Energization of Trapped Particles

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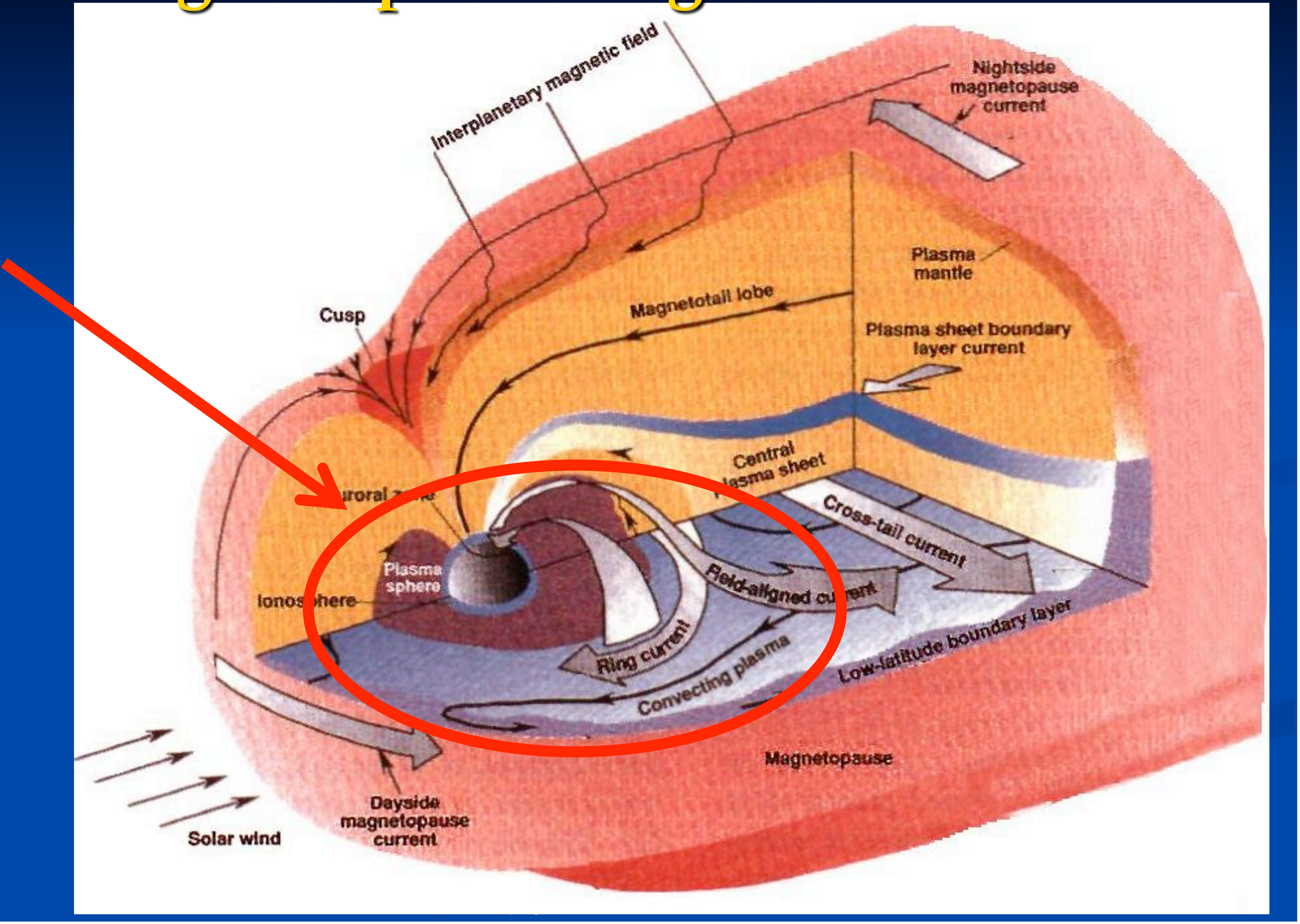
Magnetospheric Regions



The Inner Magnetosphere

- Inner magnetosphere is where space weather matters
  - This is where we fly lots of commercial and military satellites
  - Even the calm times are full of dynamic processes
- There are 3 *main* plasma populations in the inner magnetosphere
  - **Plasmasphere:** contains the mass
  - **Ring current:** contains the energy
  - **Radiation belt:** contains the dangerous particles

First Things First

- Before we talk about the plasma populations
- Let's talk about particle motion again
- In particular: **drifts, invariants, and periodicities**

Particle Motion

- Force on a charged particle in an E and B field
  - Say that E and B are perpendicular
    - Equipotential magnetic field lines…like the inner magnetosphere
  - Two forces:
    - Electric force: $\vec{F} = q\vec{E}$
      - Particle wants to move in the direction of **E**
    - Magnetic force: $\vec{F} = q\left(\vec{v} \times \vec{B}\right)$
      - Particle wants to move in a circle around **B**

Solving this equation

- Two formulas for force on a particle:
  - Total EM force and Newton's law:

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \qquad \vec{F} = m\vec{a}$$

- The steps:
  - Define your system and separate the terms/equations
  - Note that acceleration, velocity, and position are linked
  - Rewrite the equations in terms of position
  - Differential equation system: 2 equations, 2 unknowns
  - Isolate a variable and plug into the other
    - Means taking a lot of integrals and derivatives
  - Use initial conditions to get coefficients

In the Earth's Magnetosphere

- Easy math plasma sheet:
  - Magnetic field of ~ 10 nT and electric field of ~ 0.1 mV/m
  - Drift velocity: $$v_{ExB} = \frac{E}{B} = \frac{10^{-4}\,V/m}{10^{-8}\,T} = 10^4\,\frac{m}{s} = 10\,\frac{km}{s}$$
  - Gyrofrequency: $$\omega = \frac{qB}{m} = \begin{cases} \sim 1\,\frac{rad}{s} & protons \\ \sim 1800\,\frac{rad}{s} & electrons \end{cases}$$
  - Is this drift fast or slow?
    - 1 R_E ~ 6400 km and 1 h = 3600 s
    - Drift is ~ 5.6 R_E/hour
- During storms: things move faster
  - E can be ~1 mV/m and B can be ~1 nT

What about the Inner Magnetosphere?

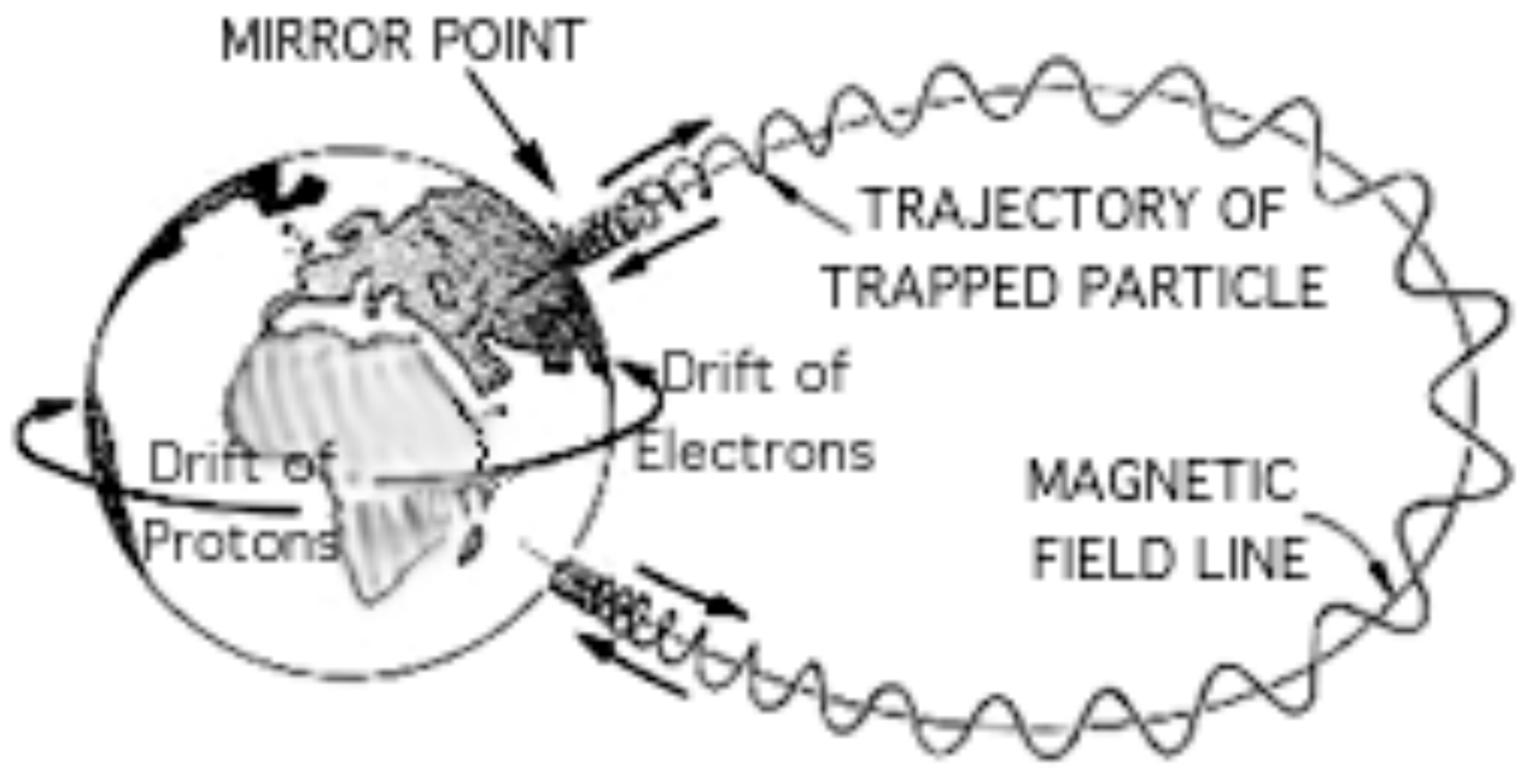
- Drift in the inner magnetosphere is different
  - Magnetic field is stronger and dipolar
    - Closer to the magnetic field source within the Earth
  - Electric fields can be shielded
    - Or intensified by localized FACs into low conductance regions
- What does it mean:
  - The calculation we just did is too simple
  - We should pay attention to motion along B field, too

The One Big Thing to Know About Particle Motion

Forces cause drift, which drives everything else

Particle Motions: Gyration, Bounce, and Drift

- Three basic motions of particles in a strong dipole magnetic field



Adiabatic Invariants

- The three main equations:
- Gyration:

$$M = \frac{W_\perp}{B} = \frac{p_\perp^2}{2m_0 B} = \frac{E\sin^2\alpha}{B}$$

- Bounce:

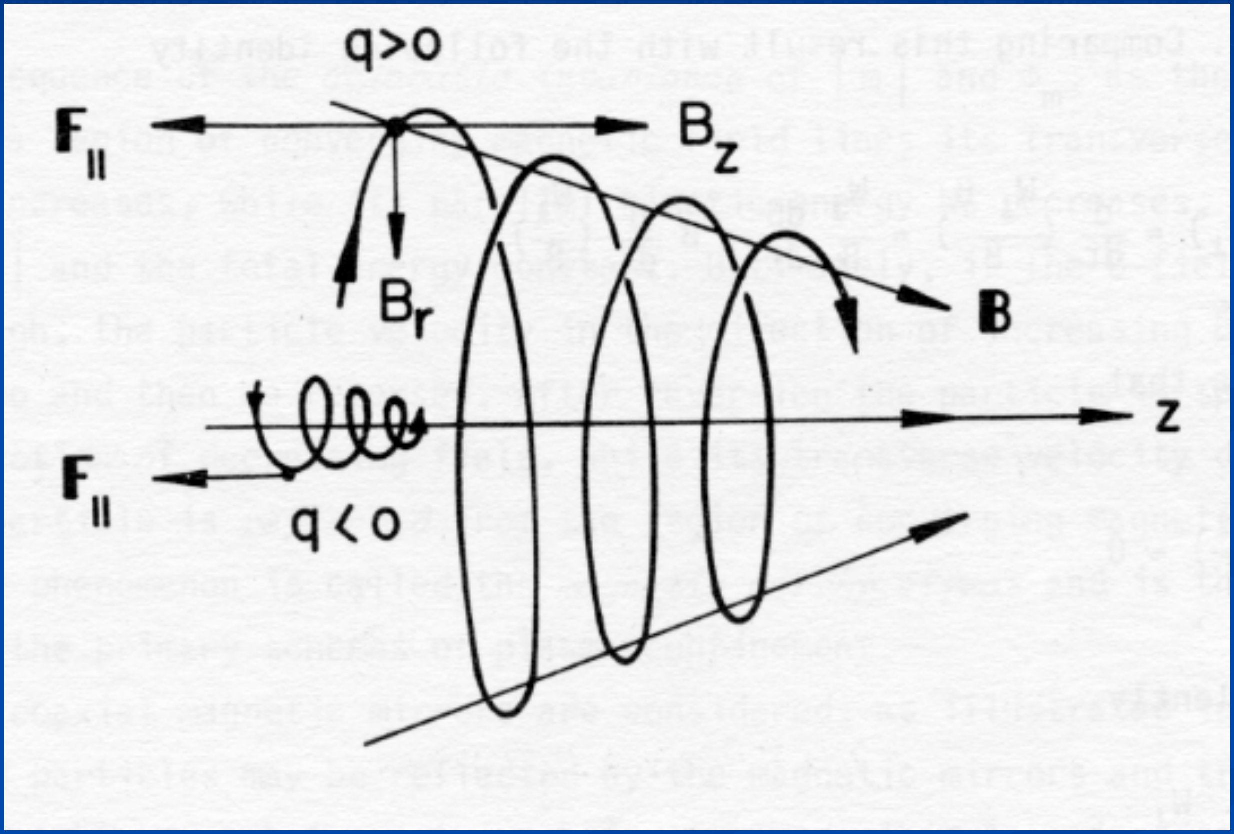
$$J = \oint_{bounce} \vec{p}\cdot d\vec{\ell} = \oint_{bounce} p_\parallel d\ell = m \oint_{bounce} v_\parallel d\ell$$

- Drift:

$$\Phi = \oint_{\substack{drift\\ path}} \vec{A}\cdot d\vec{\ell} = \int_{\substack{drift\\ area}} \vec{B}\cdot d\vec{S}$$

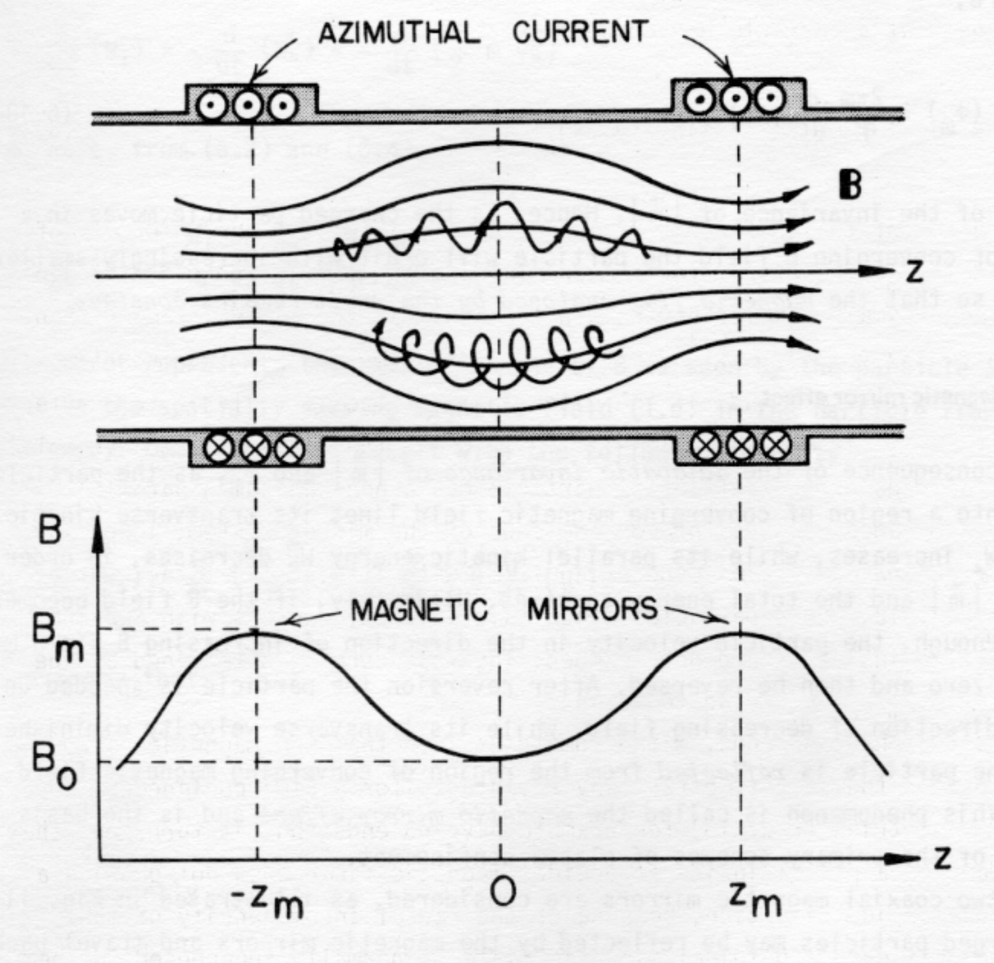
The influence of the first invariant

- Magnetic mirroring



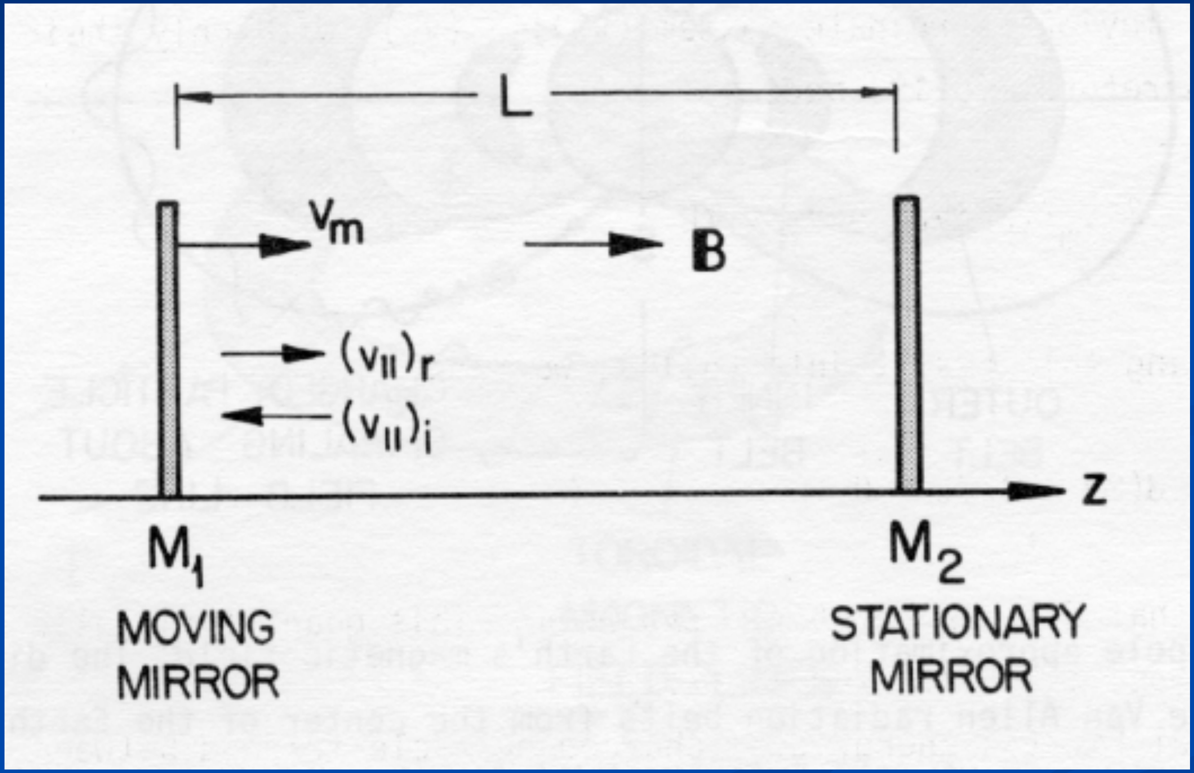
The Magnetic Bottle

- Fusion:
  - z-pinch plasma confinement
- Earth's dipole:
  - Geomagnetic trap near the equatorial plane
- Betatron acceleration
  - Energy scales with B
  - So, an L^3 dependence



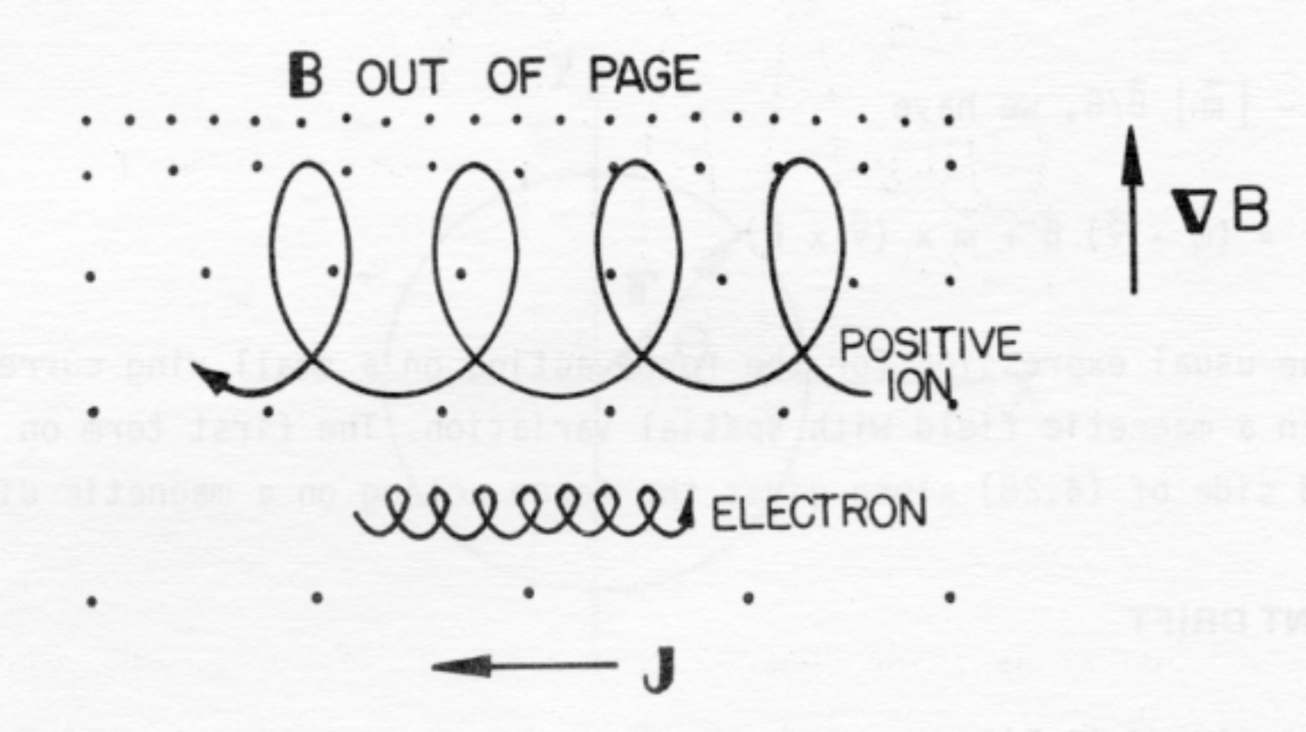
Effect of the second invariant

- First-order Fermi acceleration
  - p (or v) scales with the square root of field line length
  - Energization has an L^2 dependence



Understanding the third invariant

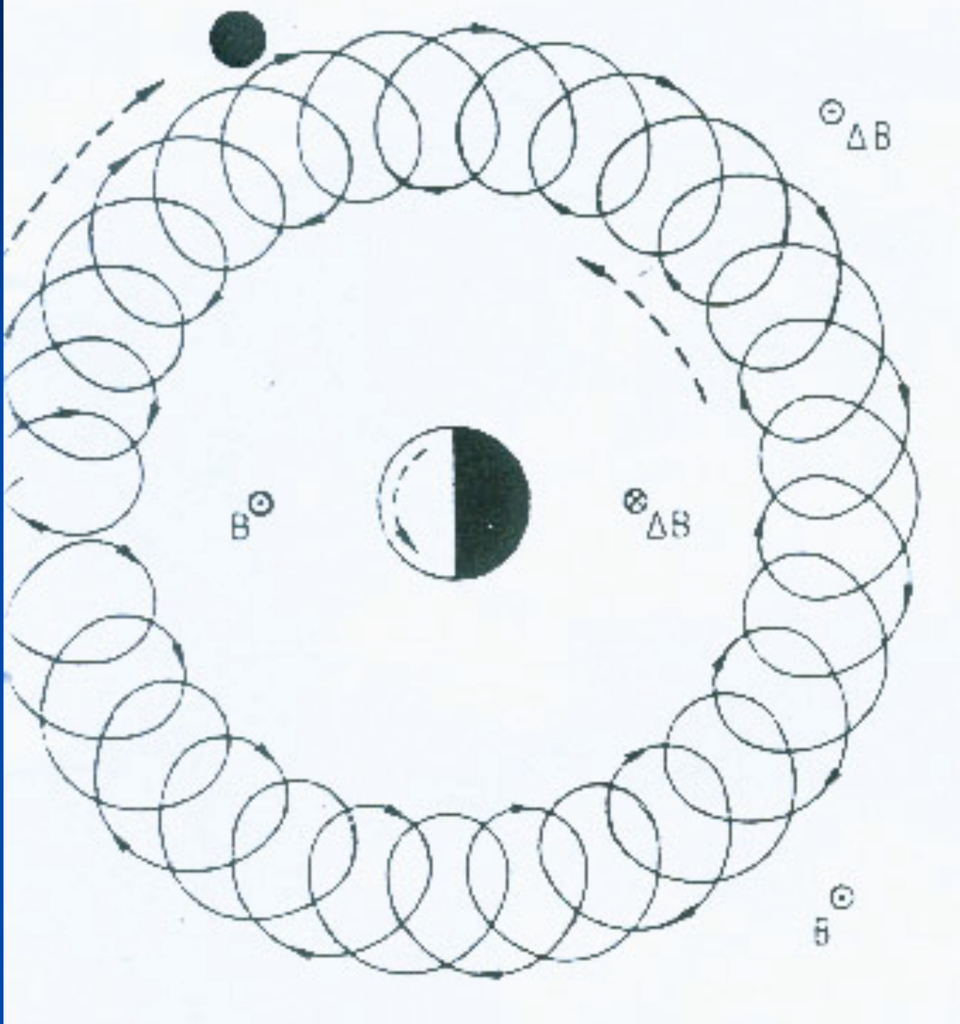
- Grad B drift: bigger particle orbits where B is smaller
- Curvature drift: very similar, and in the same direction



Schematic of proton drift at the equator

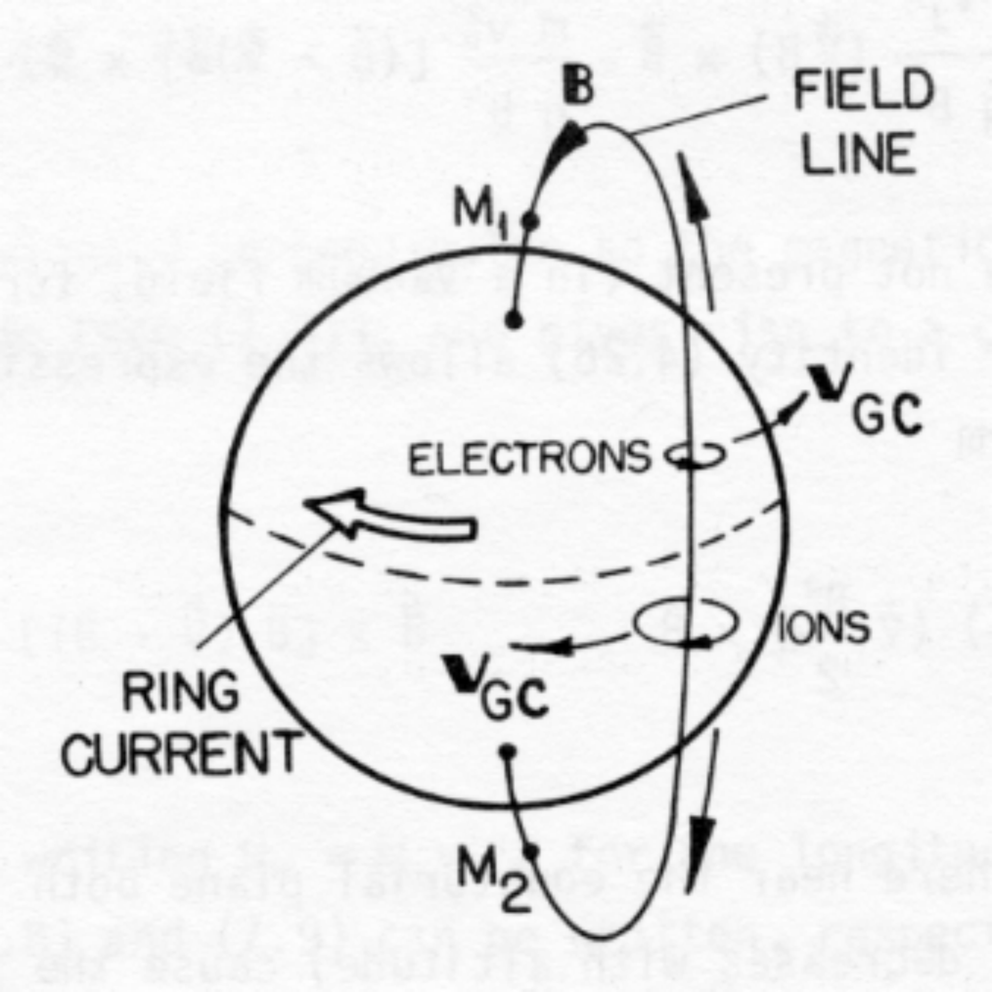
Two currents:
The dashed lines
Outer one is bigger

Net result:
Suppressed B field inside
Enhanced B field outside



Putting it all together

- One picture showing all three invariants
- Result: the westward ring current



Calculating Periods of Motion

- Gyrofrequency and period:

$$\omega_g = \frac{|q|B}{m} \qquad T_g = \frac{2\pi}{\omega_g}$$

- Bounce period:

$$T_b = \oint_{bounce} \frac{d\ell}{v_{\|}} \cong 4LR_E\sqrt{\frac{m}{2E}}\left[1.38 - 0.32\left(\sin\alpha_0 + [\sin\alpha_0]^{1/2}\right)\right]$$

  - where E is the particle energy and α_0 is the particle equatorial pitch angle, which ranges from 90° down to the loss cone edge:

$$\sin\alpha_{0,LC} \approx [\cos\lambda_m]^4 \qquad L = \frac{1}{[\cos\lambda_m]^2}$$

More Period Calculations

- Drift period:

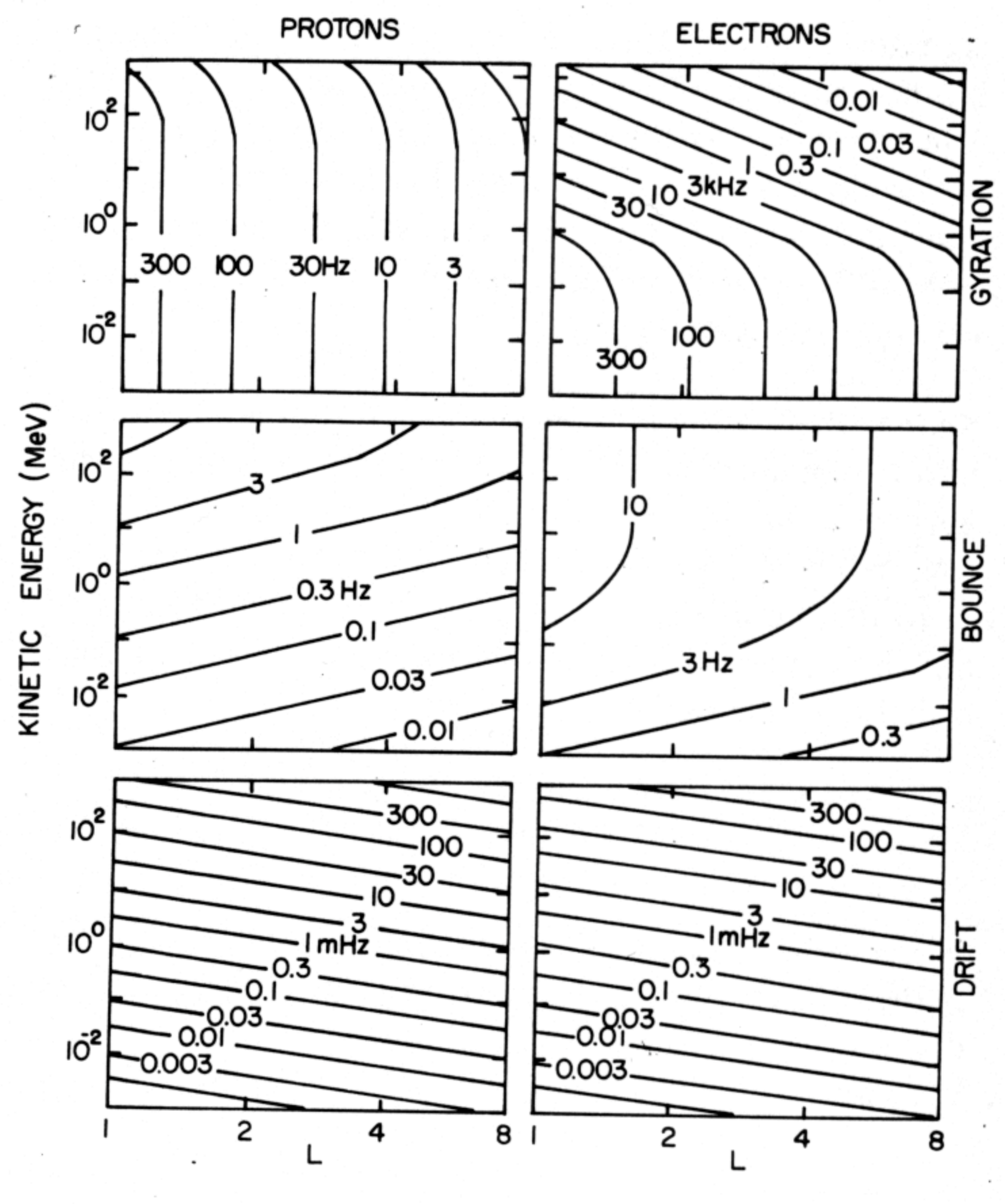
$$T_d = \oint_{drift} \frac{ds}{v_d} \cong \frac{1.43 K_t c^2}{L(1 + 0.42 \sin\alpha_0)} \left(\frac{m}{2E} \right)$$

  - where:

$$K_t = \begin{cases} 1.03 \cdot 10^4 \text{ for electrons} \\ 5.66 \text{ for protons} \end{cases}$$

$$c = 3 \cdot 10^8 \, m/s$$

$$B = \frac{3.12 \cdot 10^{-5} T}{L^3}$$



These are for $\alpha_0=90°$

In detail, the periods depend on energy and altitude

The One Big Thing to Know About Invariants and Periodicities

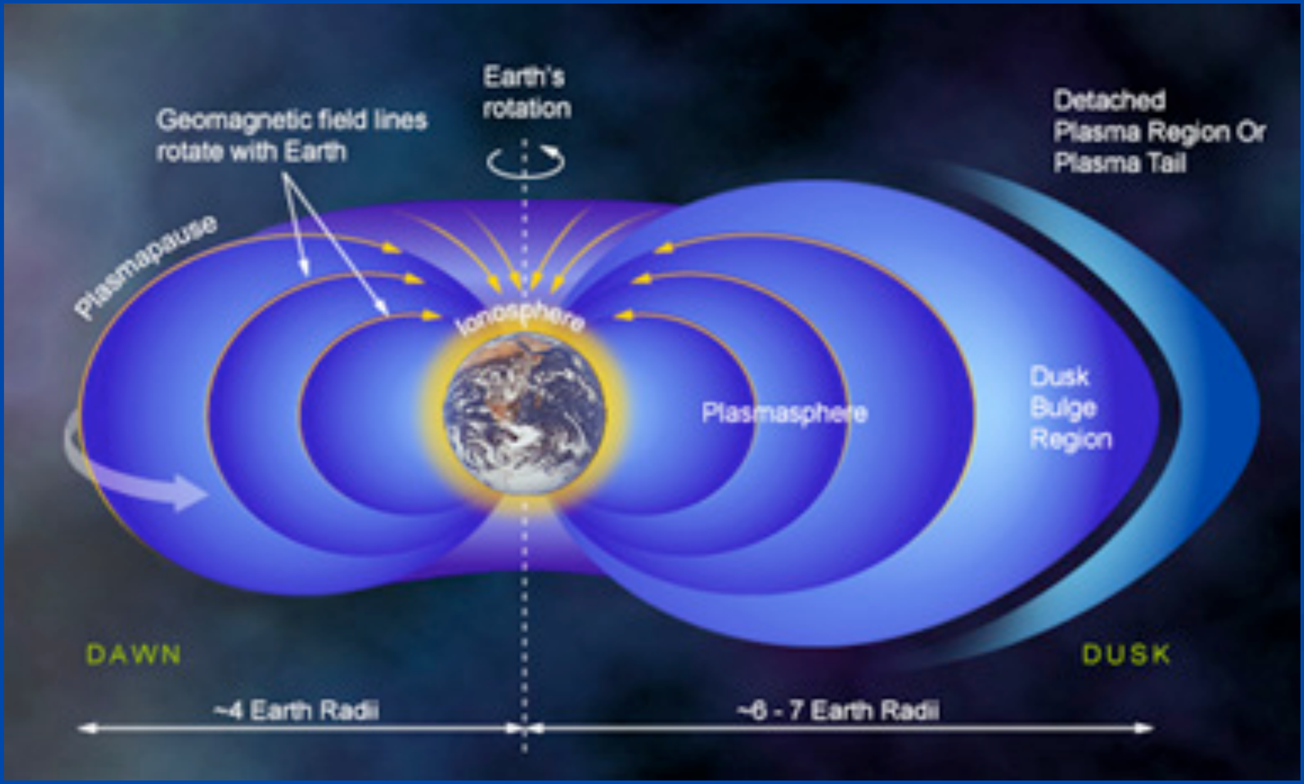
Nature hates a change in magnetic flux through a particle trajectory

Now, on to the plasma populations

- Plasmasphere, ring current, and radiation belts
- Oh my!
- There are others, too…

Schematic of the plasmasphere

- At lower latitudes, corotation dominates over convection, and the magnetic bubble fills with ionospheric material

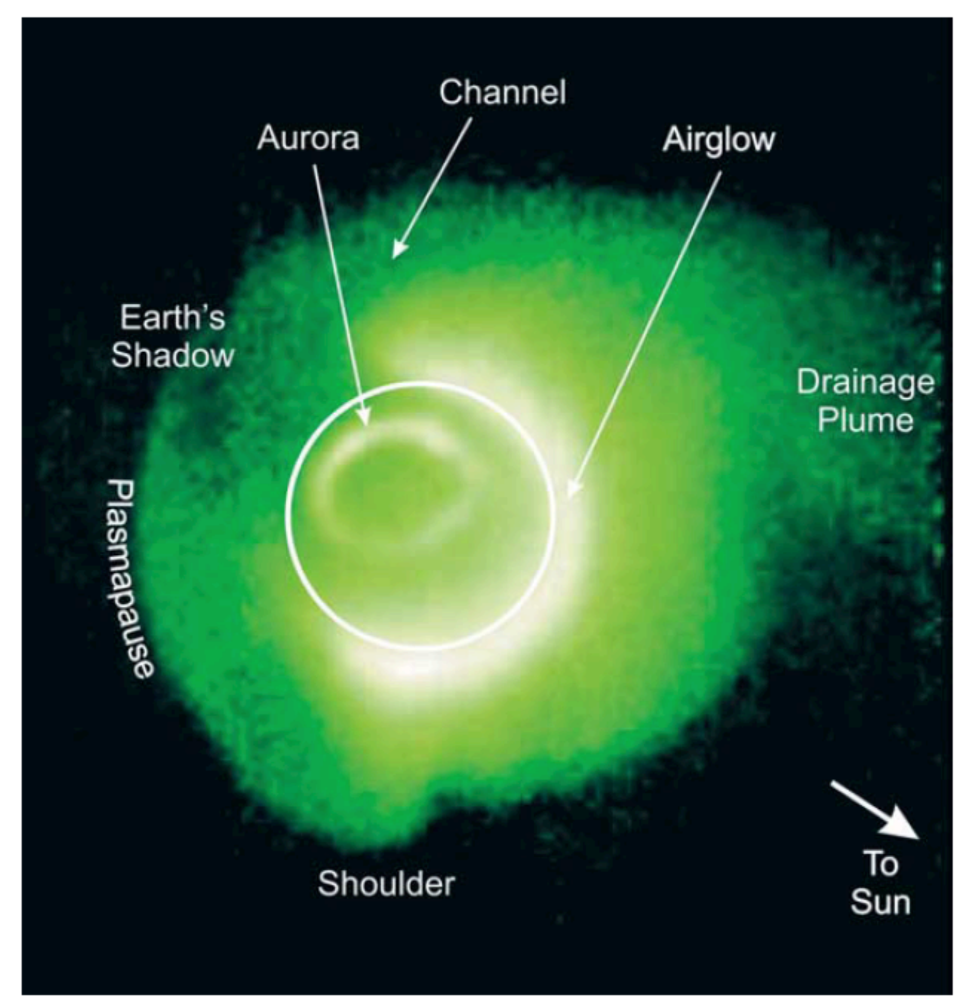


Basic Definition: Plasmasphere

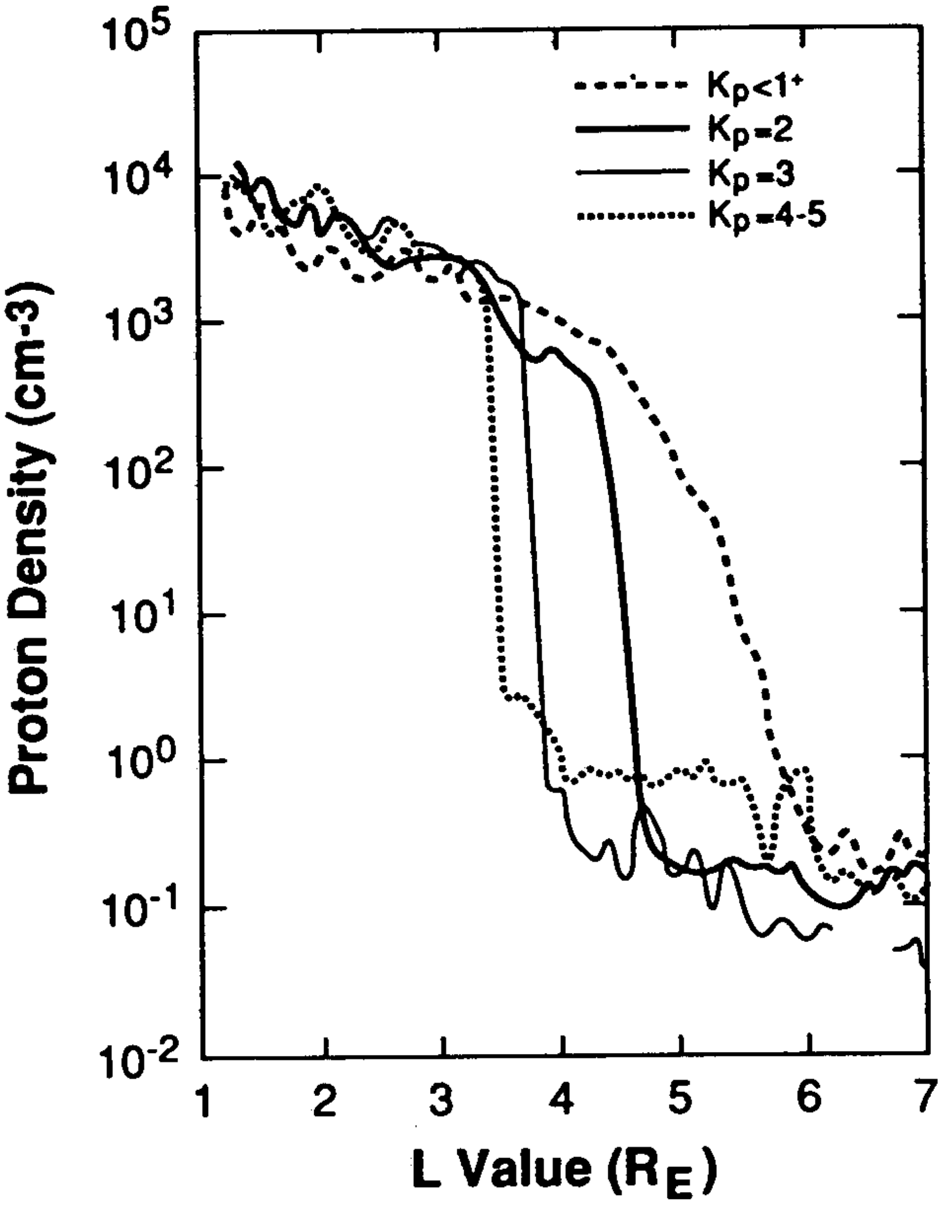
- Cold: Less than 1 eV, maybe up to 10 eV
- Dense: 100s-1000s cm^{-3}, lower out near geos.
- Ionospheric: source is the subauroral ionosphere
- Mostly Protons: some helium and bit of oxygen
  - Oft-quoted composition: 77% H^+, 20% He^+, and 3% O^+
- E-field dominated: spatial extent governed by magnetospheric electric field time history
- Two major losses: the drainage plume or the ionosphere
  - Increased convection can strip off the outer plasmasphere
  - On the nightside, ions fall back into the atmosphere
- Importance: dominates the mass density of the inner magnetosphere

Global Morphology

- IMAGE EUV has shown the plasmasphere to be a lumpy and bumpy creature
    - Tracer of the time-history of inner mag. fields (mostly E, also a bit by B variations)

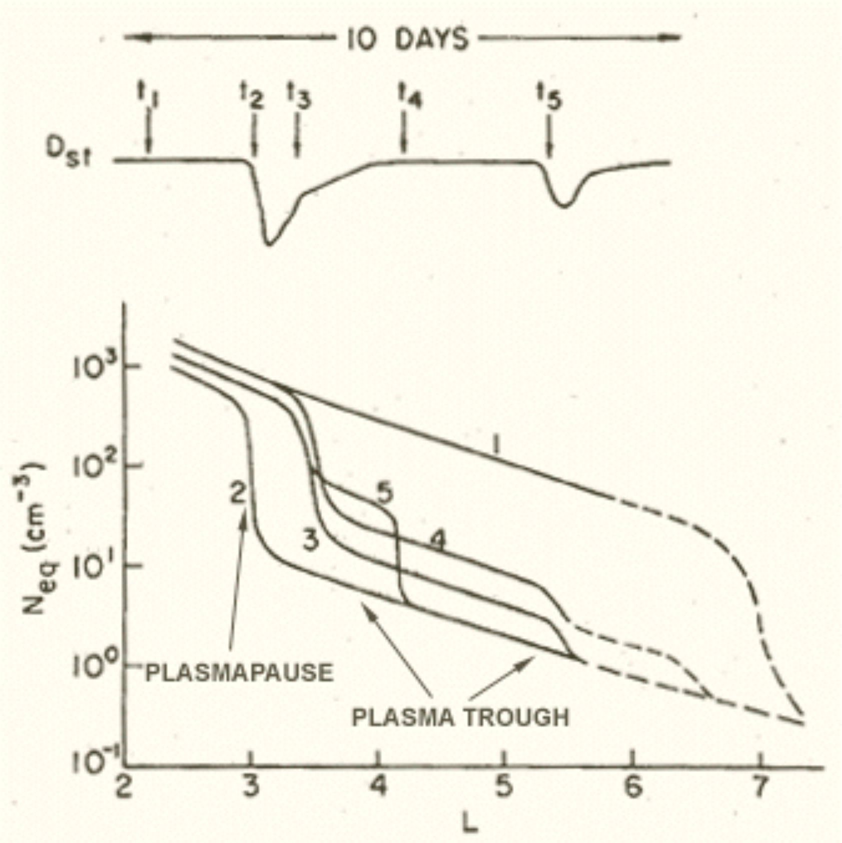


The plasmapause moves



Reaction of the nightside plasmasphere to geomagnetic activity.

Similar result, different source



Total Mass Content of the Plasmasphere

- Plasmasphere dominates the mass of the inner magnetosphere...just how big is it?
- Total number density integral:

$$N = \int_V n d^3 v$$

- What are d^3v and n?
  - Equatorial plane area of $\pi L^2 dL$ from L=1 to L_{pp}
  - Field line length is about 2L
  - Area contracts away from equator proportional to B^{-1}
  - Assume density along field line proportional to B
    - Cancels the B^{-1} of the cross sectional area decrease
  - Density in equatorial plane: assume $n=n_0 L^{-a}$
- Total mass: multiply by average kg/particle
  - Assume all protons or some mixture of H^+, He^+, and O^+

The One Big Thing to Know About the Plasmasphere

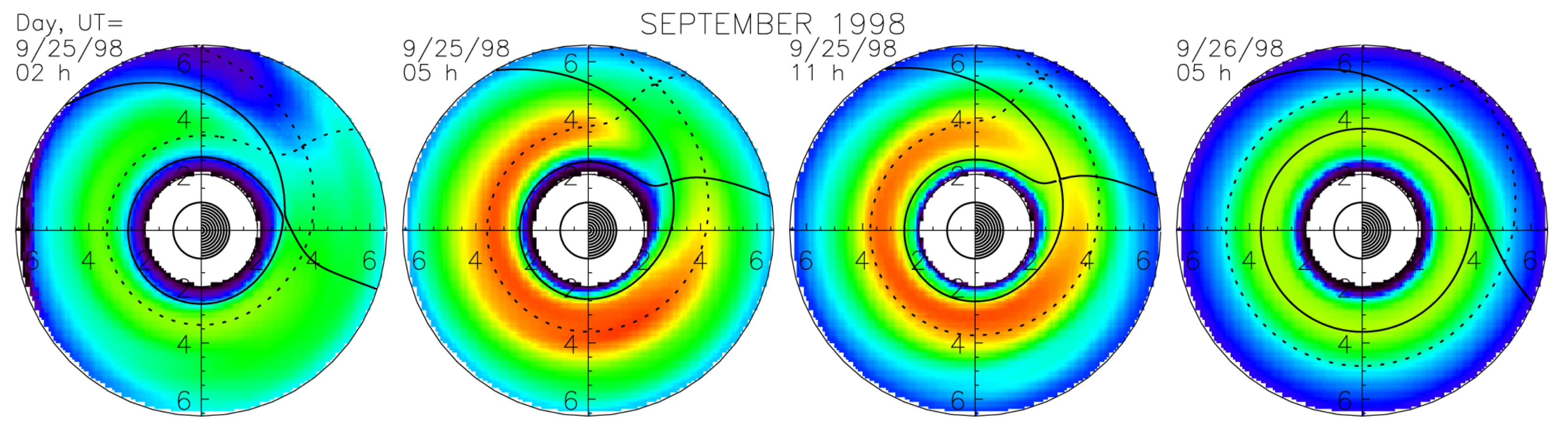
The plasmapause

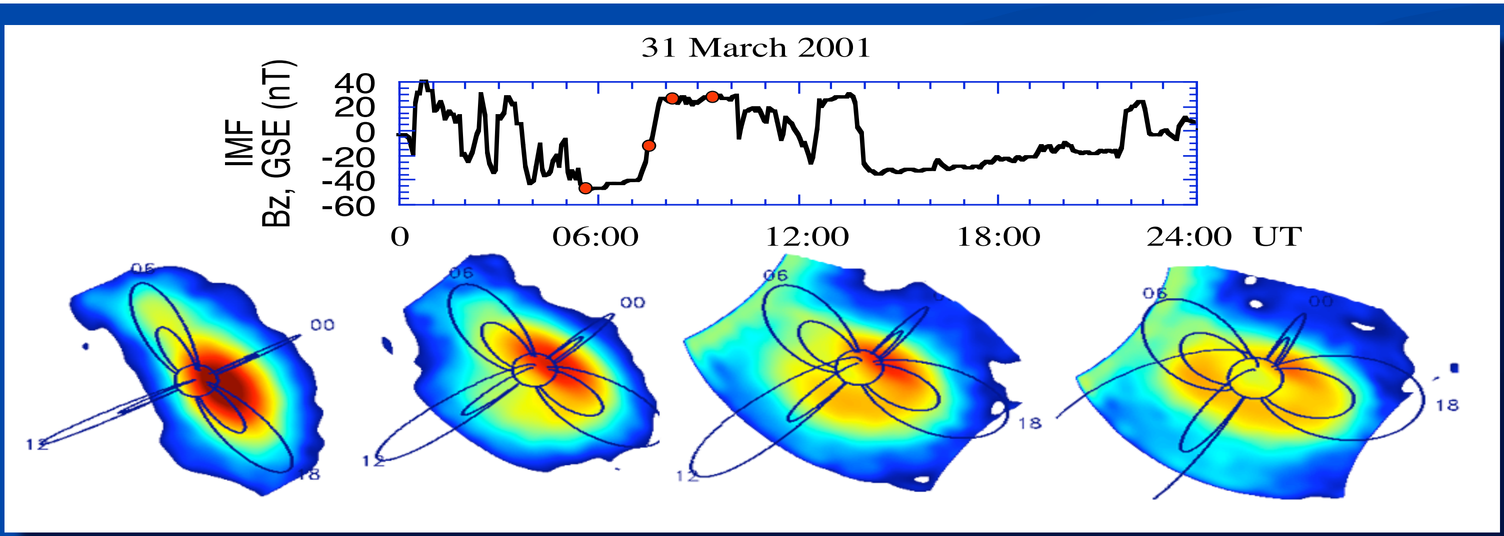
$\neq$

The Alfven boundary

Ring Current Morphology

- The ring current is not a ring during storms





Basic Definition: Ring Current

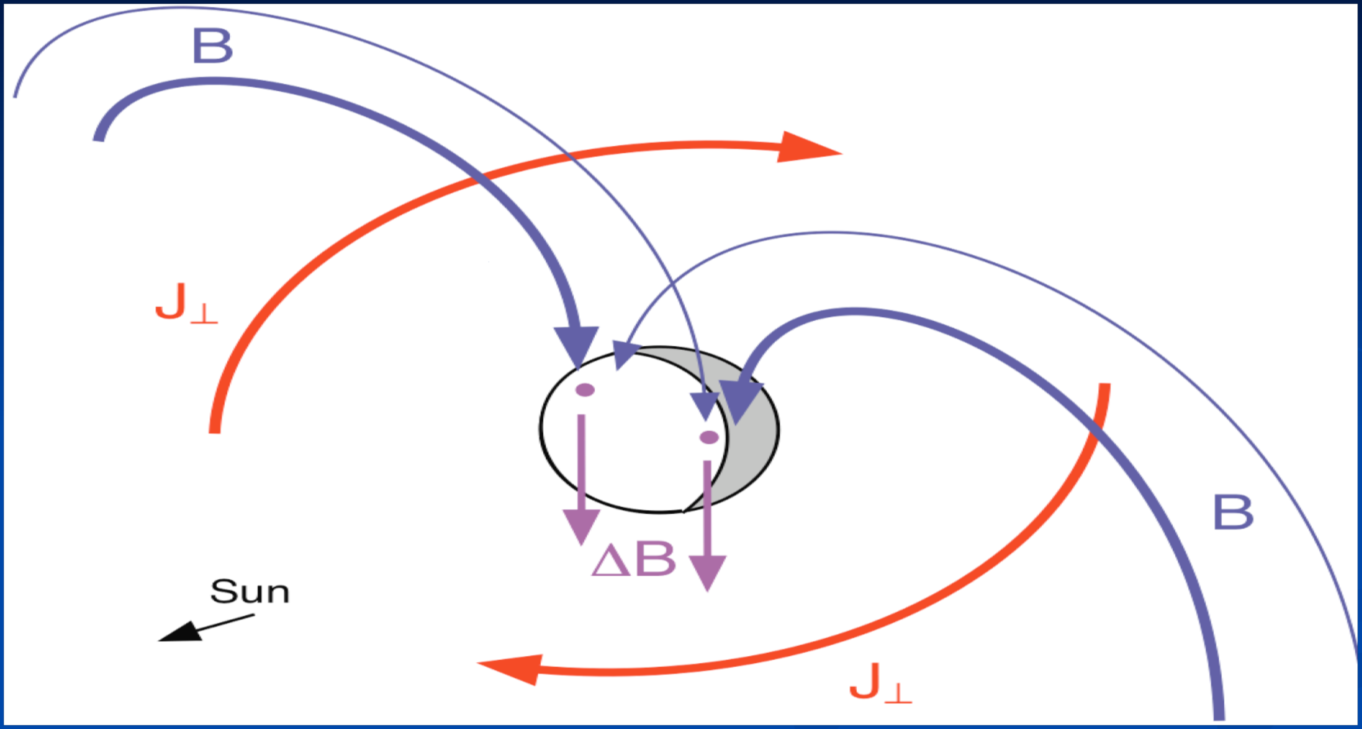
- Hot: 1-400 keV
- Tenuous: quiet, 1 cm^{-3}; active, maybe 10s cm^{-3}
- Plasma sheet: source is near-Earth magnetotail, wherever that comes from
- Mostly Protons: During big storms, O^+ can dominate
- Complicated Drift: E-field, B-field, Gradient-curvature terms
- Two major losses: Flow through or charge exchange
  - They drift out of the inner magnetosphere
  - They collide with the extended upper atmosphere of Earth
- Important: Dominates the energy density of the inner magnetosphere

The Biot-Savart Law

$$\Delta \mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_V \frac{\mathbf{J}(\mathbf{r}') \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} d\mathbf{r}'$$

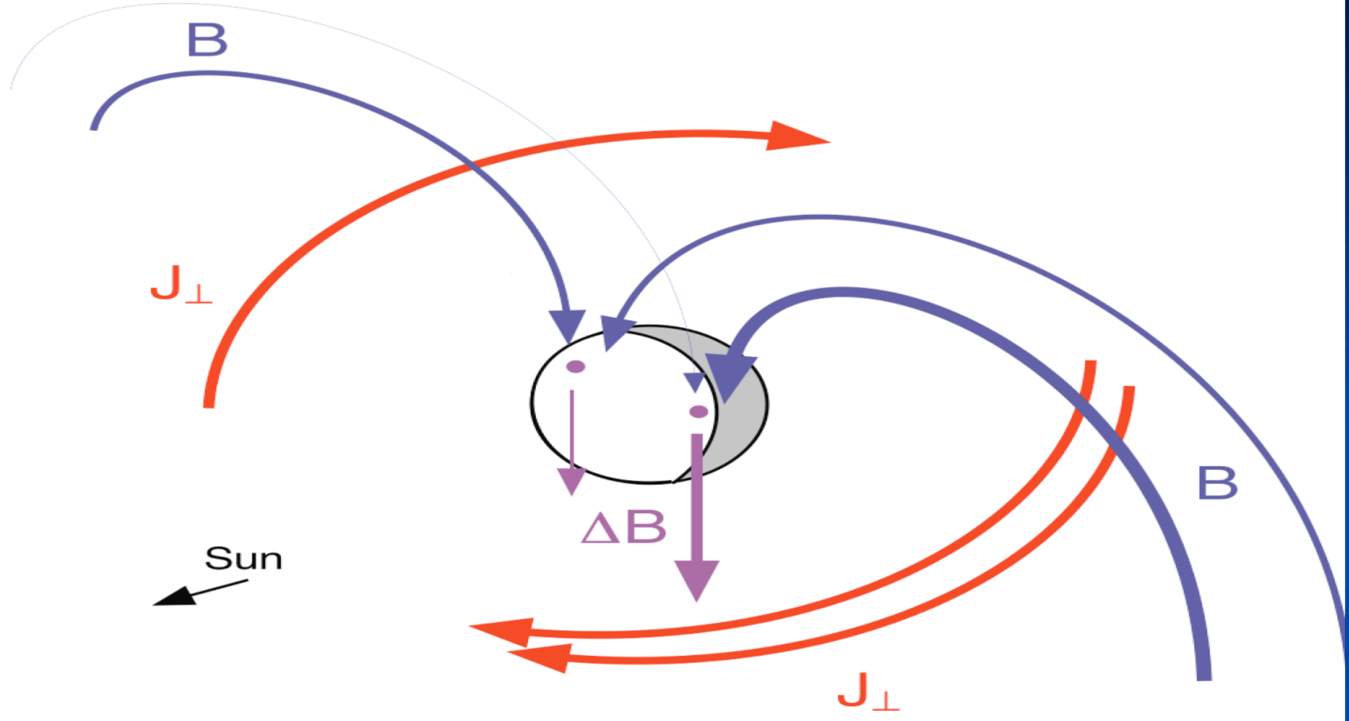
- Integral form says perturbation is:
  - Proportional to intensity of current density
  - Proportional to the volume of space filled by the current
  - Proportional to angle between current and relative position vector
  - Inversely proportional to the square of the relative distance
    - But the exponent is 3 in the equation… ?

Symmetric Ring Current



- RC-B Relationship
  - Right-hand rule (Biot-Savart Law): westward current produces a southward magnetic field at Earth
- RC-ΔB Relationship
  - Symmetric current produces a symmetric perturbation

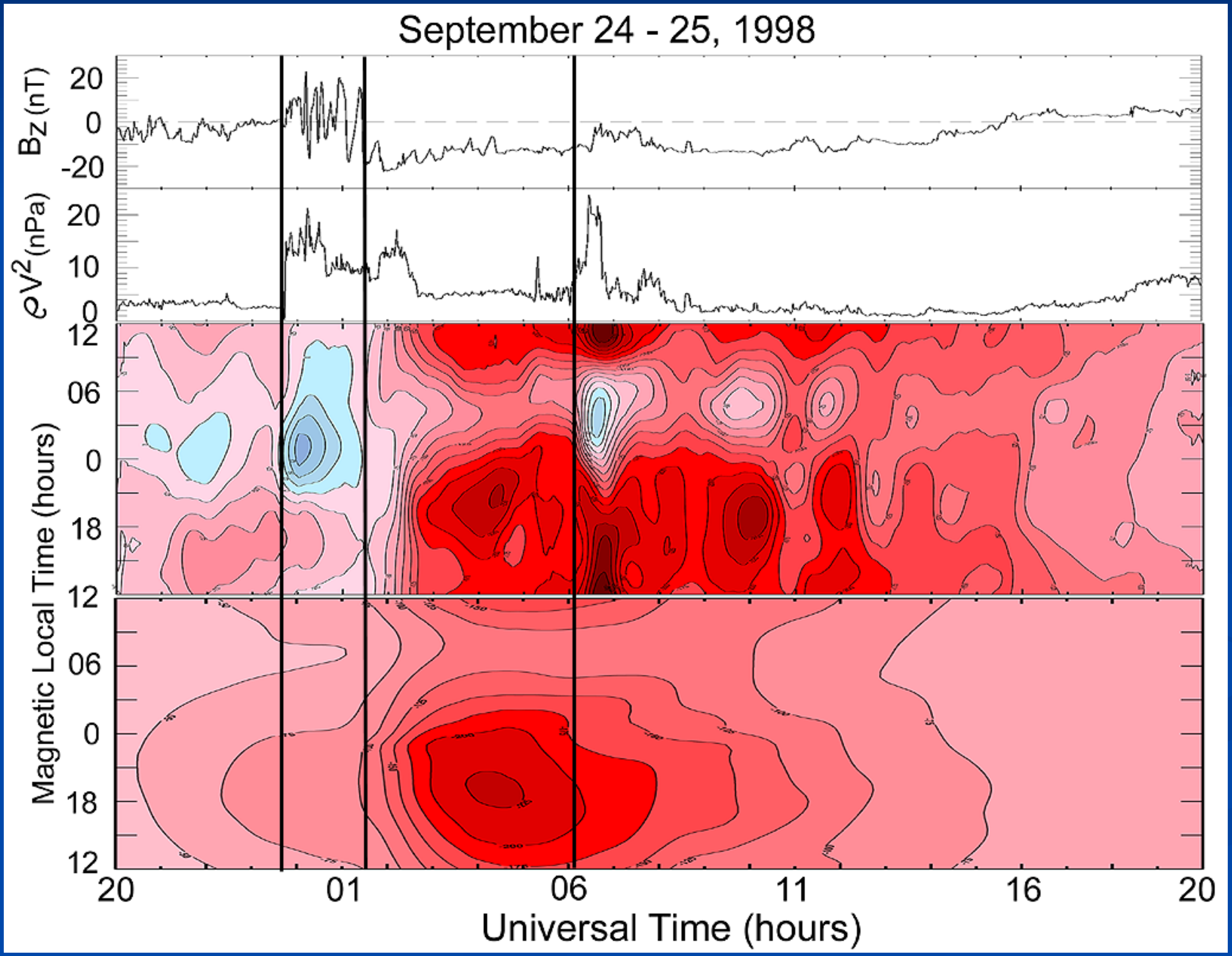
Asymmetric Ring Current



- RC-ΔB Relationship
  - Weaker RC at some local time makes the perturbation asymmetric
  - A completely asymmetric RC will still produce a symmetric component to the perturbation

Ground-based measurements of ΔB

- LT-UT magnetic perturbation maps



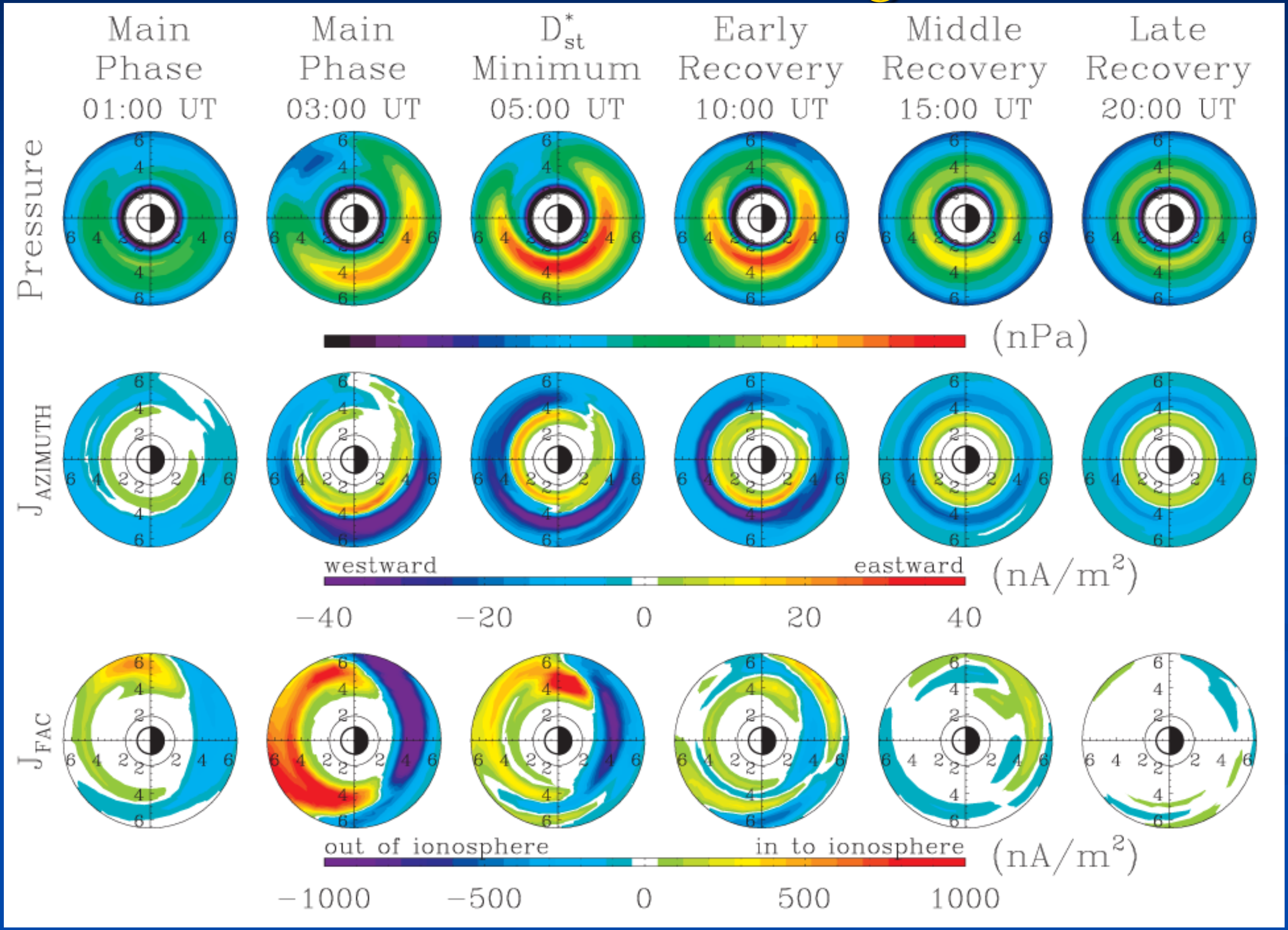
Solar wind

Data

Model

Ring Current Simulation Results

- Pressure and currents from a ring current model



Total Energy Content of the Ring Current

- Ring current dominates the energy content of the inner magnetosphere...just how big is it?
- Total energy integral:

$$W = \int_V n\overline{E}d^3v$$

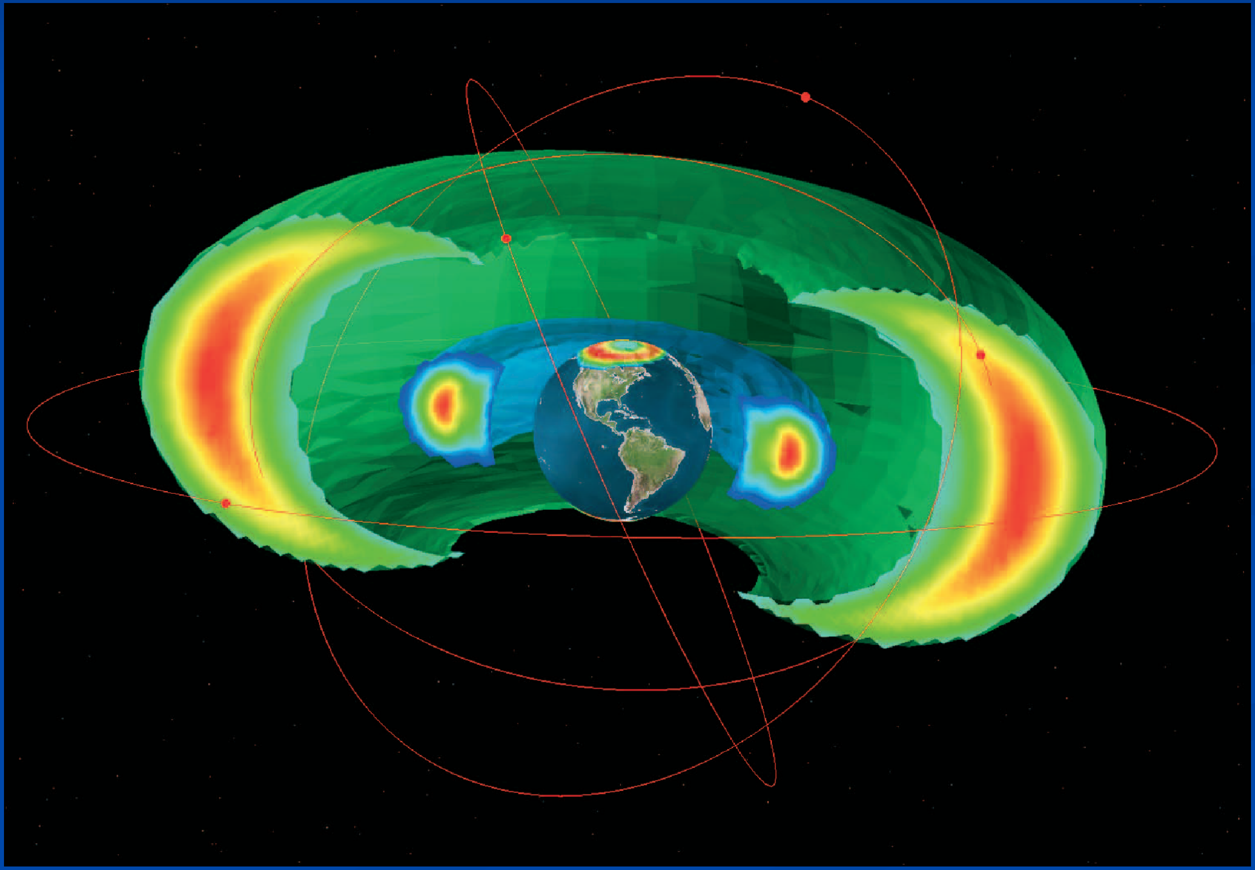
- What are d^3v, n, and E-bar?
  - Equatorial plane area of $\pi L^2 dL$ from $L=L_{min}$ to L_{max}
  - Take it to be a slab/wedge of thickness L
  - Assume a constant density everywhere in the slab
    - Density in equatorial plane: constant or some function of L
  - Assume a constant average energy (or some function of L)
- Composition?
  - Assume all protons or some mixture of H^+, O^+, and e^-

The One Big Thing to Know About the Ring Current

The ring current is usually not a ring

View of the Radiation Belts

- Two belts: inner and outer
- Slot region: severe losses at that altitude

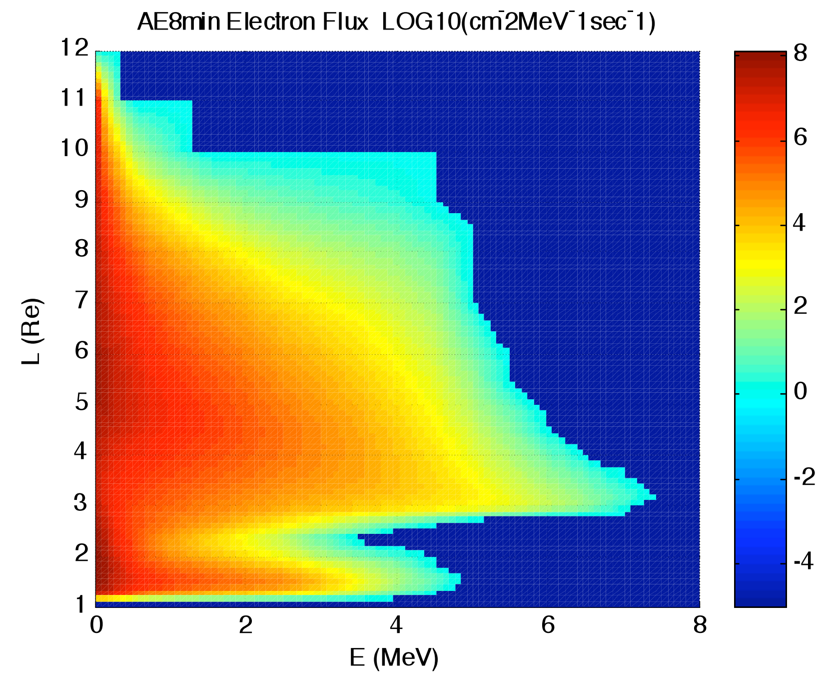


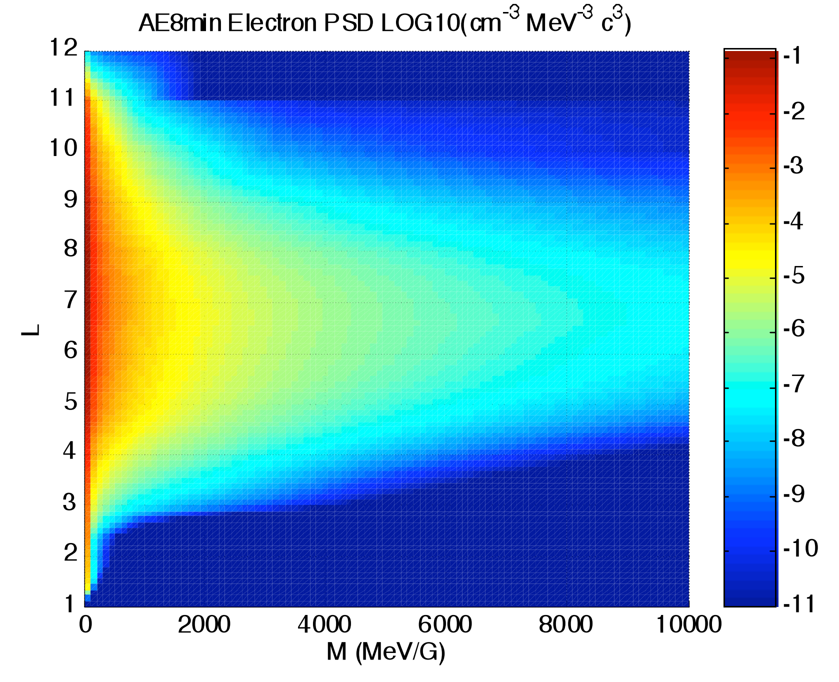
Basic Definition: Radiation Belts

- Extremely hot: 100s of keV to MeV
- Extremely tenuous: <<1 cm^{-3} all the time
- Plasma sheet/heliosphere: source is either
  - Energetic particles from the near-Earth magnetotail
  - Locally accelerated ring current particles
  - Captured SEPs or cosmic rays (or GCR byproducts)
- Mostly electrons: H^+ is significant in the inner belt
- B-field dominated: Topology governs trajectories
- Lost by wave interactions: Eventually scattered out of their stably trapped orbits into the atmosphere
- Important: Dominates the reasons for spacecraft anomalies, damage, and failures

The AE-8 Model (solar min values)

- AE-8 and AP-8: Engineering models for spacecraft designers

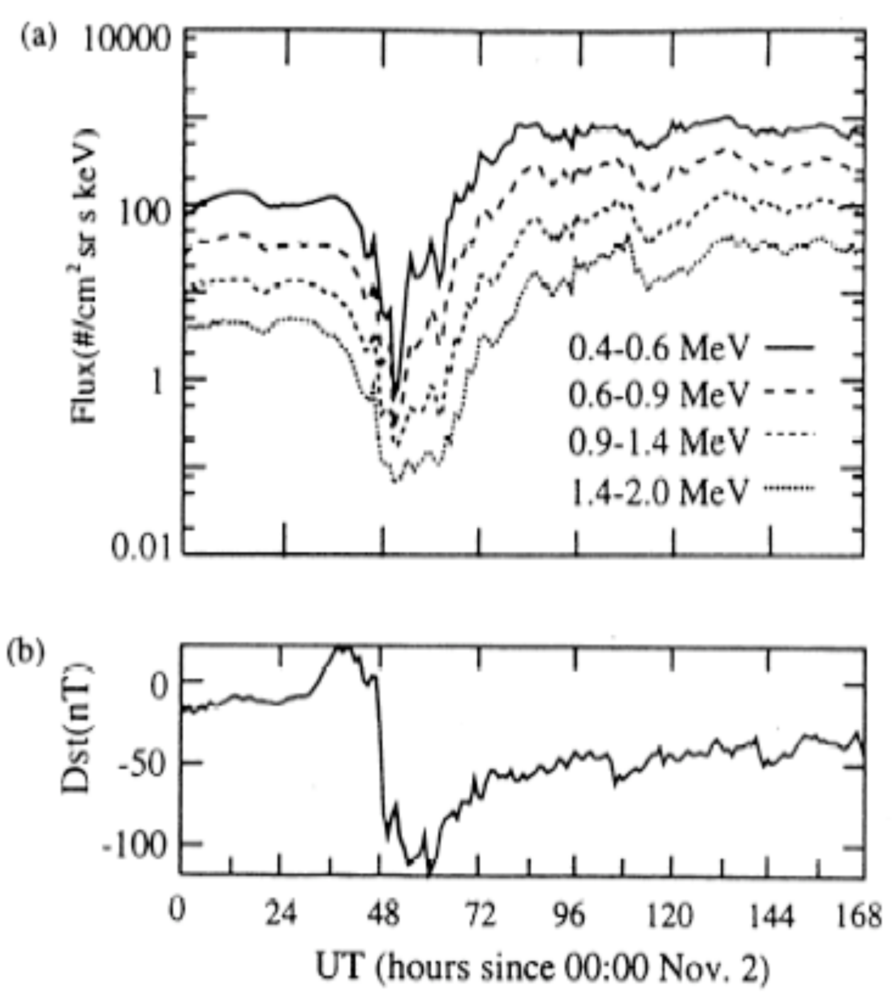




$$PSD = \frac{Flux}{p^2}$$

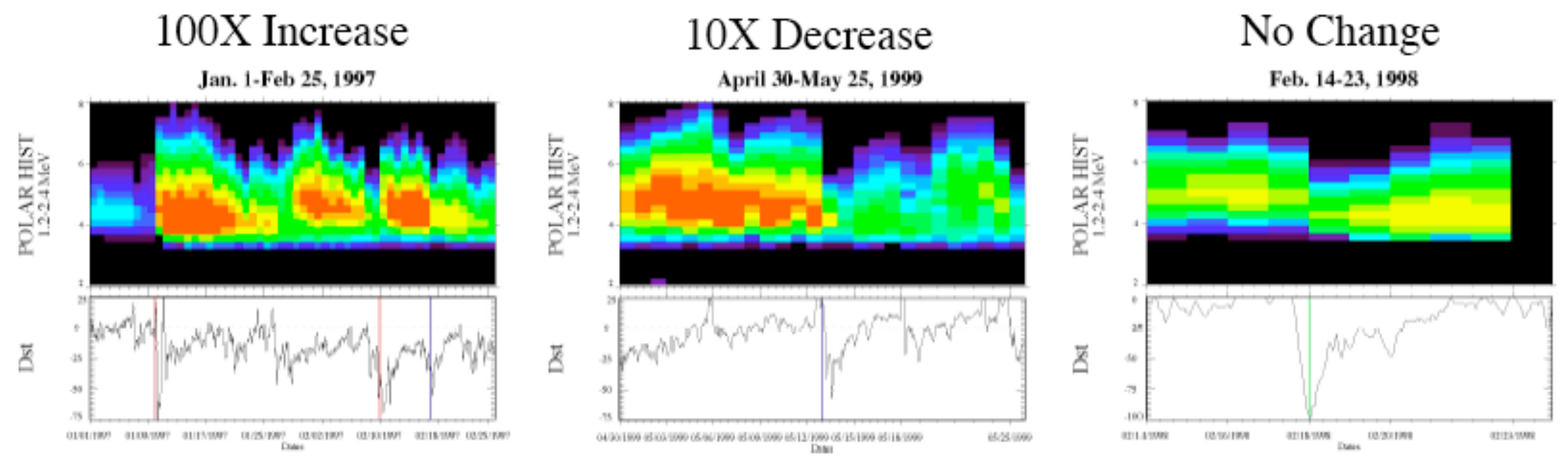
Radiation belts are modulated by the ring current through the B-field

- Standard response during magnetic storms:
  - The Dst effect
  - Flux dropout due to inflation of magnetic field
- If no other losses, flux should fully recover



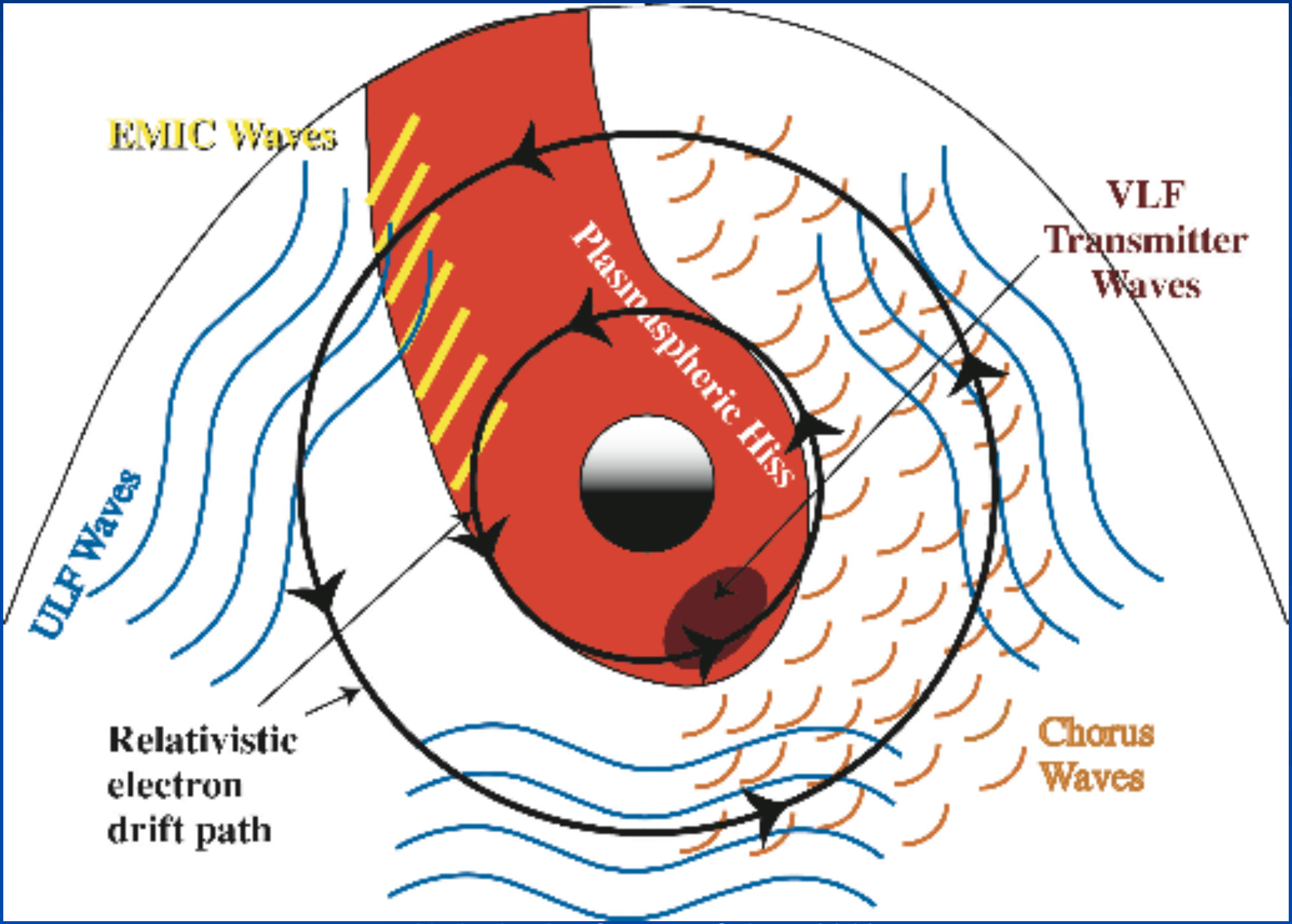
Radiation belts have their own drivers, related but separate from ring current

- Magnetic storms are ring current increases
- Radiation belts can increase, decrease, or show no change after a such a storm



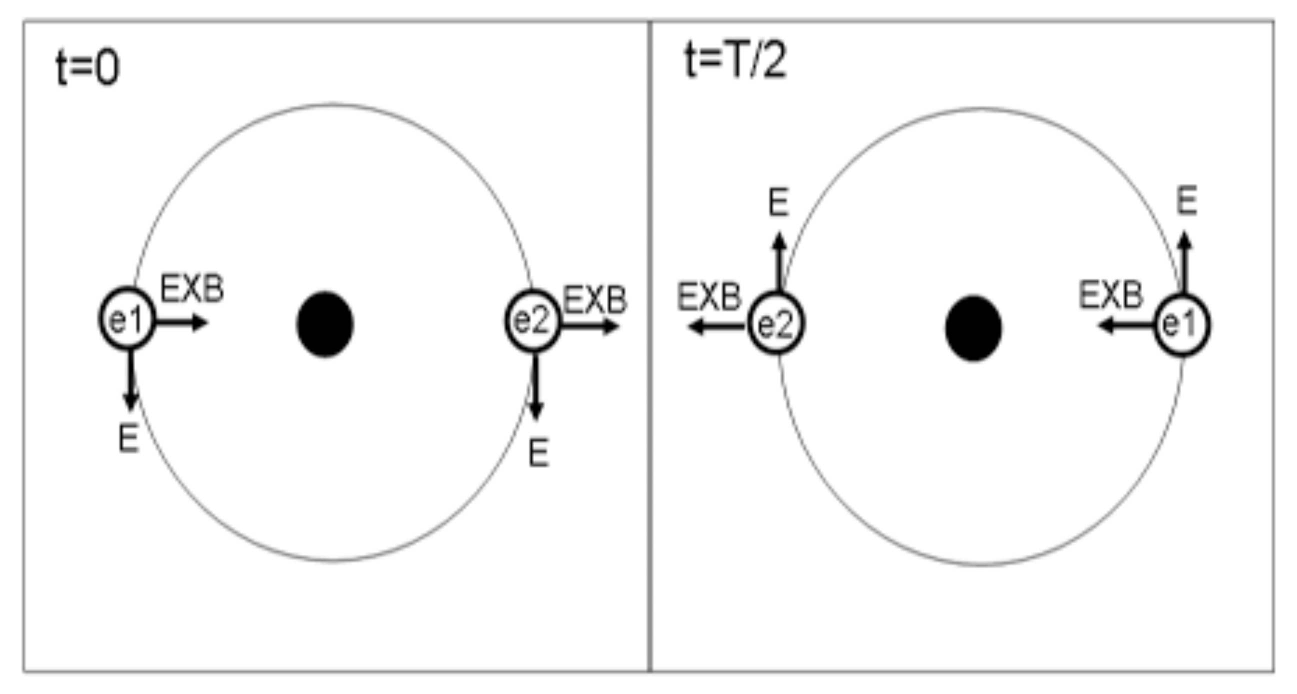
Plasma Waves and RB Electrons

- Plasma waves are critical for RB e- dynamics



Radial Diffusion

- Drift period of the particle in resonance with the wave frequency
  - Or multiples of one or the other frequency
  - Happening on the mHz scale (tens of minutes)



Calculating Radial Diffusion

- Energetic plasma sheet electrons pushed inward
- The basic radial diffusion equation:

$$\frac{df}{dt} = \left(L^*\right)^2 \frac{\partial}{\partial L^*}\left[\frac{D_{LL}}{\left(L^*\right)^2}\frac{\partial f}{\partial L^*}\right] - \frac{f}{\tau}$$

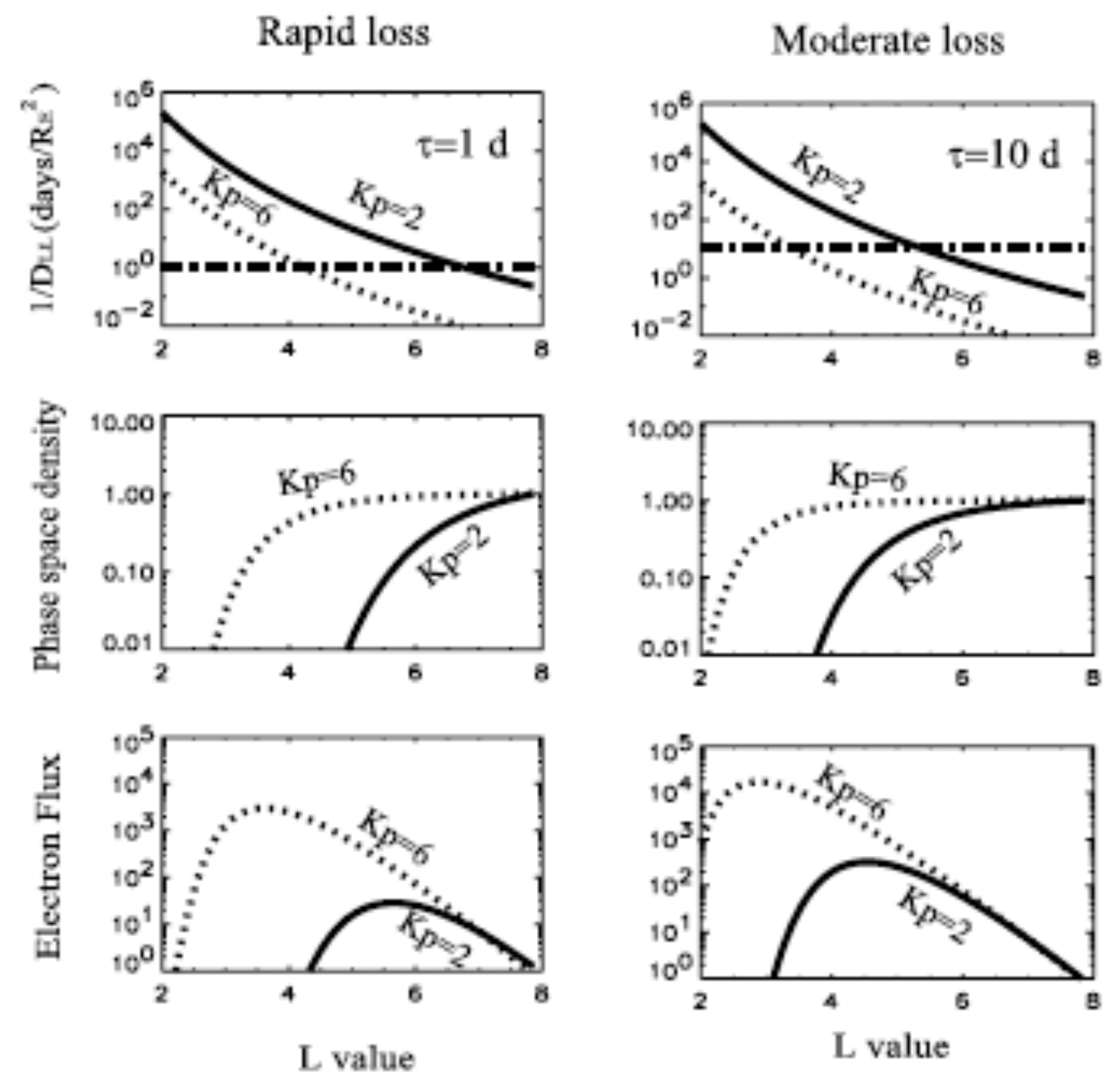
- Expanding the derivative:

$$\frac{df}{dt} = \left[\frac{\partial D_{LL}}{\partial L^*} - \frac{2D_{LL}}{L^*}\right]\frac{\partial f}{\partial L^*} + D_{LL}\frac{\partial^2 f}{\partial L^{*2}} - \frac{f}{\tau}$$

- D_{LL} is a function of L*, M, K, solar wind conditions

Acceleration by Radial Diffusion

- From Shprits et al. [2004]
- External source is pushed inward
- Intensity depends on push strength (D_{LL}) and loss strength (τ)
  - D_{LL}: increases with L
  - τ: who knows

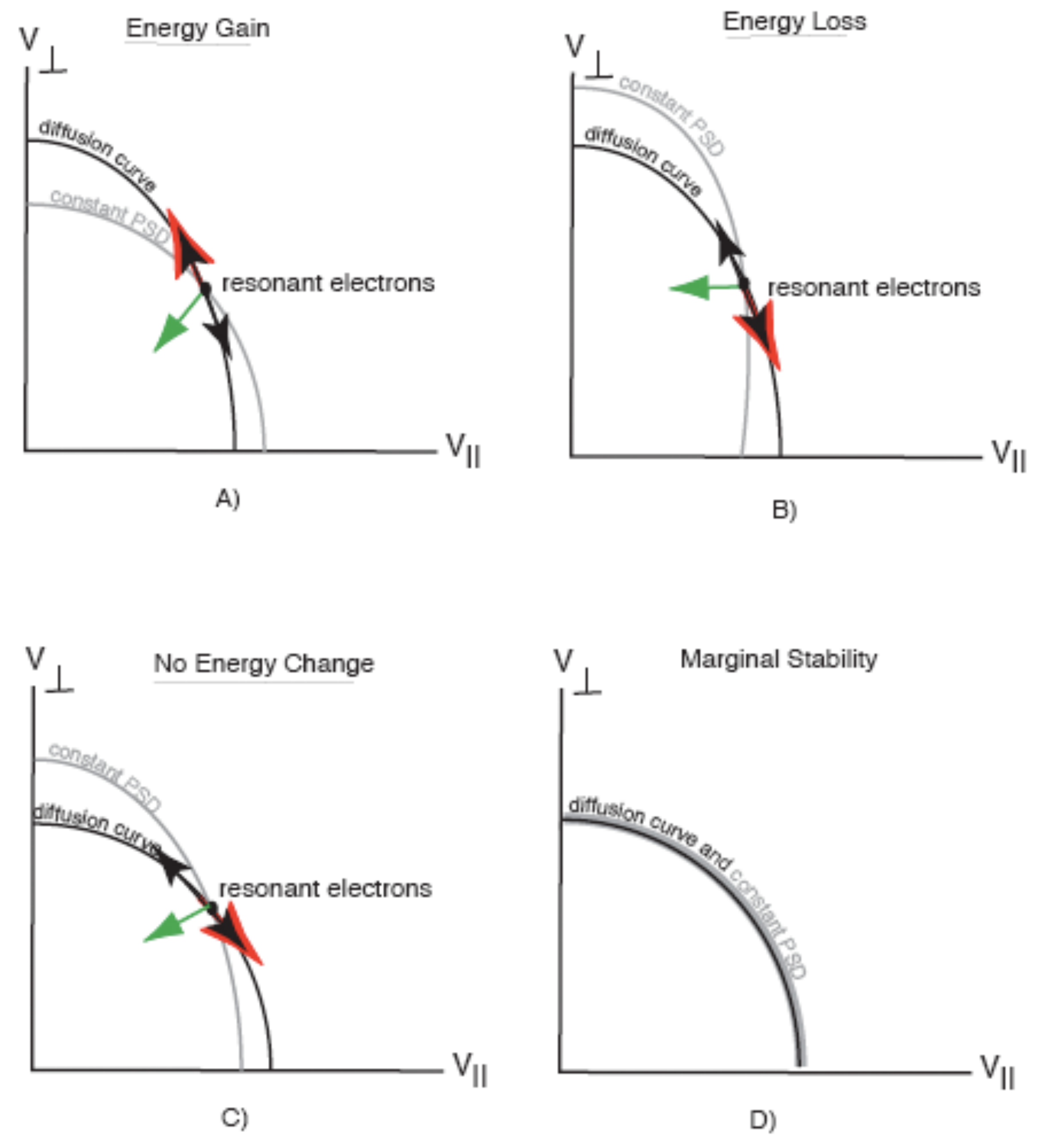


Local Acceleration

- Gyration of particle in resonance with the wave frequency
    - Again, or multiples of either of these
    - Same picture as before, but now on the gyration scale
    - Happening on the Hz to kHz scale (ms to s range)
- The usual suspect: nightside VLF chorus
    - Created by freshly-injected plasma sheet electrons
    - Energies around a keV (± a factor of 10)
    - Unstable "loss cone" distribution -> excites plasma waves
    - Intended consequence: pitch-angle scatter keV e-
    - Unintended consequence: accelerate 100s of keV e-

Resonance Curves

- A confusing intersection…
- Black lines: resonance curves
    - Particles are scattered on these lines
- Gray lines: constant PSD
    - Higher PSD inside, lower PSD outside
- Perfect circular arcs: constant energy
    - Scattering on a circle means no energy gain or loss for particles



Calculating Energy Diffusion

- Again, a diffusion equation:

$$\frac{\partial f}{\partial t} = \frac{1}{\sqrt{E}}\frac{\partial}{\partial E}\left[\sqrt{E}\,D_{EE}\frac{\partial f}{\partial E}\right]$$

- The hard part: finding D_{EE}
  - D_{EE} is a function of E, α, L, MLT, and activity
  - You have to average D_{EE} also...it's a function of latitude
- Also: this is energy, not an invariant quantity
  - Complicates the issue of combining it with L diffusion

Radiation Belt Losses

- Lots of losses
  - Adiabatic de-energization
    - Tail and ring currents are inflating the field
    - Reversible process: particles not actually lost
  - Magnetopause flow-out
    - Drift paths can cross this boundary on the dayside
    - Particles are gone: fly off to deep space
  - Scattering into the loss cone
    - Pitch angle scattering by various waves
- Lots of waves can cause this:
  - Plasmaspheric hiss
  - EMIC waves
  - Magnetosonic waves
  - Dayside VLF chorus
  - Lightning whistlers

Calculating Pitch Angle Scattering

- Again, a diffusion equation, but in pitch angle:

$$\frac{\partial f}{\partial \mu_o} = \frac{1}{h(\mu_o)\mu_o}\frac{\partial}{\partial \mu_o}\left[\langle D_{\mu_o \mu_o}\rangle h(\mu_o)\mu_o \frac{\partial f}{\partial \mu_o}\right]$$

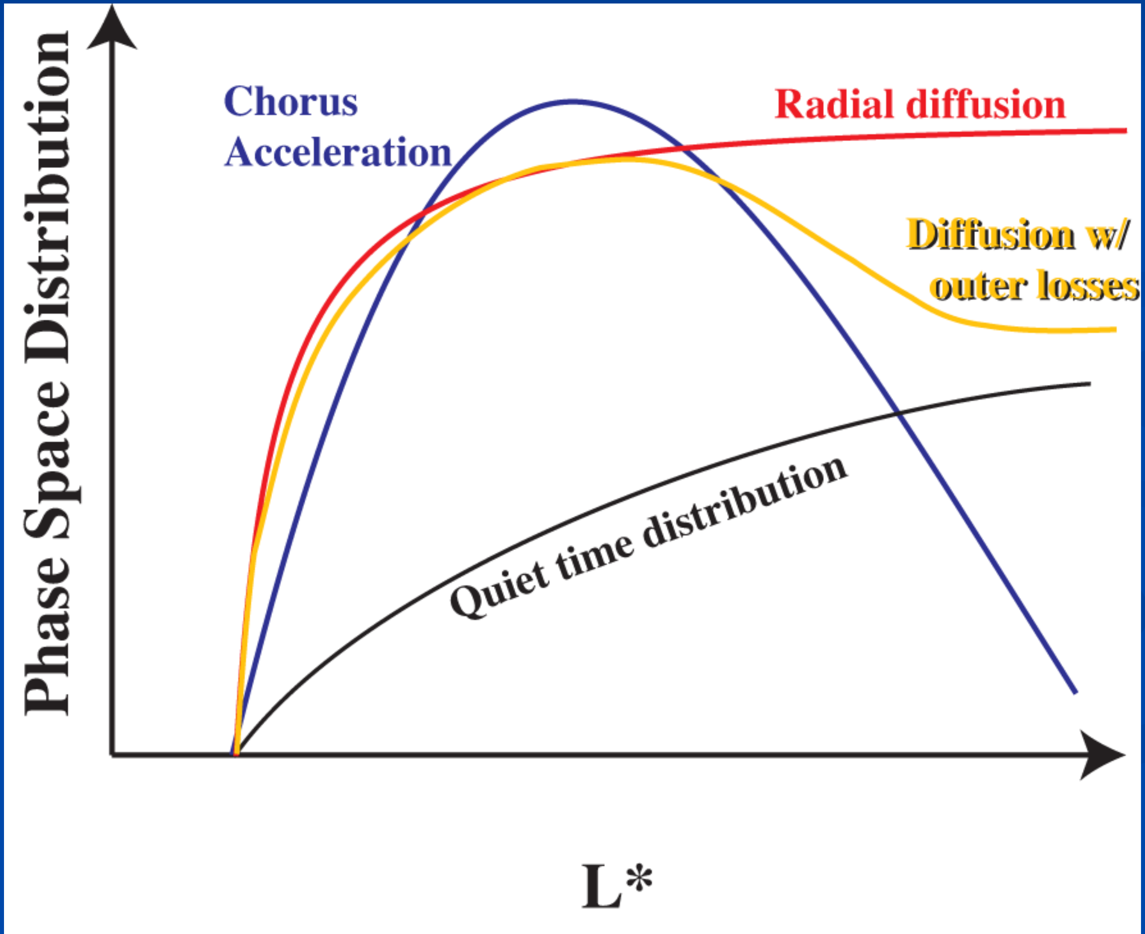
  - where h is a bounce-average term and μ_0 is related to α_0:

$$\mu_0 = \cos \alpha_0$$

- Again, the tough thing is calculating $D_{\mu\mu}$
- Also, this is a local variable, not an invariant quantity

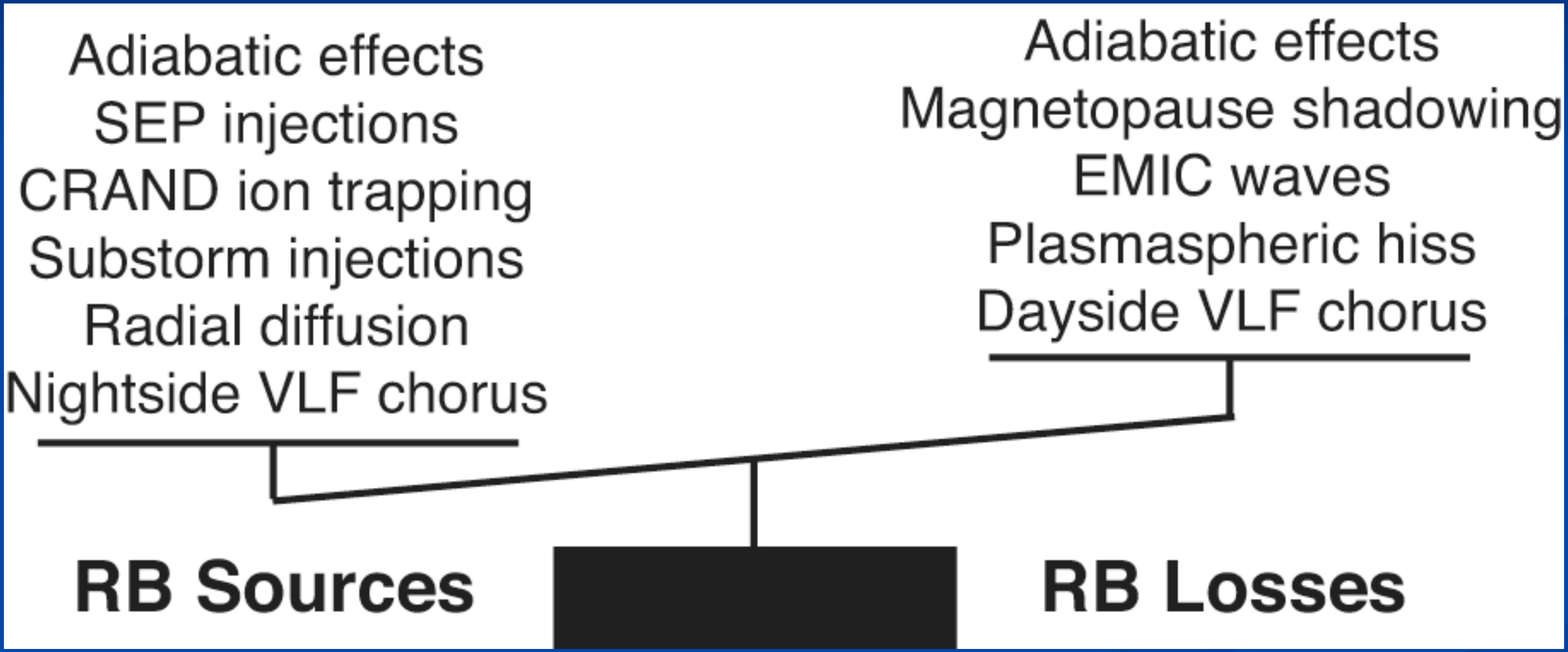
Hard to interpret RB e- acceleration

- Different ways to get the same radial profile
- In fact, it's hard to even observe PSD
  - Measurements done in energy flux



Lots of processes in the radiation belts

- Like a big balancing act between sources and losses



Adiabatic effects
SEP injections
CRAND ion trapping
Substorm injections
Radial diffusion
Nightside VLF chorus

RB Sources

Adiabatic effects
Magnetopause shadowing
EMIC waves
Plasmaspheric hiss
Dayside VLF chorus

RB Losses

The One Big Thing to Know About the <u>Radiation Belts</u>

The radiation belts do their own d#mn thing

Summary of inner magnetospheric characteristics

- Here's a nice little table of facts

Population	Density	Temperature	Source	Composition	Driver	Importance
Plasma-sphere	100s – 1000s cm^{-3}	<1 eV	Subauroral ionosphere	H^+, some He^+ and O^+	E fields	Dominates mass density
Ring Current	<1 to 10s cm^{-3}	1-400 keV	Plasma sheet	H^+ and e^-, O^+ in storms	E and B fields	Dominates energy density
Radiation Belts	<<1 cm^{-3}	100s keV to MeV	Plasma sheet, ring current, SEPs	Mostly e^-, some H^+ (inner belt)	B fields	Dominates S/C damage

Inner Magnetosphere Summary

- All three particle populations are...
  - coupled together
  - controlled by the electric and magnetic field
  - influenced by external source/driver terms
  - important for understanding space weather
  - drastically modified during <u>magnetic storms</u>
- What about storms...
  - How is it modified?
  - Depends on the type of driver for the storm
  - Two main drivers:
    - ICMEs: interplanetary coronal mass ejections
    - CIRs: corotating interaction regions

Magnetic storms

- Big convection events within the magnetosphere
- The typical components:
    - Formation of a partial, and then symmetric, ring current (defining element = Dst perturbation)
    - Reduction (and subsequent enhancement) of radiation belts
    - Plasmaspheric drainage plume creation
    - Multiple substorm expansion phase auroral intensifications and magnetic dipolarizations
- Driving conditions last for hours, effects last for days

Different types of storm activity

- A bit of the Borovsky-Denton chart

Parameter	ICME-Driven Storm	CIR/HSS-Driven Storm
Solar cycle phase when dominant	Solar maximum	Declining phase
Occurrence pattern	Irregular	27-day periodicity
Ring current	Stronger	Weaker
Radiation belts	Less severe	More severe

The One Big Thing to Know About Magnetic Storms

Know Your Driver Conditions

Final Side Trip: Plasma Waves

- Read "*Waves in Plasmas*" Thomas Stix
    - Not an easy read, but worth it
- Two big concepts:
    - Excitation of plasma waves
        - Playing with the dispersion relation and resonance conditions
    - Wave-particle interactions
        - Basic approach: quasilinear theory and diffusion coefficients
        - To do it right: nonlinear wave-particle interactions

Plane Waves and Wave Growth

- Assume a plane wave:

$$\Psi = \Psi_0 \exp\left[i\left(\mathbf{k}\cdot\mathbf{r} - \omega t\right)\right]$$

  - Frequency ω and wave normal vector **k**
- Then, for wave amplitude growth or decay:

$$\omega = \omega_R + i\omega_I$$

$$\exp\left[-i\omega t\right] = \exp\left(-i\omega_R t\right)\cdot\exp\left(\omega_I t\right)$$

  - If $\omega_I > 0$, then the wave amplitude will grow

Dispersion Relation

- How do we know if $\omega_I>0$?
- Solve the dispersion relation
  - Plug plane wave formula into Maxwell's equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

  - Magnetostatic waves: $B=B_{wave}$, $E=0$
  - Electrostatic waves: $E=E_{wave}$, $B=0$
  - Electromagnetic waves: $E=E_{wave}$, $B=B_{wave}$
- Solve for ω (real and imaginary)
  - Real part: gives you what frequencies can exist
  - Imaginary part: gives you wave excitation/damping rate

Wave-Particle Resonance

- The resonance condition:

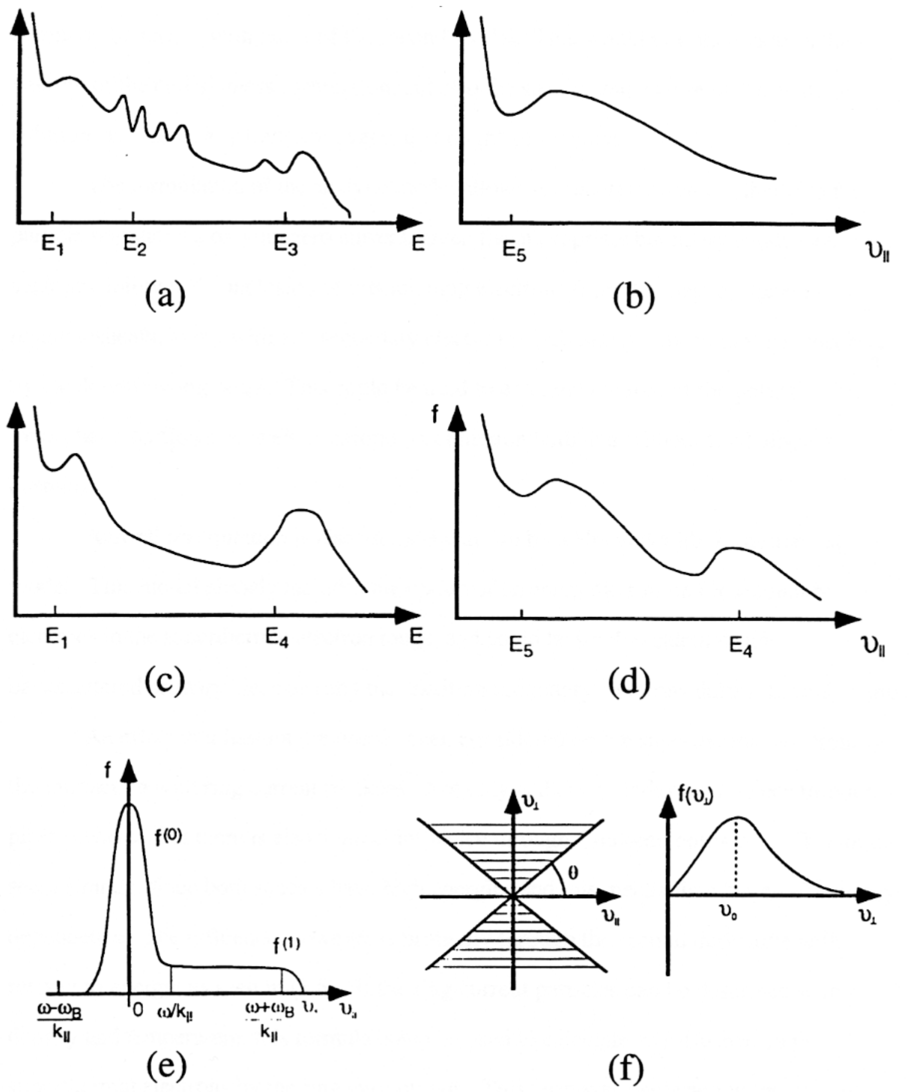
$$\omega - k_{\parallel} v_{\parallel} + n\omega_g = 0$$

  - Particles interact with waves of very specific frequencies
  - "n" is any integer: zero, positive, or negative
    - n=0: Landau resonance (also called Cherenkov resonance)
    - n≠0: cyclotron resonance
- What does it mean?
  - Particle and wave are *in phase* with each other
  - Energy easily exchanged between the two
    - Particle is emitting EM oscillations of this frequency
    - Particle is absorbing EM radiation of this frequency

Wave Growth and Decay

- Bumps in the particle distribution function are flattened
- Example: photoelectron distributions
  - Many possibilities for bumps in the distribution function
  - Electrons will excite plasma waves that will then scatter the electrons
- Important Sidenote:
  - Once a wave is excited, it can propagate and/or interact with other plasma populations

Photoelectron Instabilities



Quasilinear Theory

- Basic assumption: small-amplitude waves
  - B_{wave} is much smaller than B_0
  - Change of *f* is much slower than change in B_0
  - So: perturbation on a static background
- Reduces to a diffusion equation

$$\frac{\partial f}{\partial t} = \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2 D_{pp}\frac{\partial f}{\partial p} + pD_{p\alpha}\frac{\partial f}{\partial \alpha}\right] + \frac{1}{p\sin\alpha}\frac{\partial}{\partial \alpha}\left[D_{\alpha p}\frac{\partial f}{\partial p} + \frac{D_{\alpha\alpha}}{p}\frac{\partial f}{\partial \alpha}\right]$$

- Finding the diffusion coefficients is the hard part
  - Typically involves lots of nested integrals/loops
  - Particle energy and pitch angle, wave frequency and wave normal angle, location along a drift path

The One Big Thing to Know About Plasma Waves

Plasma waves are everywhere

The One Big Thing to Know About <u>This Talk</u>

The inner magnetosphere is a highly coupled system, and there is still a lot to learn