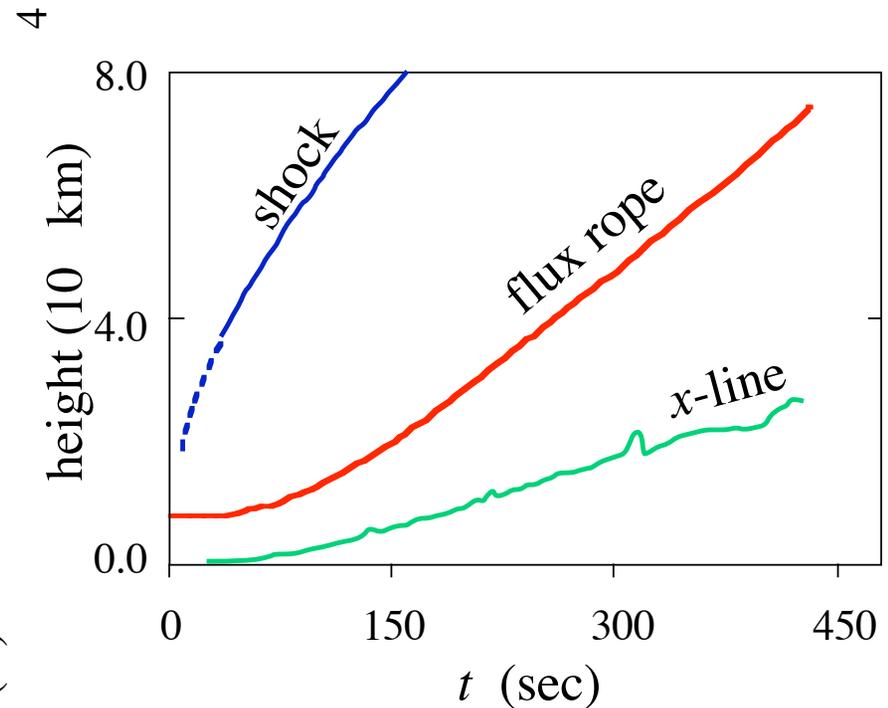
Numerical Simulation of Critical Point Configuration



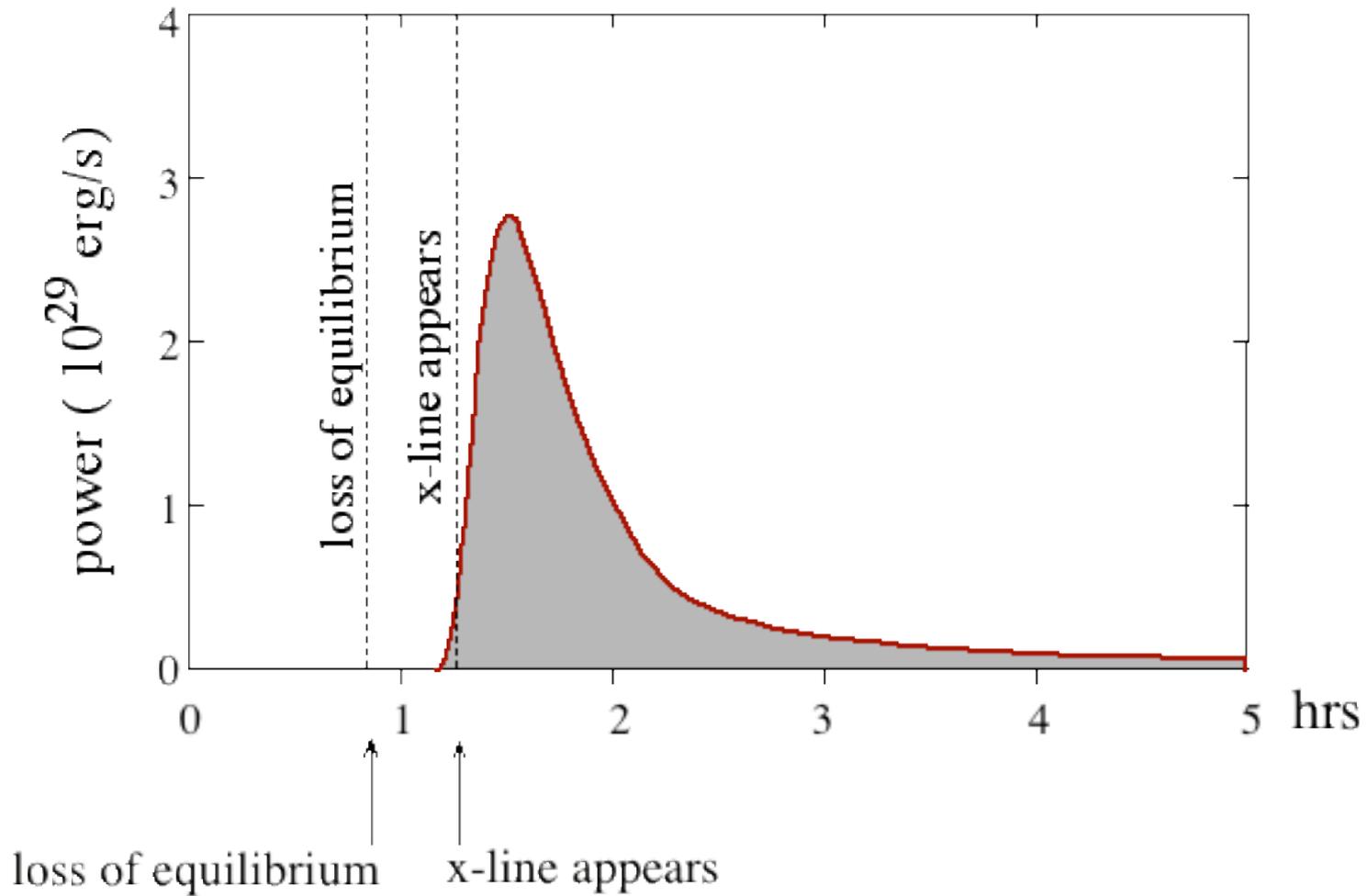
initial condition: $\mathbf{V} = 0$

energy equation: Ohmic heating
no cooling

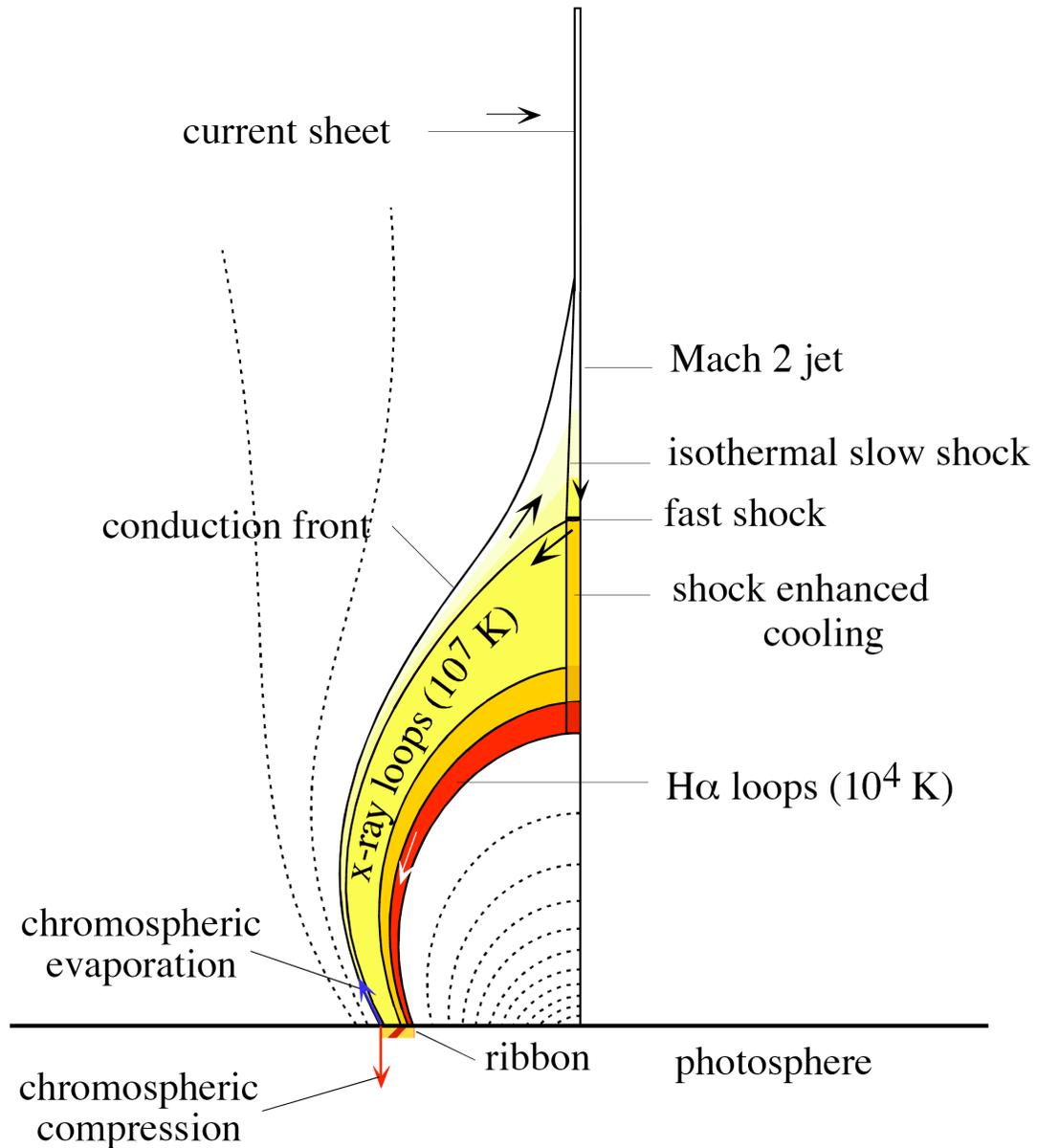
resistivity: uniform, $S = 500$

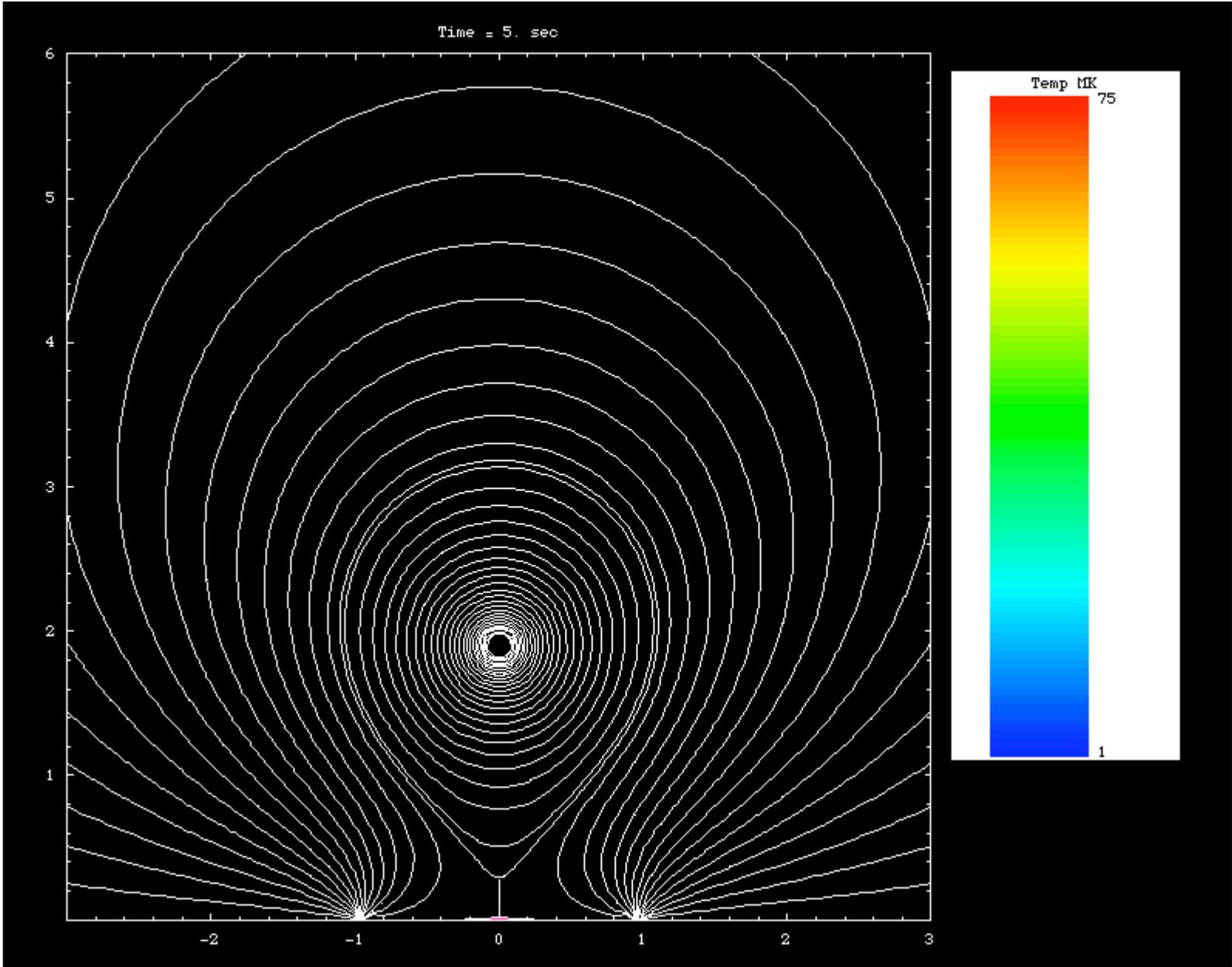


Power Output



Chromospheric Evaporation

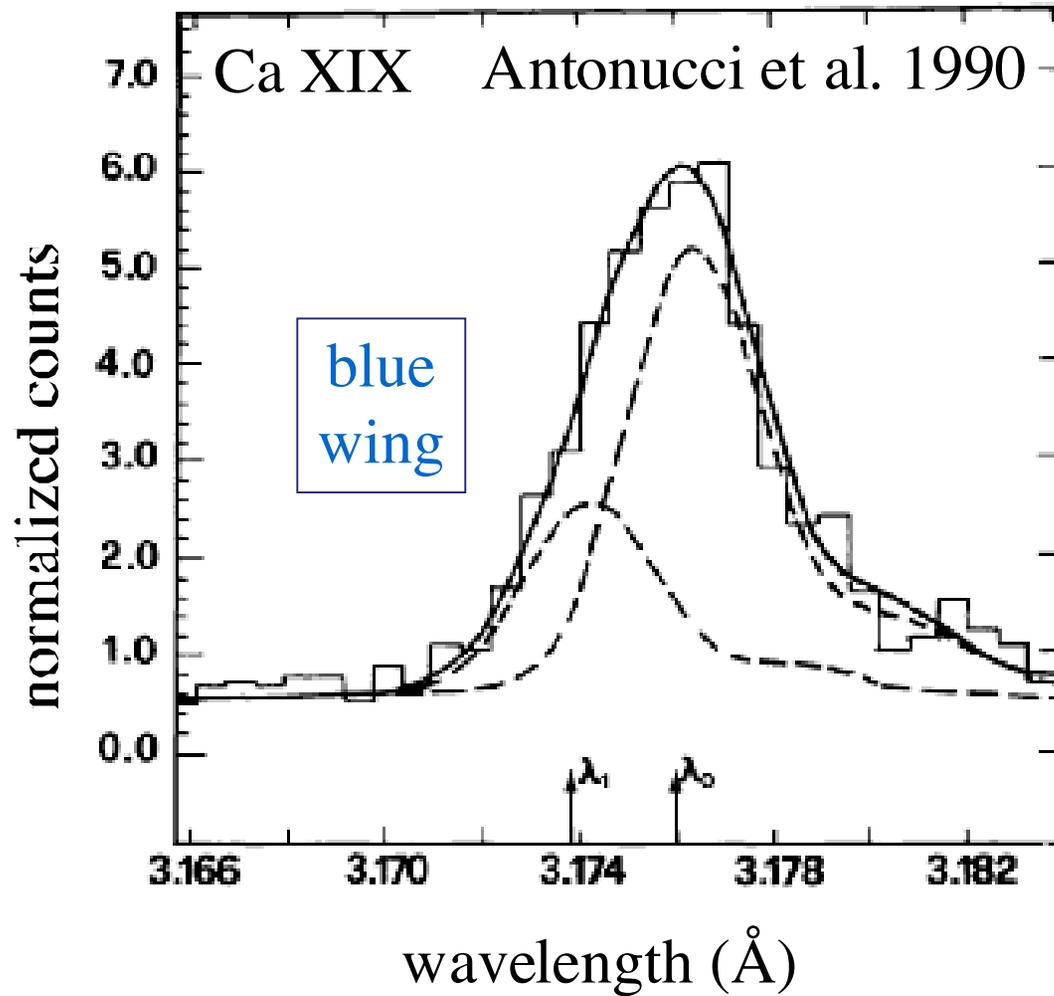


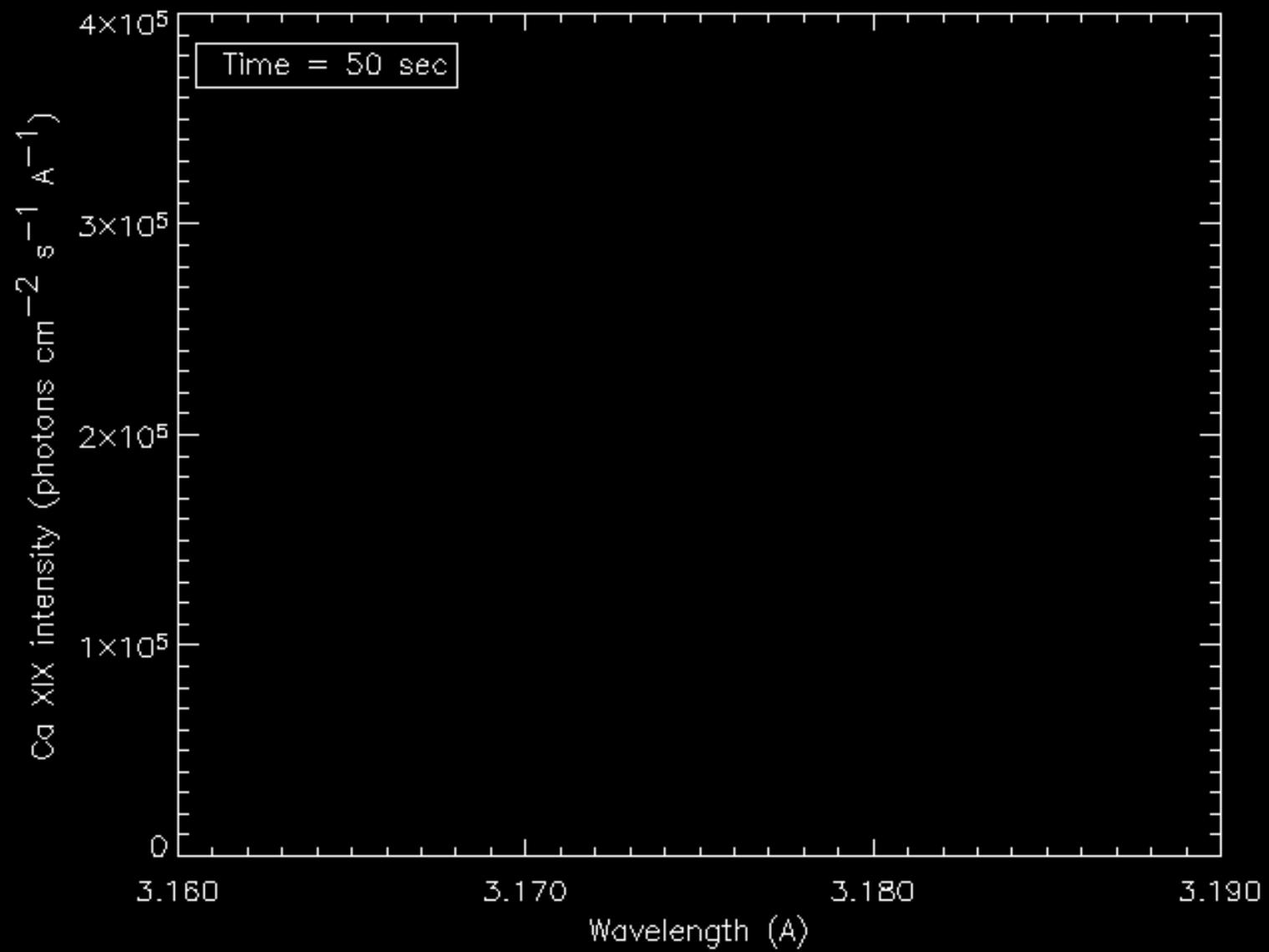


Evaporation Doppler Shift Puzzle

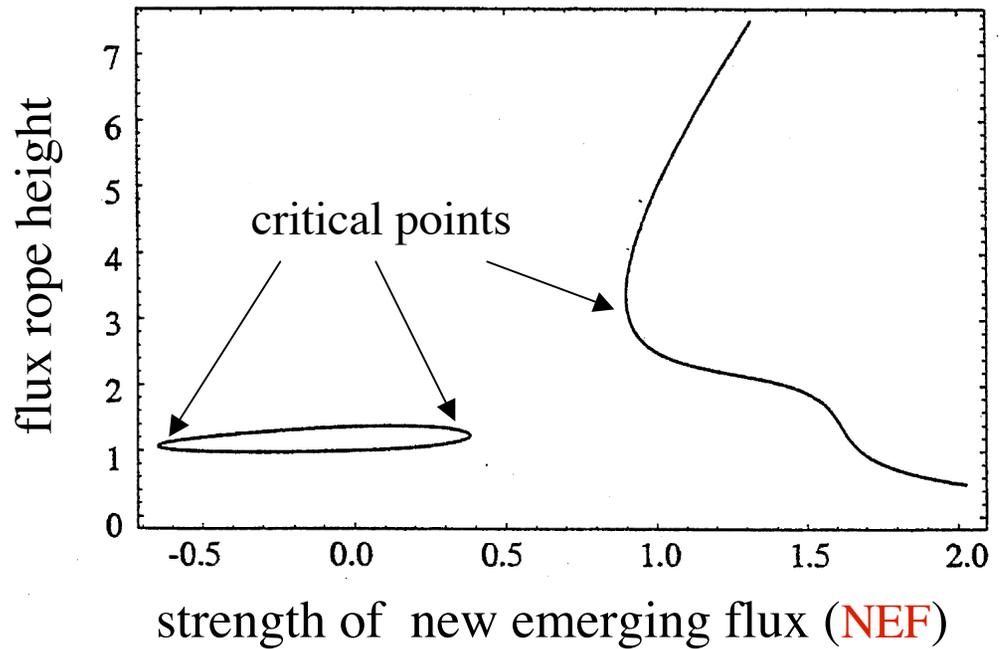
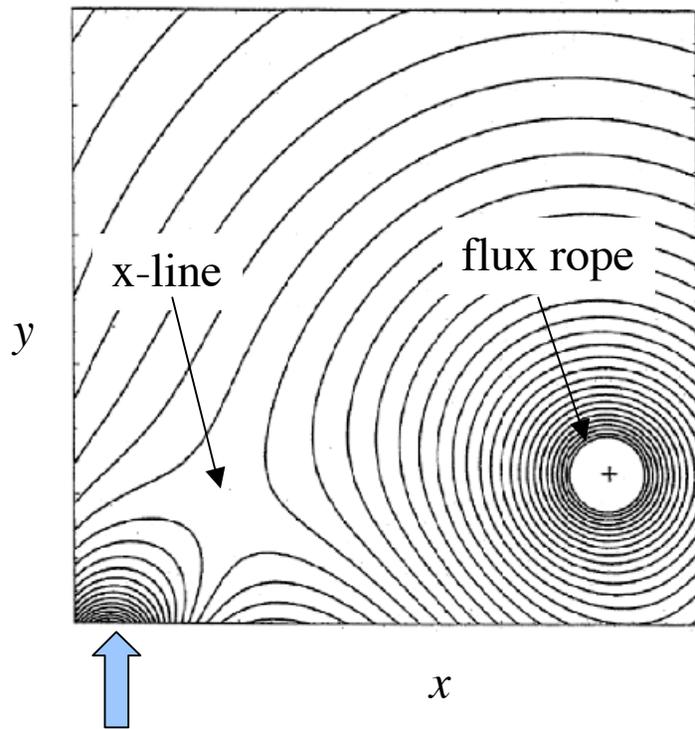
24 April 1984

BCS/SMM





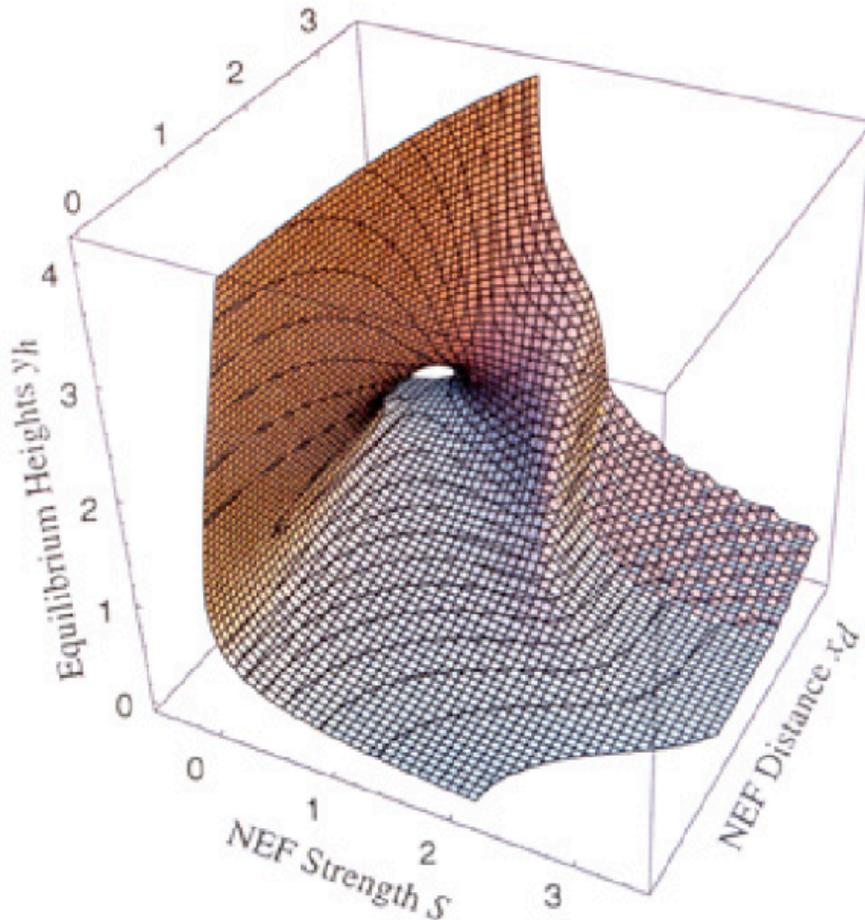
2D Asymmetric Quadrupole Model



NEF

test of “tether-cutting” concept

Equilibrium Manifold in 5D Parameter Space of Model



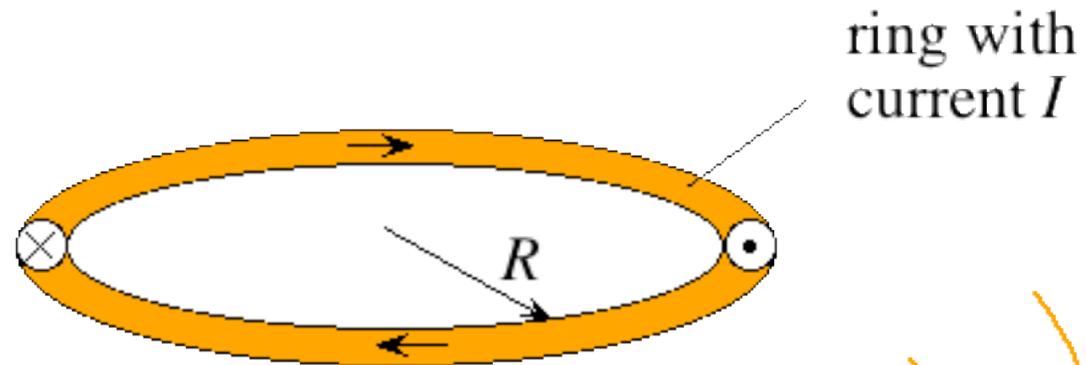
3D “cross section”

1. normalized radius of flux rope
2. normalized main arcade field
3. new emerging flux strength (NEF)
4. normalized depth of NEF
5. normalized distance of NEF

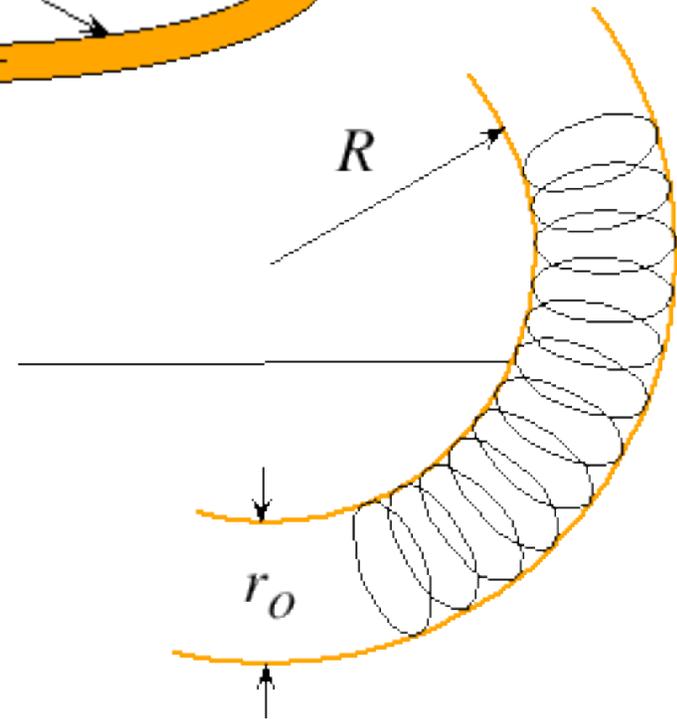
2nd order umbelic catastrophe

Basic Principles I

Driving Force:



inner edge is pinched
by curvature of rope

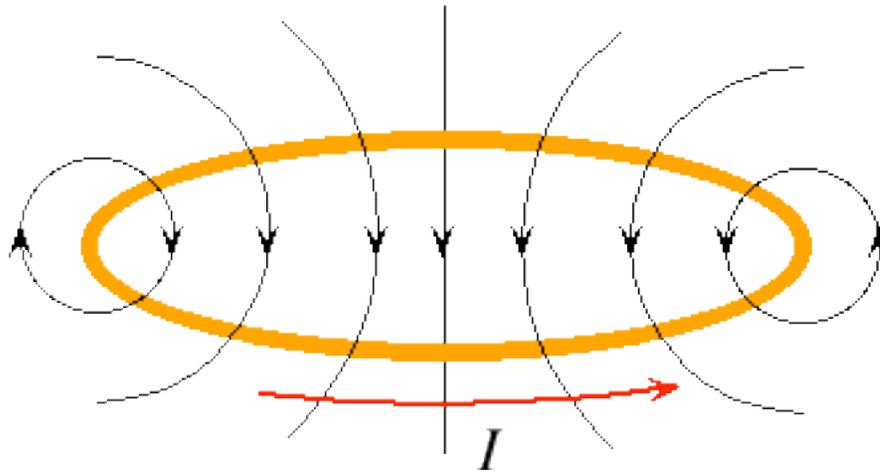


repulsive force:

$$F \propto \frac{I^2}{R} \ln(R/r_0)$$

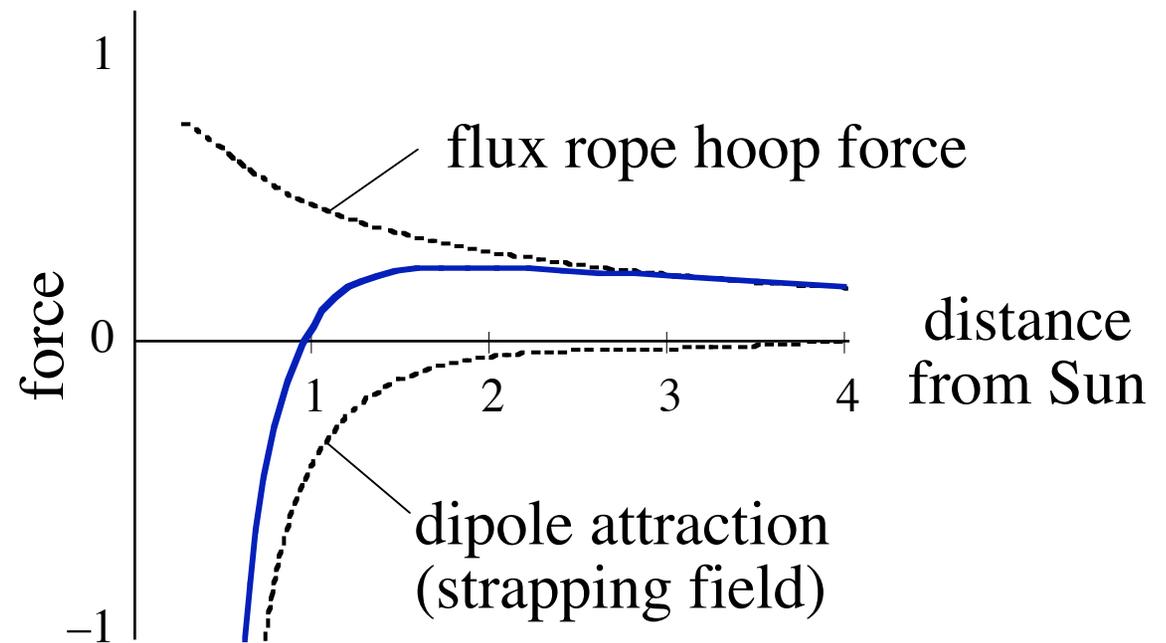
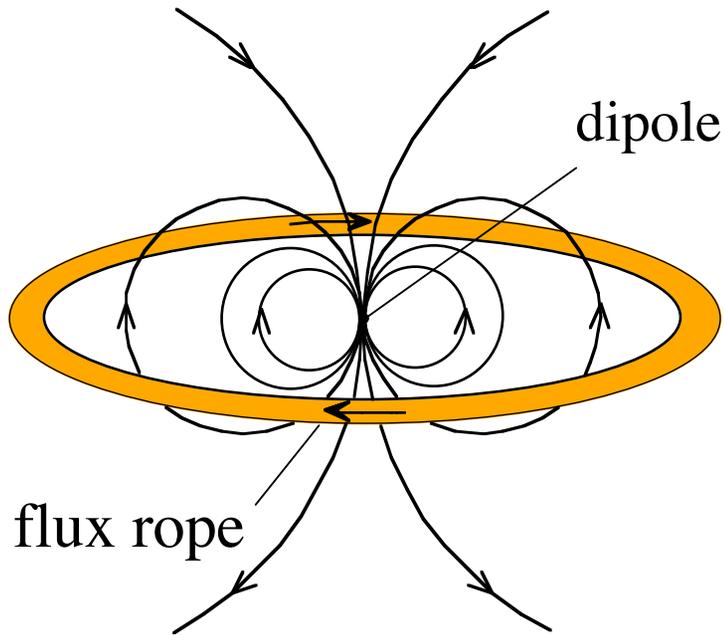
Basic Principles II

Flux Conservation:



$$I \propto 1/[R \ln(R/r_0)]$$

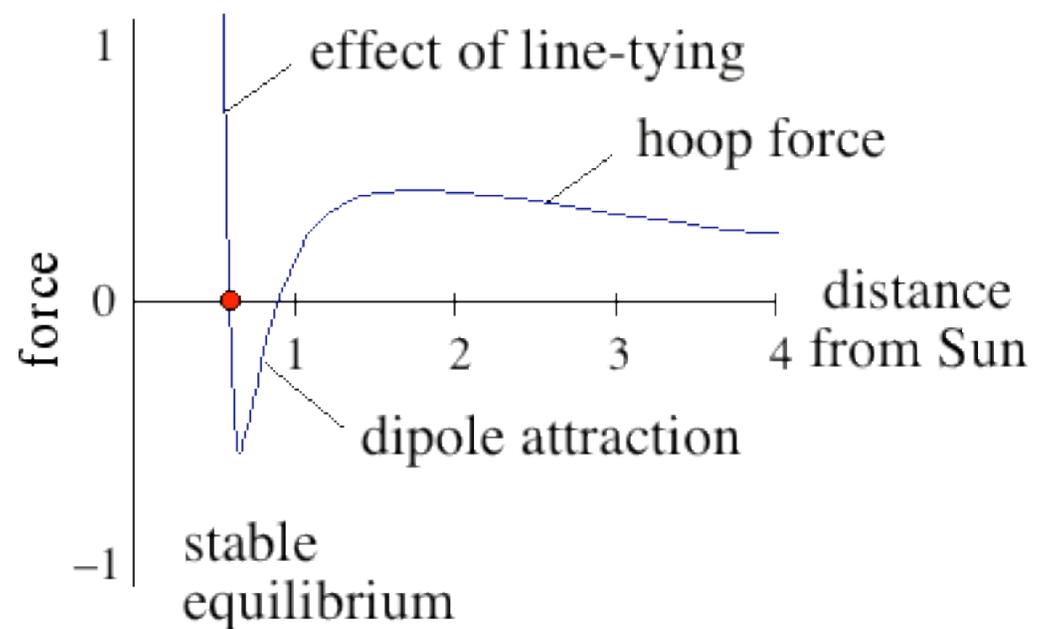
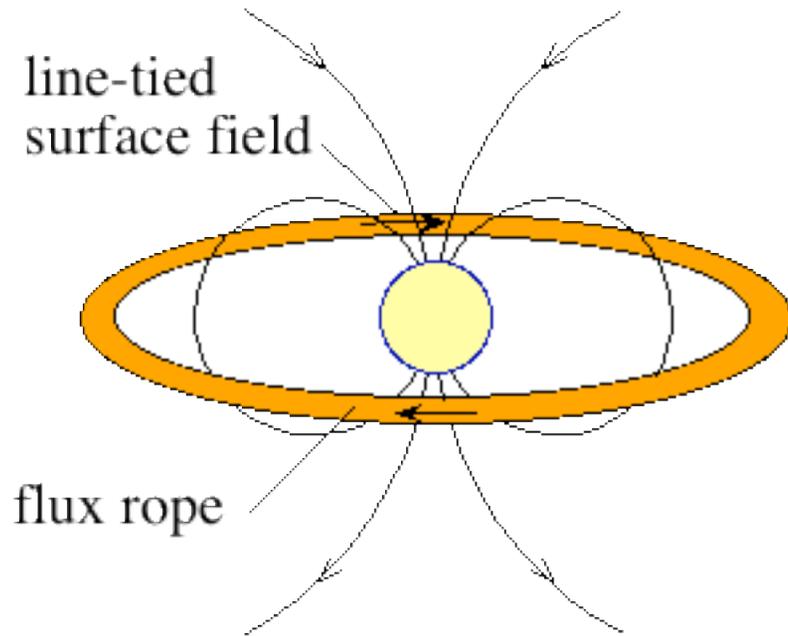
How to Achieve Equilibrium



However, such an equilibrium is unstable!

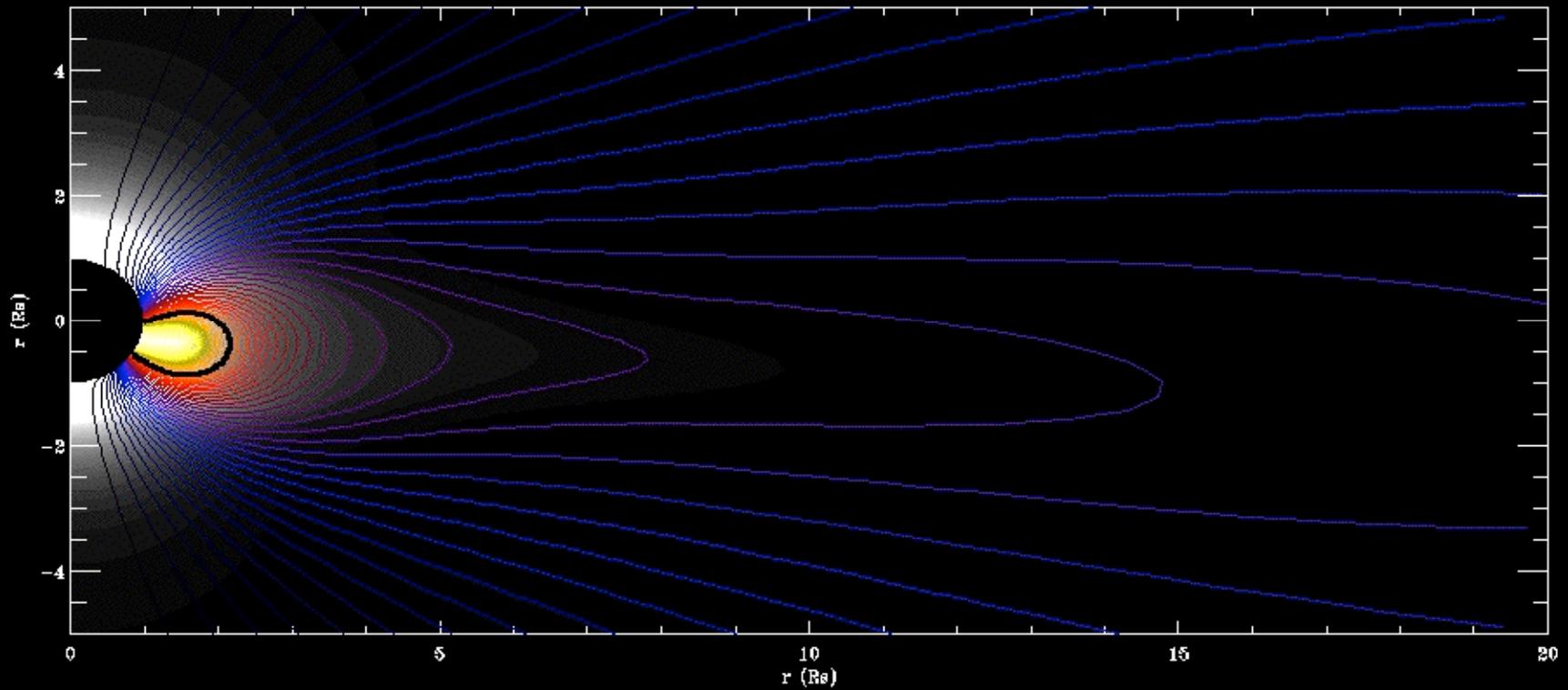
How to Achieve a Stable Equilibrium

Key factor: Line-tying



Line-tying creates a second, stable equilibrium

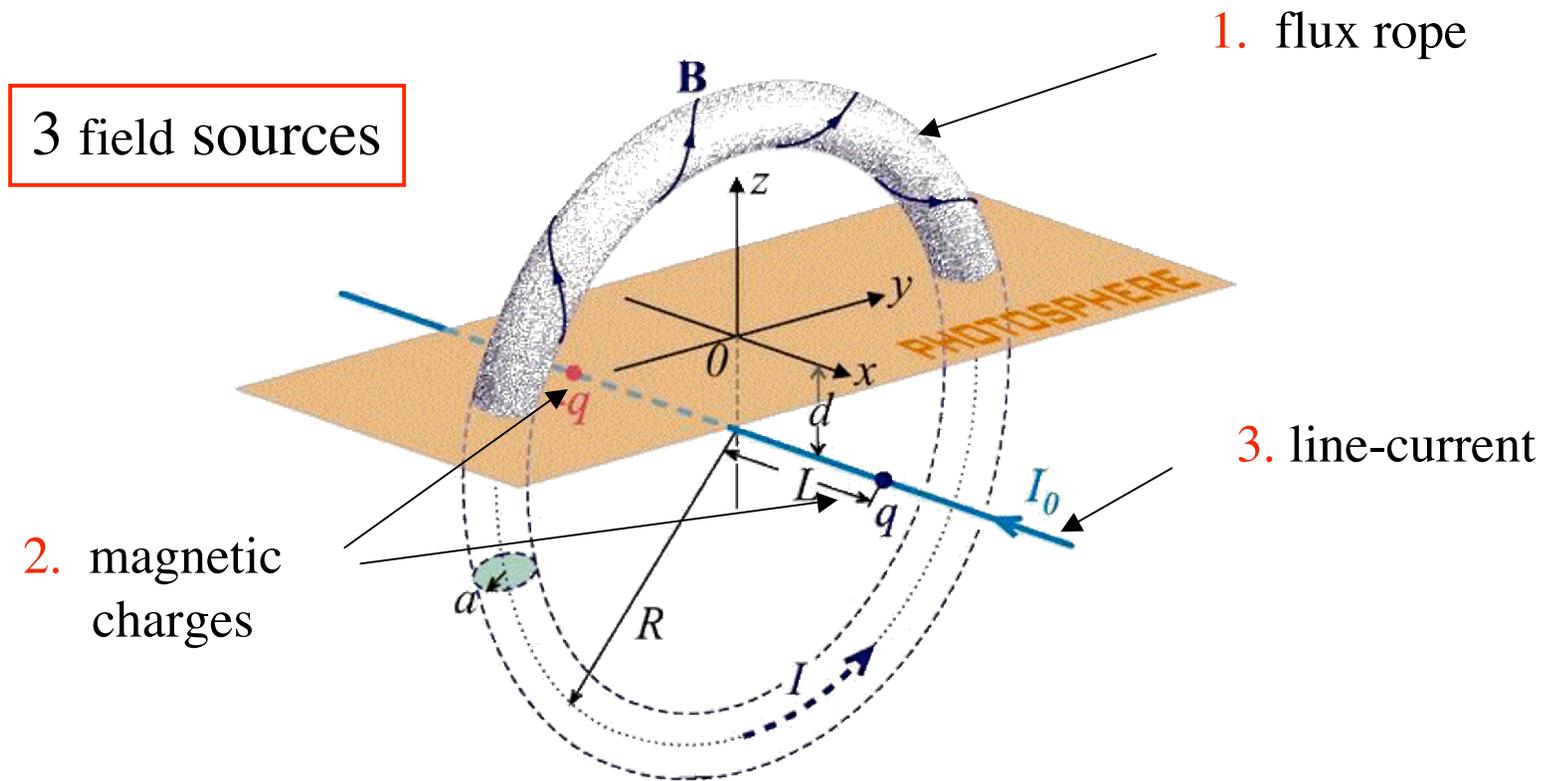
SAIC CME Simulation



Linker et al. (2001)

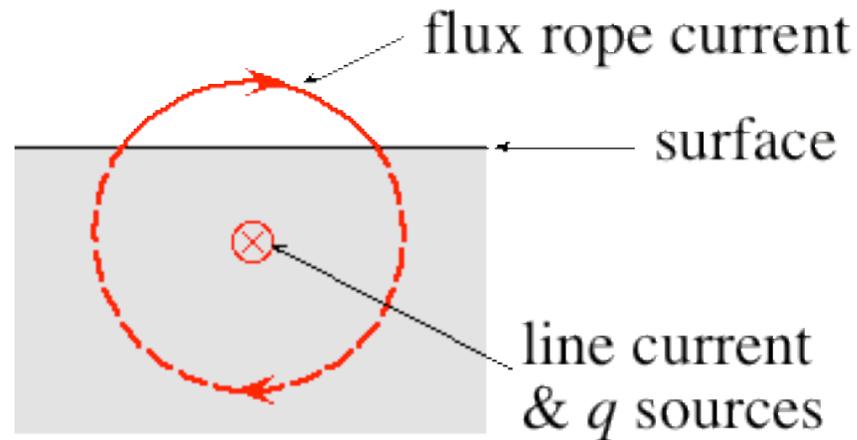
3D Loss-of-Equilibrium Model

Titov & Démoulin (1999)

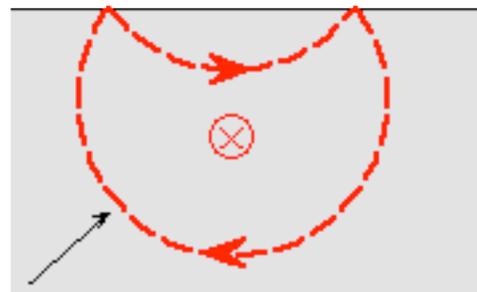


3D Line-Tied Solution by Method of Images

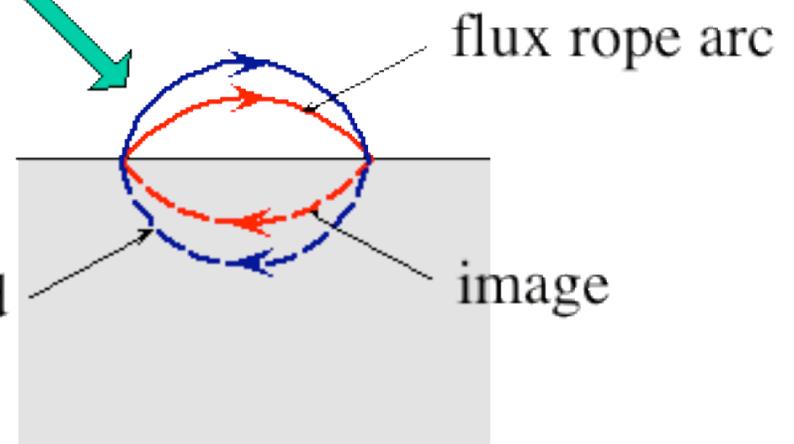
Solution for \mathbf{B} in terms of incomplete elliptical integrals



stationary background source

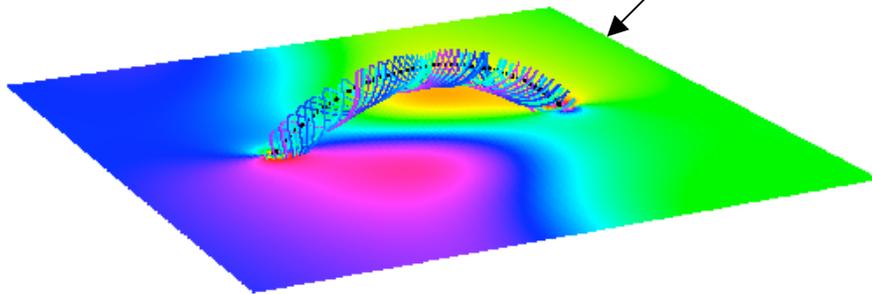


perturbed position



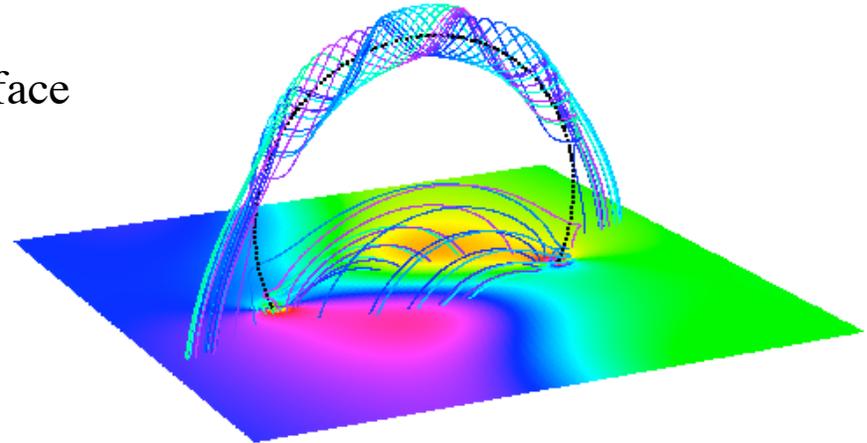
Line-Tied Evolution

initial
configuration

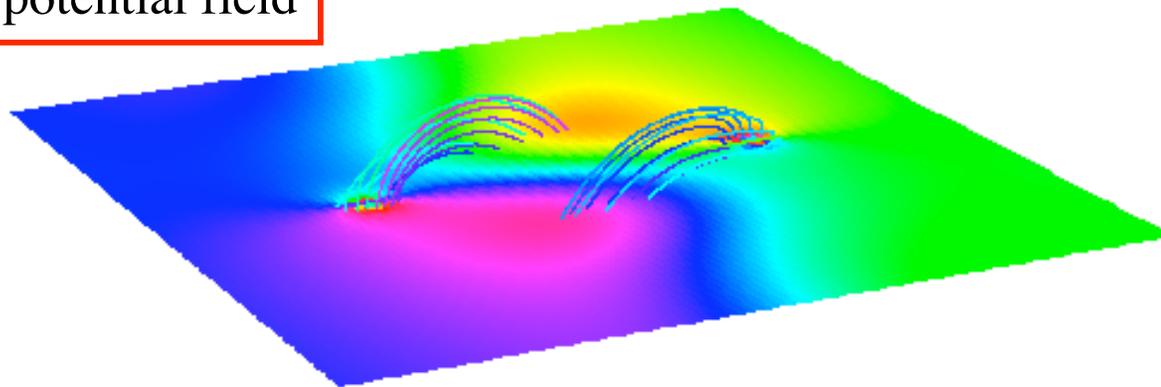


vertical field at surface

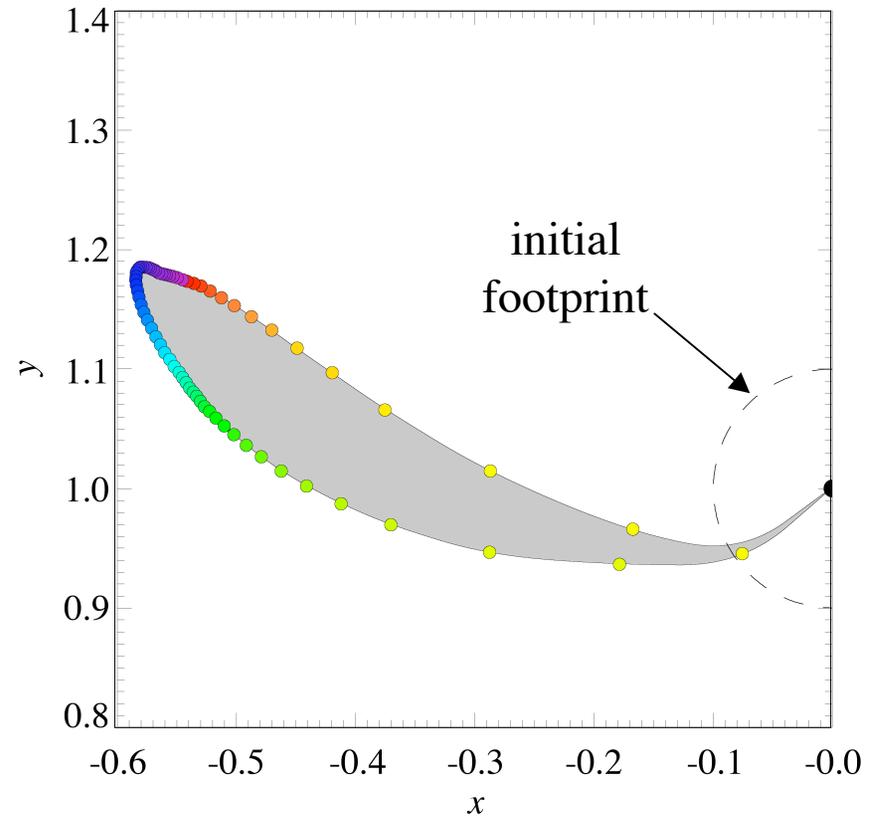
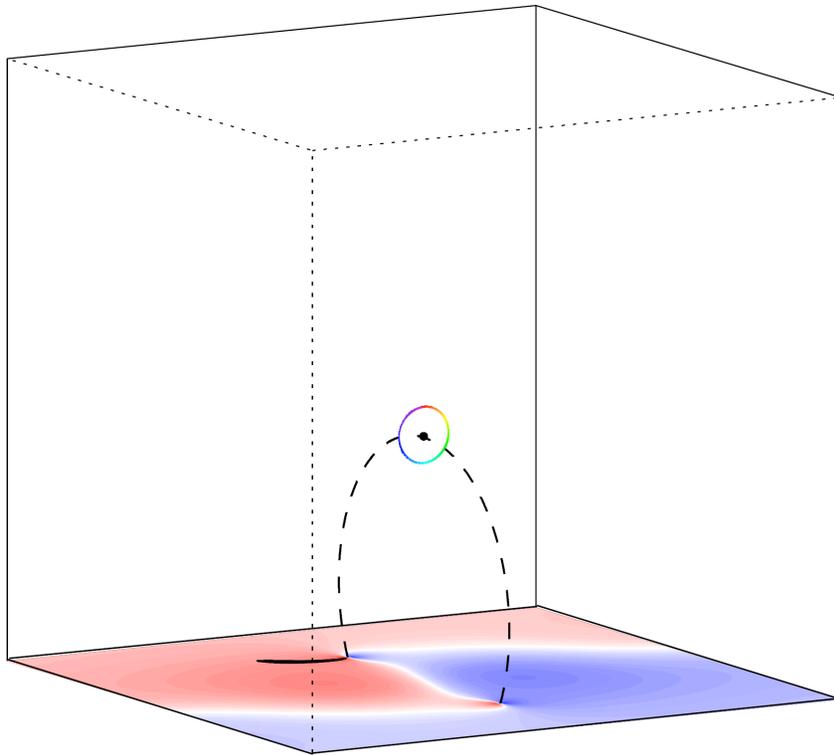
erupted configuration



potential field

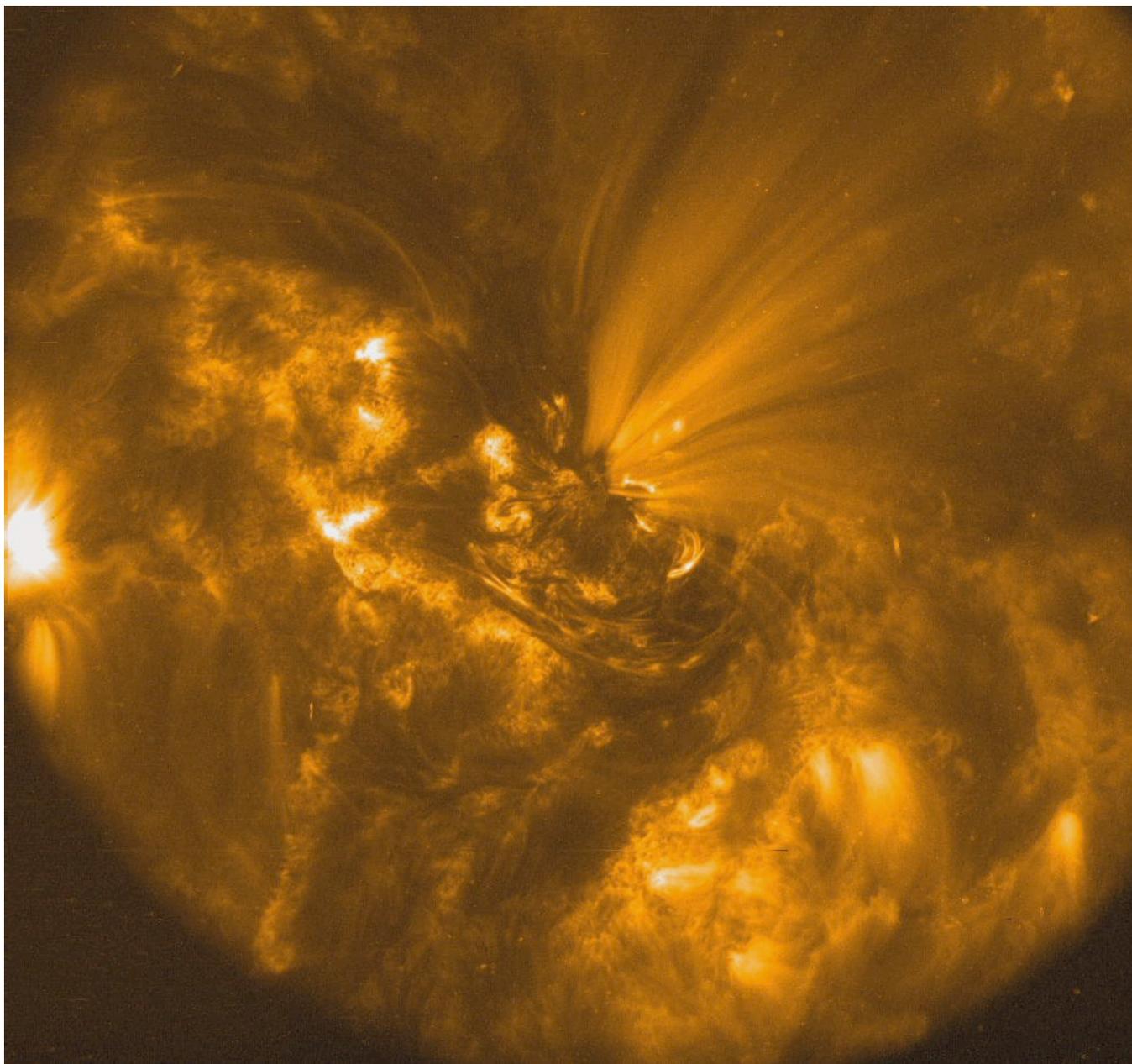


Flux-Rope Footprint

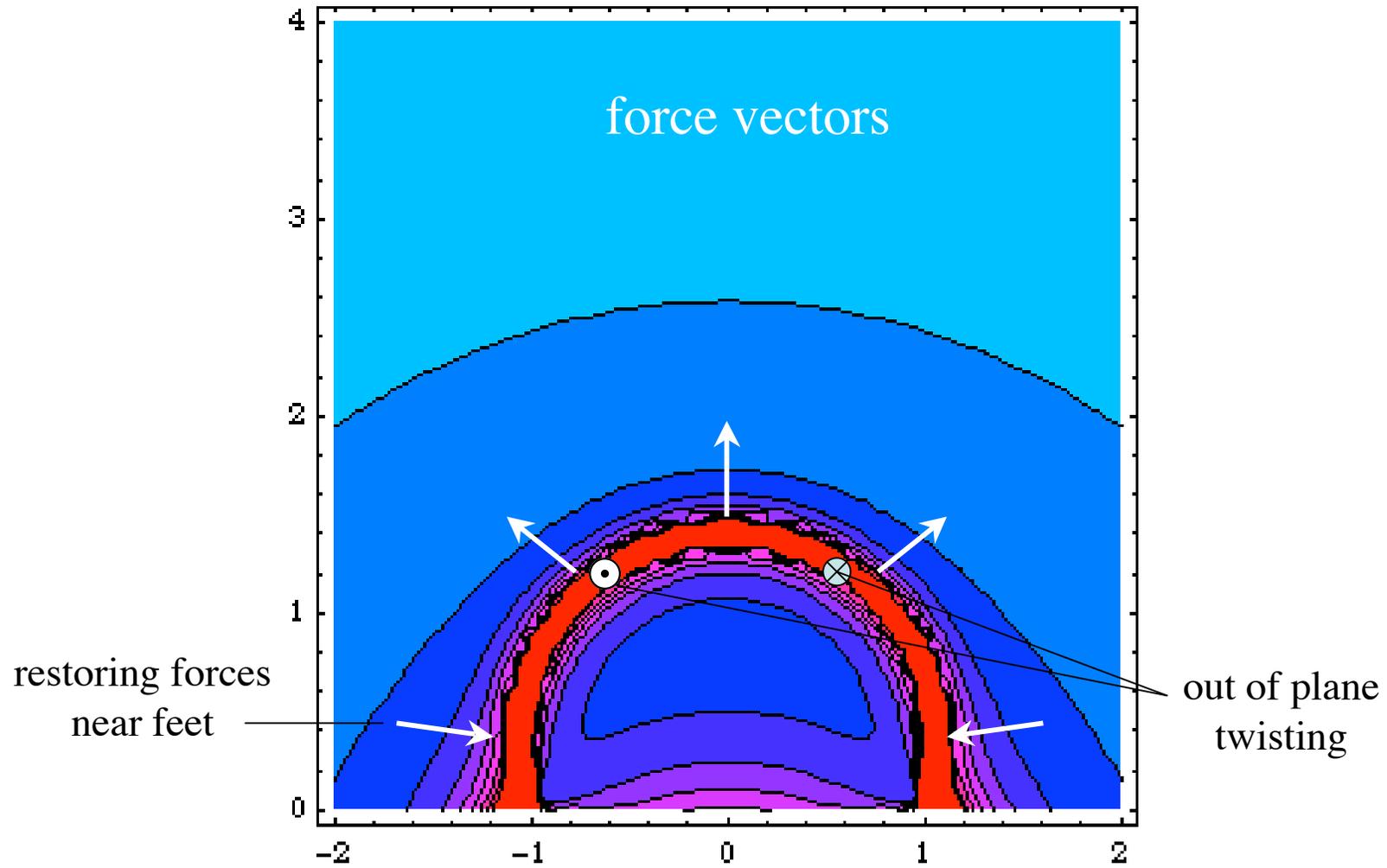


images courtesy of B. Kliem

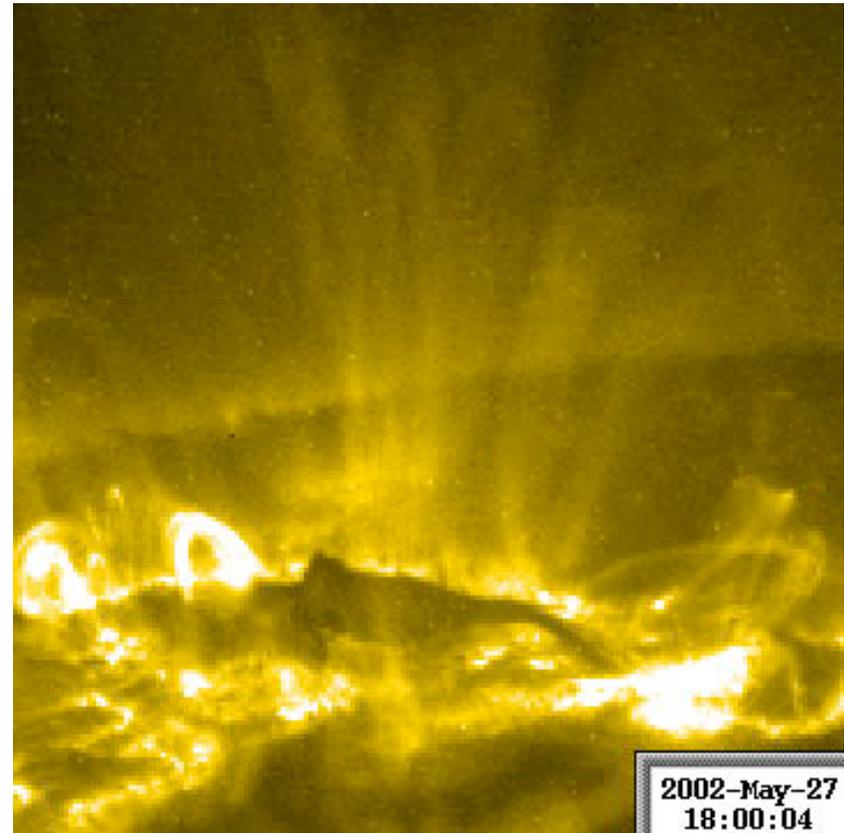
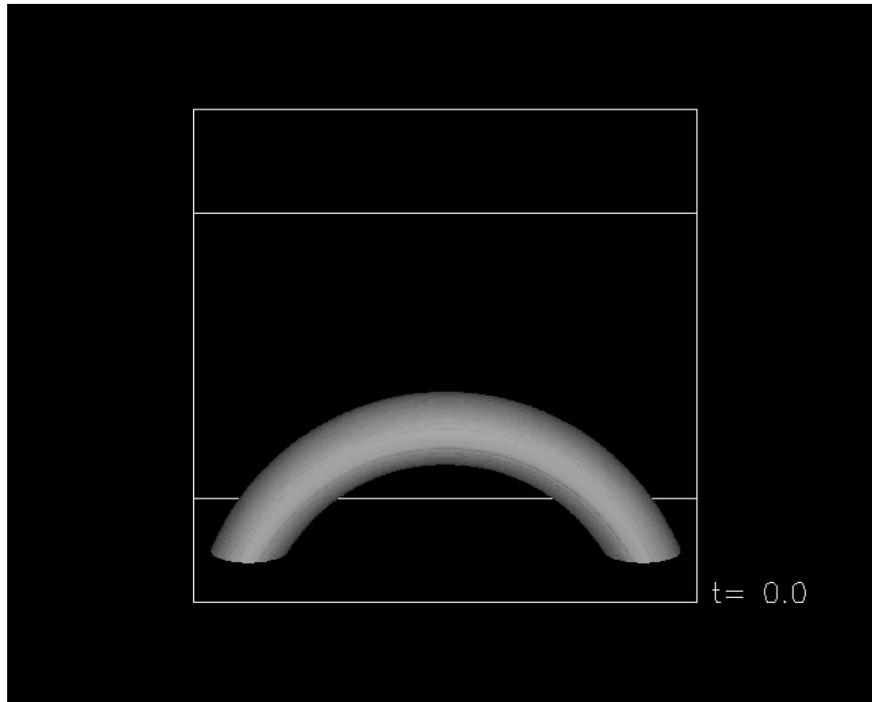
Transient Coronal Holes as Seen by TRACE



Forces Acting on Flux Rope

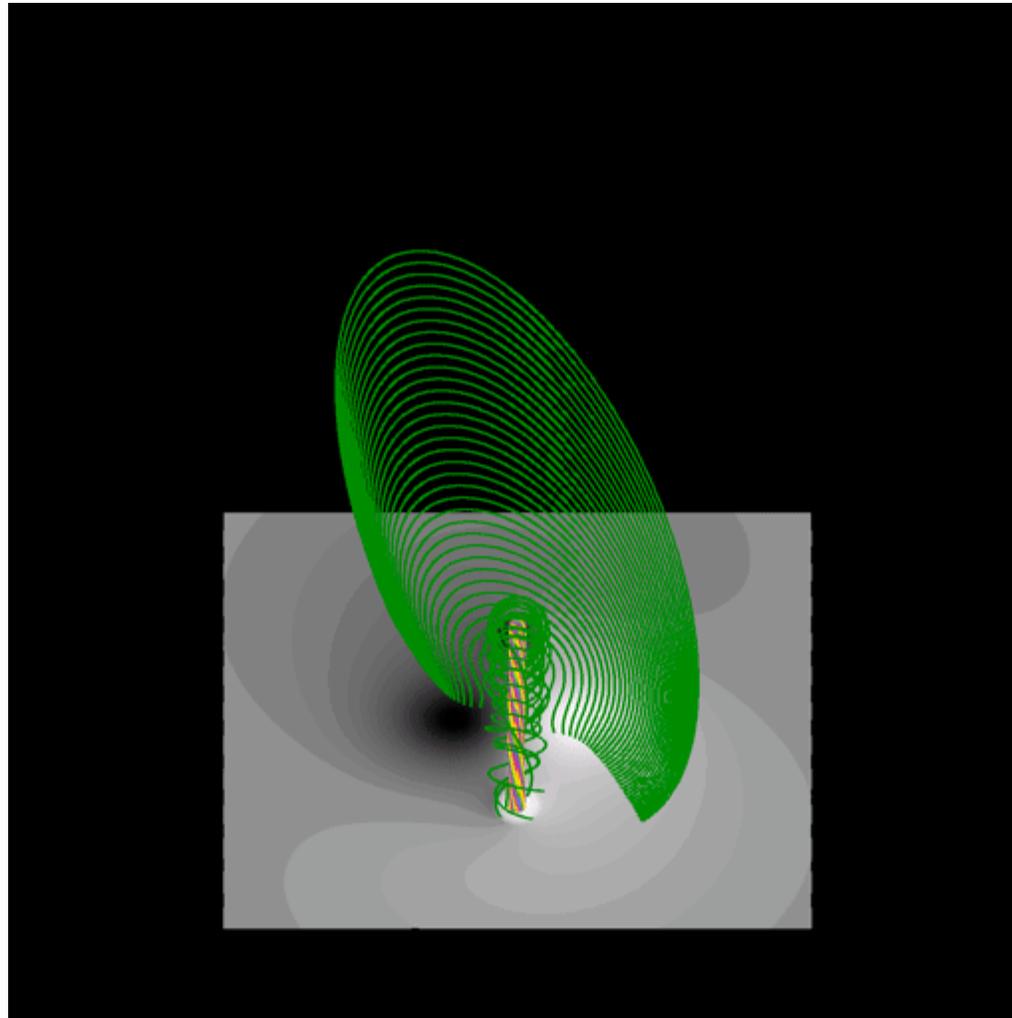


current density



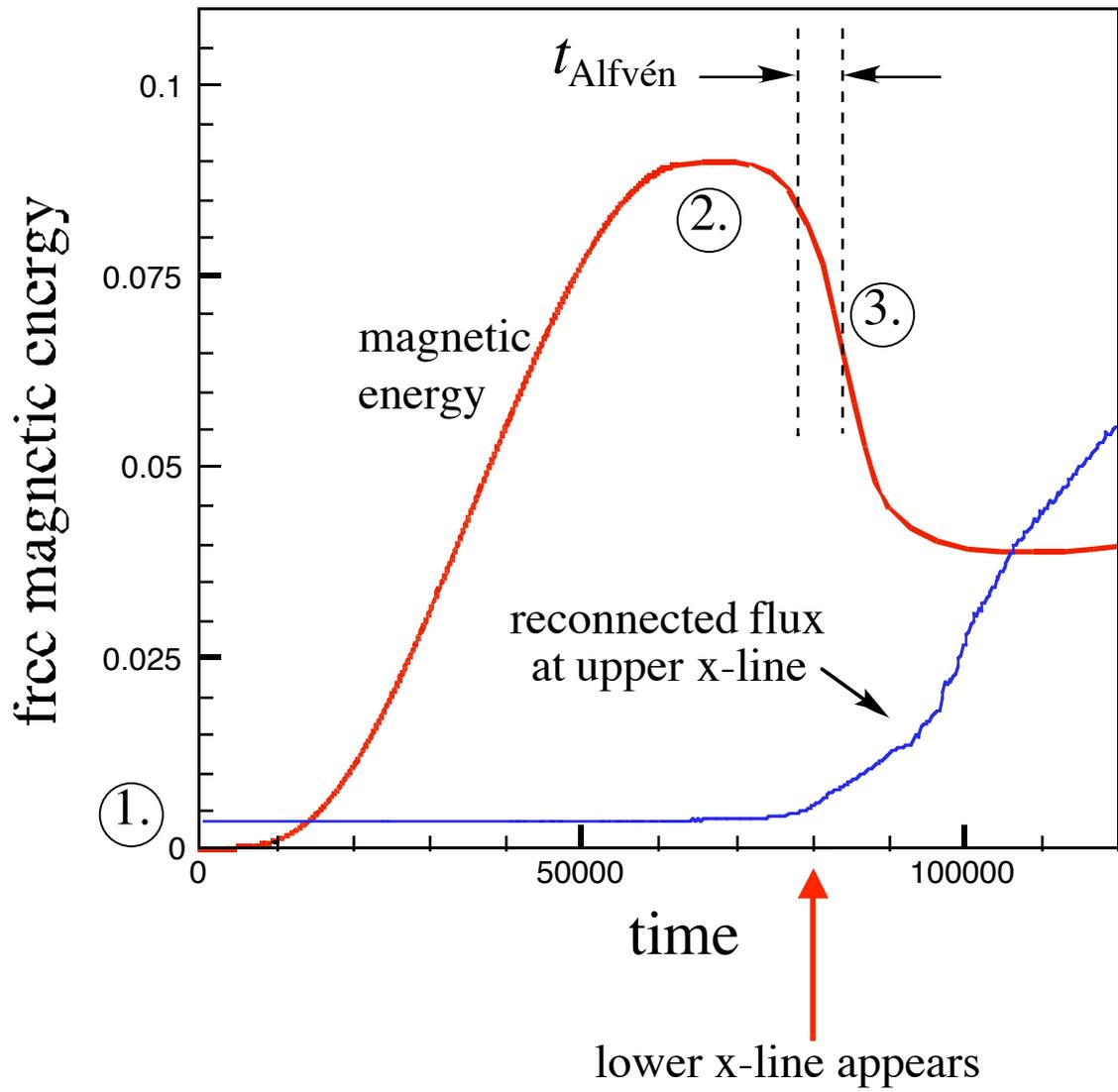
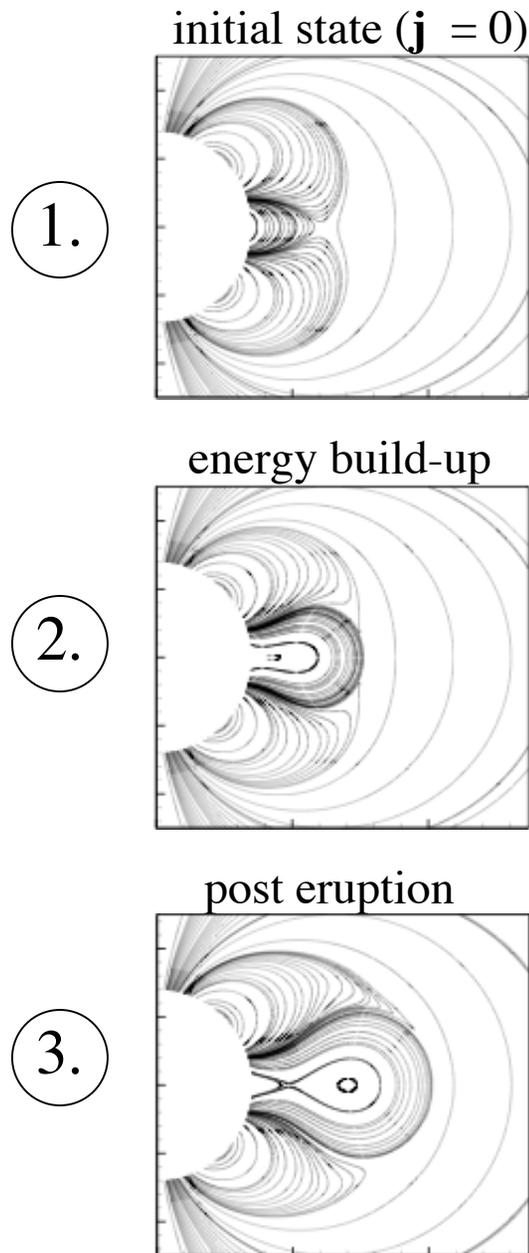
Kliem & Török (2004)

Simulation of Kliem & Török

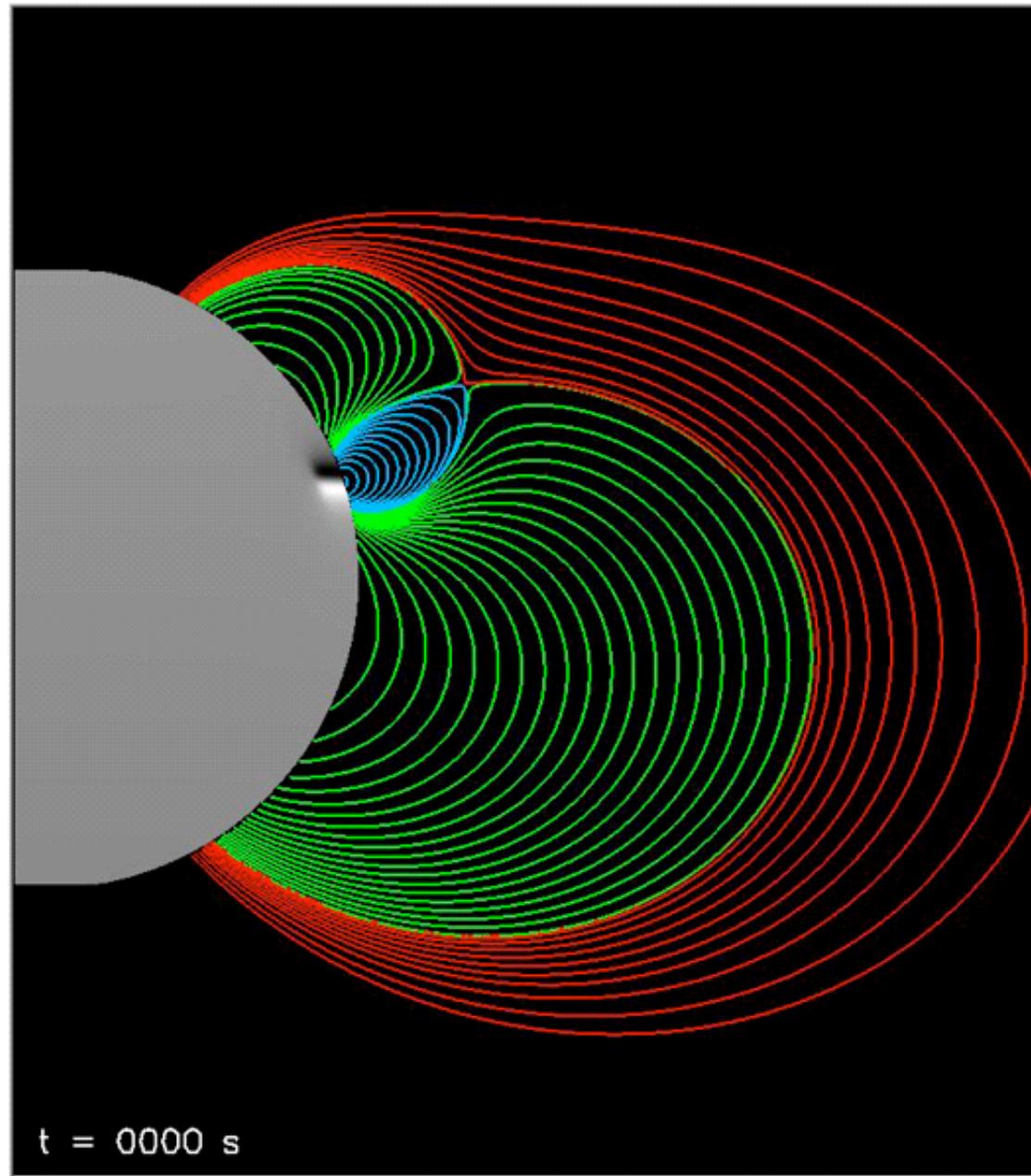


1. line current replaced by quadrupole
2. subcritical twist for helical kink
3. torus center near surface

What is the Trigger Mechanism in the Breakout Model?

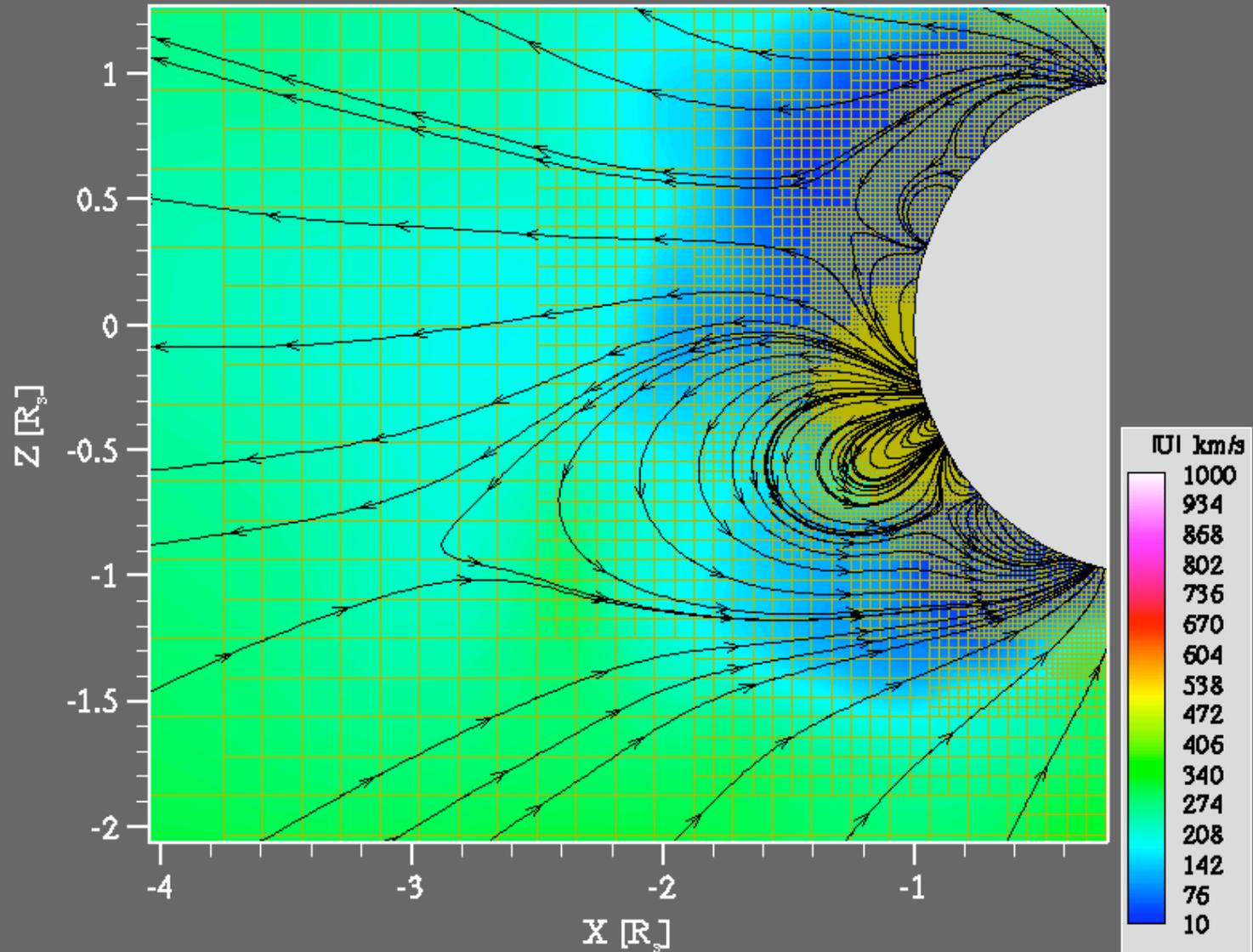


Role of Reconnection in the Breakout Model





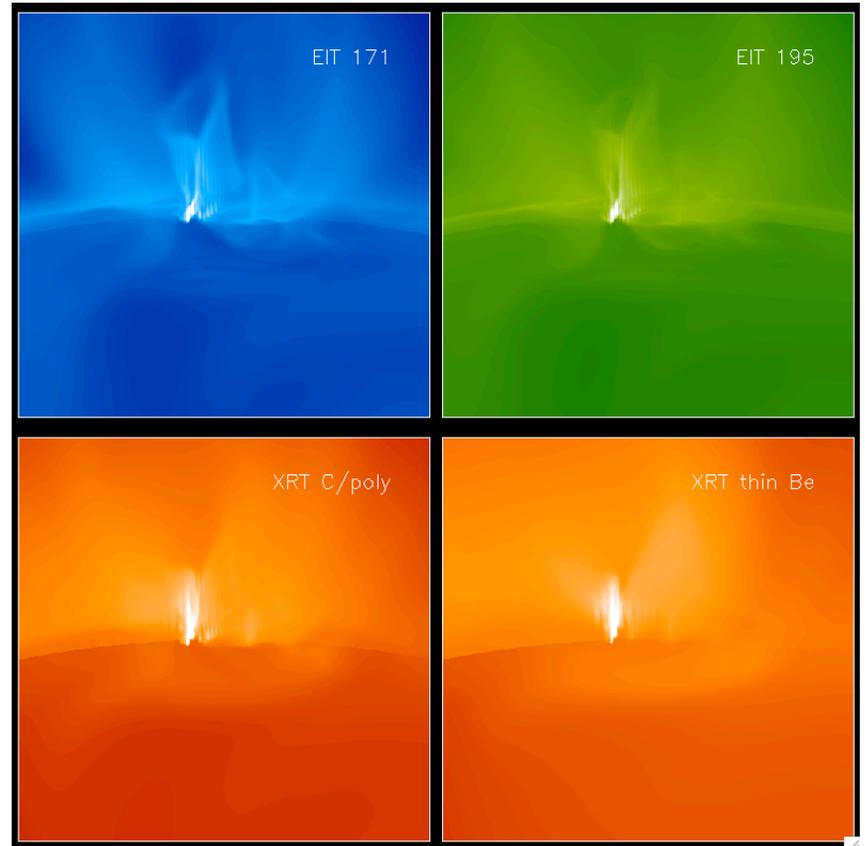
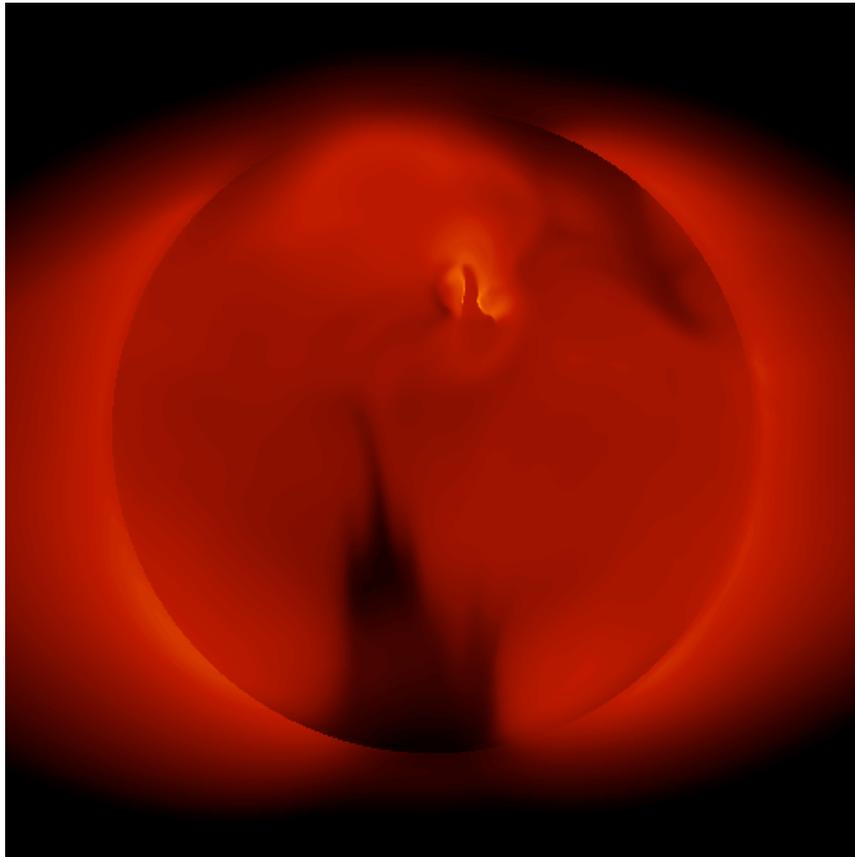
Center for Space Environment Modeling
University of Michigan
Roussev et al. 2004



Simulation of Flux-Rope Eruption in 3D MHD

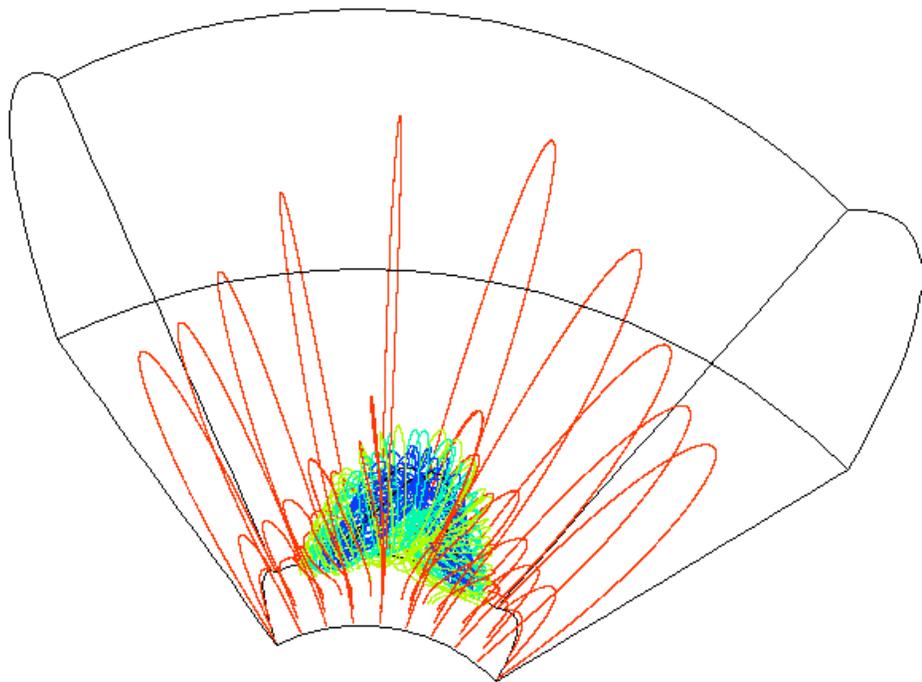
top view (XRT)

side views

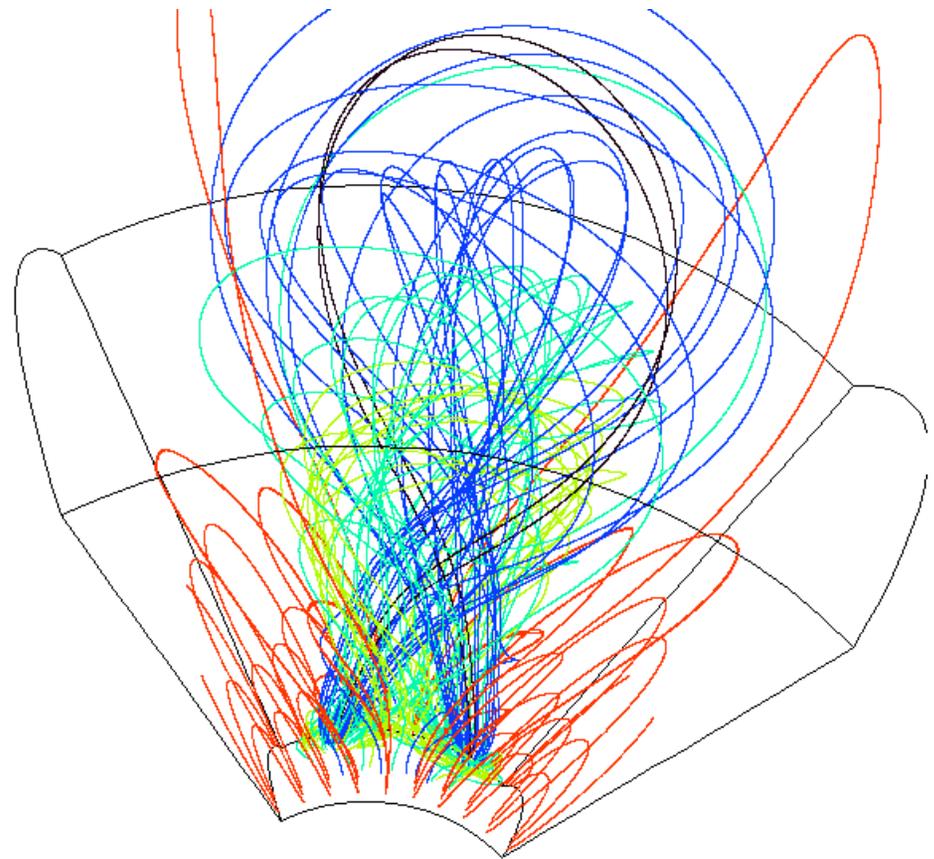


courtesy of John Linker at Predictive Science

Flux Rope Emergence & Eruption



t=143.



t = 94 (R_{\odot}/V_{AD})

3D simulations of Fan & Gibson (2006)

Flux Ropes Are Characteristic of Low β Plasmas

van Ballegooijen & Mackay 2007

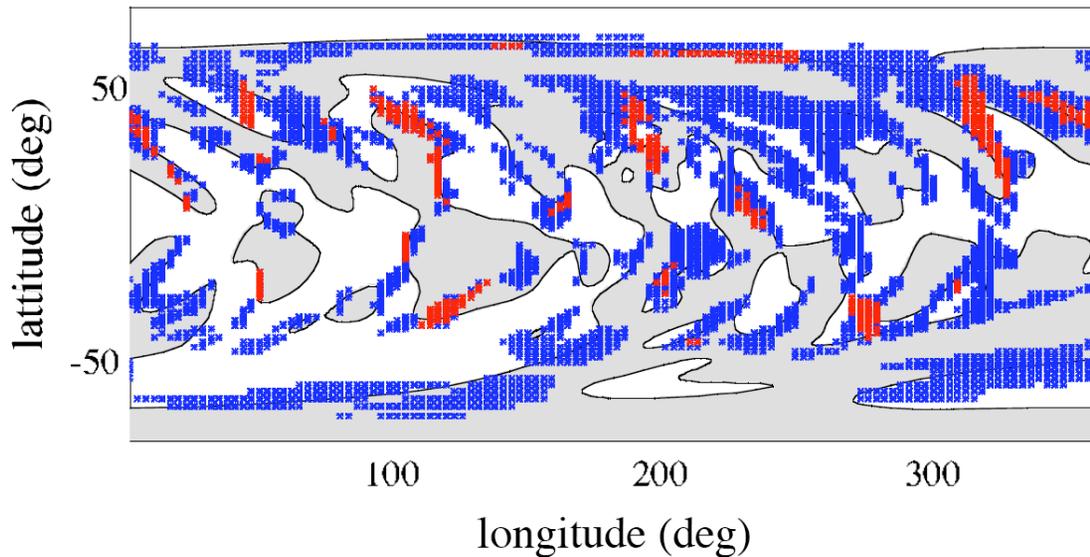
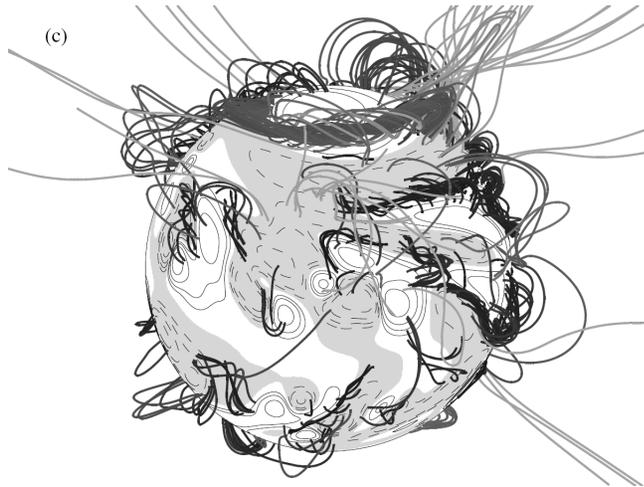
prominence plasma $\beta \ll 1$

$$\nabla P \approx 0 : \mathbf{j} \times \mathbf{B} \approx 0 \quad \mathbf{j} \parallel \mathbf{B}$$

\mathbf{j} along \mathbf{B} produces twist

flux rope defined as enough twist
to produce inverse polarity

(about 1 turn)



blue: flux ropes

red: flux ropes that erupted

Yeates & Mackay 2009

Some Unanswered Questions

1. How are stressed magnetic fields formed?
 - magnetic energy storage —
2. What determines the rate of reconnection?
 - kinetic processes —
 - turbulence —
3. To what extent are flares & CMEs predictable?
 - loss of equilibria —
 - loss of stability —