Cosmic Ray Transport in the Heliosphere

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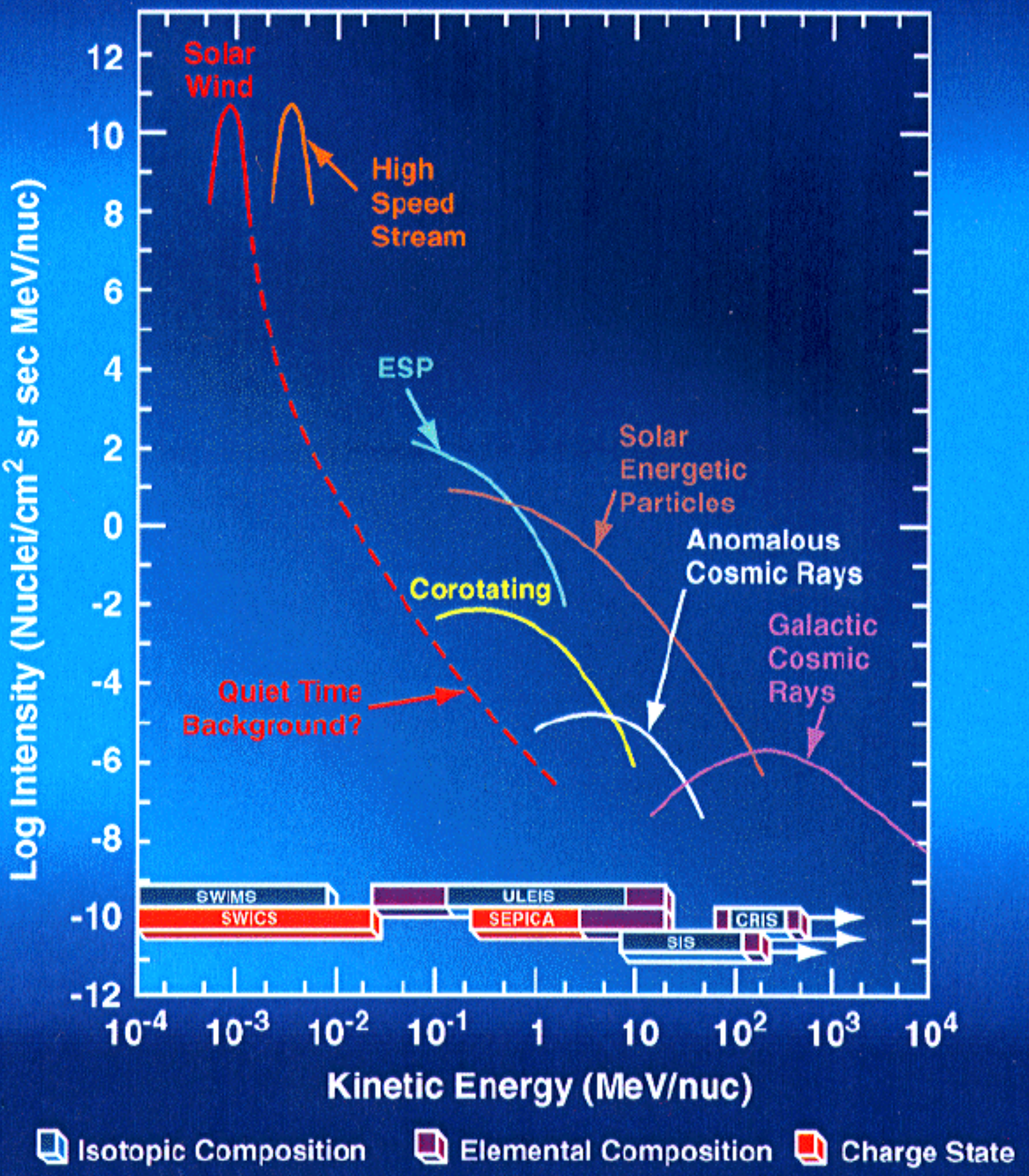
Heliophysics Summer School, Boulder, CO, July 16, 2013

Outline

- Lecture 1: Background
  - The heliosphere
  - Cosmic Rays in the heliosphere
  - Record-intensity cosmic rays during the last sunspot minimum
  - Has Voyager 1 entered the interstellar medium?
- Lecture 2: Basic theory of charged-particle transport
  - Equations of motion, large-scale drifts, resonances
  - Restricted motions
  - Diffusion, Convection, Energy Change
  - The Parker transport equation
- Recitation/Problem Sets: Applications

Spectra of Energetic Oxygen Nuclei



The distribution function

- We define the phase-space distribution function f as the number of particles within a given phase-space volume
- Phase space consists of 6 dimensions: 3 spatial coordinates, 3 velocity coordinates
- The normalization is such that the *number density,* $n(\mathbf{r},t)$, is given by:

$$n(\boldsymbol{r},t) = \iiint d^3p \, f(\boldsymbol{r},\boldsymbol{p},t)$$

Cartesian coordinates

$$n(x,y,z,t)= \int_{-\infty}^{\infty} dp_x \int_{-\infty}^{\infty} dp_y \int_{-\infty}^{\infty} dp_x \;\; f(x,y,z,p_x,p_y,p_z,t)$$

Spherical coordinates

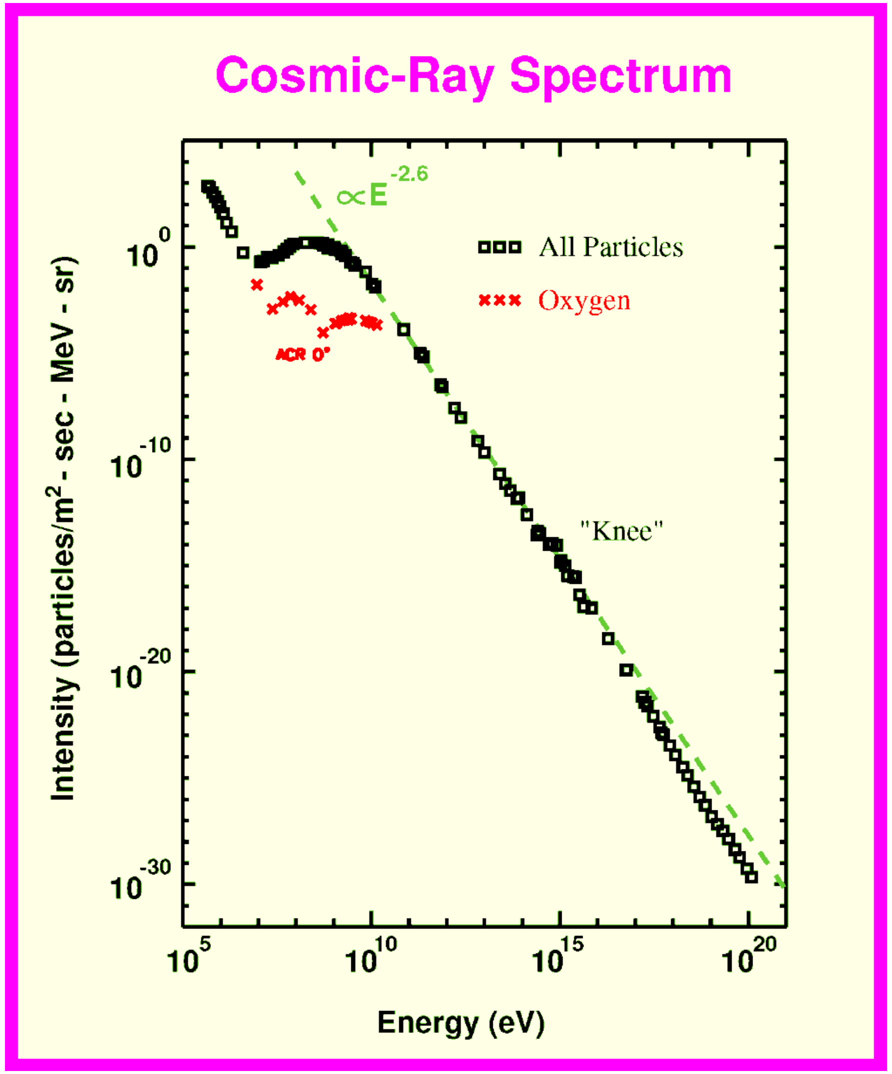
$$n(\boldsymbol{r},t)= \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta \, d\theta \int_0^{\infty} p^2 dp \;\; f(\boldsymbol{r},p,\theta,\varphi,t)$$

Note that this last form is most convenient for cosmic rays in most situations because cosmic rays are observed to have very little anisotropy meaning that f can be taken as independent of θ (pitch angle) and φ (phase angle)

The differential intensity

- Observers commonly represent their data using the differential intensity, or sometimes referred to simply as "the energy spectrum." It is essentially the flux per energy, per solid angle and is related to the phase-space distribution function by

$$dJ/dE = p2\ f$$



Forces acting on energetic charged particles

- Lorentz force

$$\frac{d\mathbf{p}}{dt} = q\mathbf{E} + \frac{q}{c}\mathbf{w} \times \mathbf{B}$$

- Where $\boldsymbol{w} = \boldsymbol{p}/m$ is the particle velocity vector, and $\boldsymbol{p}$ is the momentum. q is the particle's charge, $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric and magnetic fields, respectively.
- Other forces are generally small, but can be added as needed (e.g. gravity, radiation pressure, etc.)

Constant Electric and Magnetic Fields

Case A: $\boldsymbol{B}$ = constant

$\boldsymbol{E}$ = 0

One gets simple gyromotion

To begin, examine the motion of individual charged particles

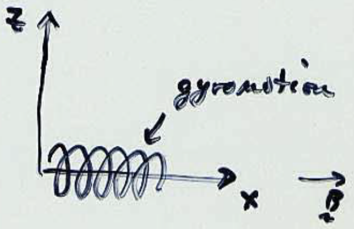
$$\mathbf{F}_L = q\mathbf{E} + \frac{q}{c}\mathbf{w}\times\mathbf{B}$$

Force on a single particle

Case A

$\mathbf{B}$ = constant

$\mathbf{E}$ = 0



$$m\frac{d\mathbf{w}}{dt} = q\mathbf{E}^{\,0} + \frac{q}{c}\mathbf{w}\times\mathbf{B} \qquad \mathbf{B} = B_0\hat{x}$$

$$w_x = w_{\parallel} = \text{constant}$$

$$w_y = -w_{\perp}\cos(\Omega t - \phi)$$

$$w_z = w_{\perp}\sin(\Omega t - \phi)$$

where $\Omega = \frac{qB}{mc}$ = gyrofrequency

$w_{\perp} = (w^2 - w_{\parallel}^2)^{1/2}$

$\phi = \tan^{-1}(w_{0y}/w_{0z})$ phase

Constant Electric and Magnetic Fields Case

Case A: $\boldsymbol{B}$ = constant

$\boldsymbol{E}$ = constant

One gets gyromotion + drift
(the electric field drift)

define α = pitch angle - angle between $\underset{\sim}{w}$ & $\underset{\sim}{B}$

$w_{\parallel} = w \cos\alpha$

$w_{\perp} = w \sin\alpha$

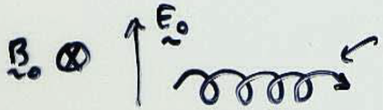
Case B

$\underset{\sim}{E}$ = constant = $\underset{\sim}{E}_0$

$\underset{\sim}{B}$ = constant = $\underset{\sim}{B}_0$

define this ∋ $\underset{\sim}{E}_0 \cdot \underset{\sim}{B}_0 = 0$

then we get a "drift" of particle



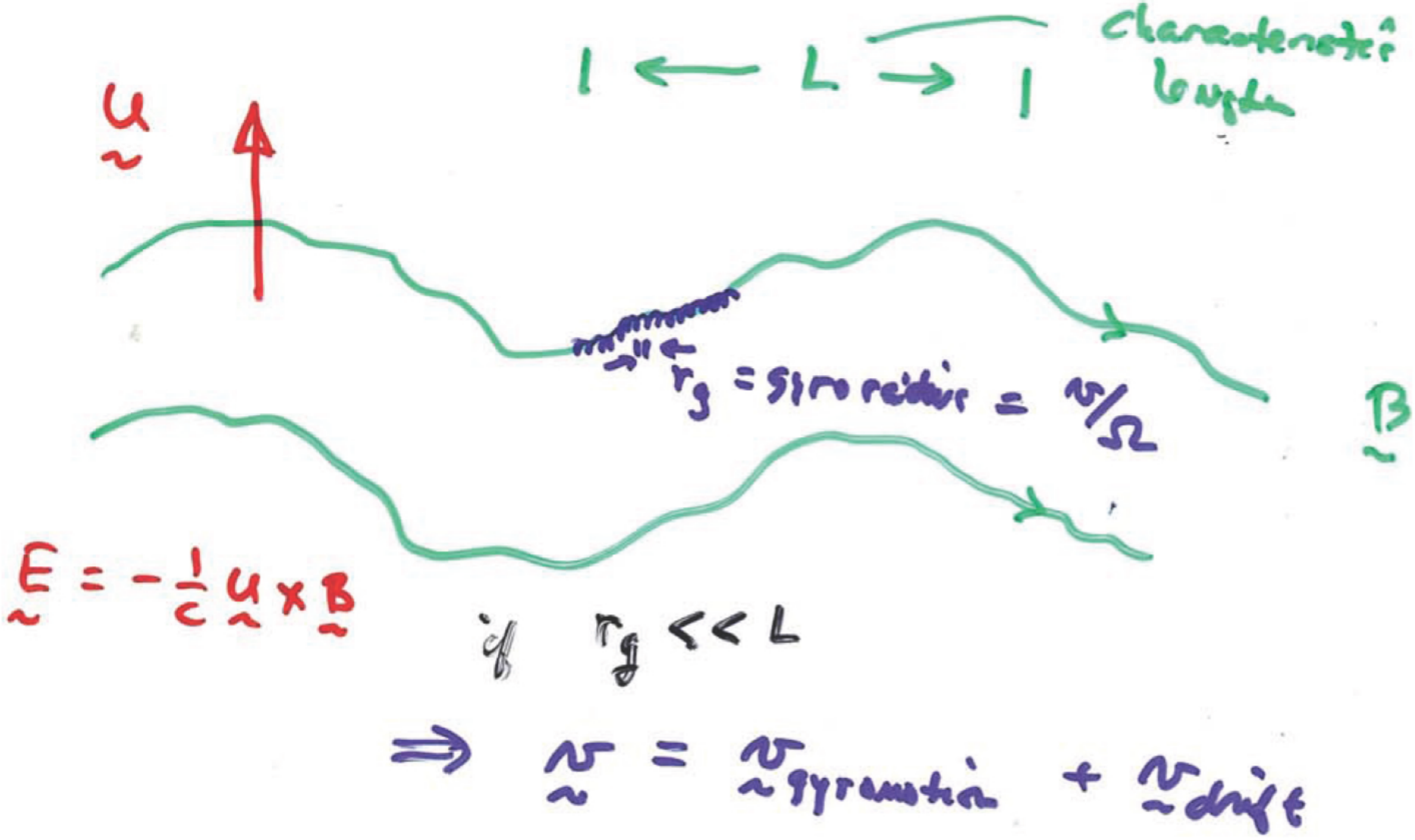
drift + gyromotion

$$\underset{\sim}{w}_D = c\,\frac{\underset{\sim}{E}_0 \times \underset{\sim}{B}_0}{B_0^2}$$ = Electric Field Drift

$$\underset{\sim}{w} = \underset{\sim}{w}_D + \underset{\sim}{w}'$$

↑ drift ↑ gyromotion

Varying E and B fields
Case 1: Scale of variation >> gyroradius of particles



the drifts are:

Electric Field drift $\quad \underset{\sim}{v}_E = \frac{c}{B^2} \underset{\sim}{E} \times \underset{\sim}{B}$

if $\nabla \times \underset{\sim}{B} = 0$ (Special Case!)

curvature drift $\quad \underset{\sim}{v}_c = \frac{2c}{qB^3} W_{\parallel} \underset{\sim}{B} \times \nabla \underset{\sim}{B}$

gradient drift $\quad \underset{\sim}{v}_g = \frac{c}{qB^3} W_{\perp} \underset{\sim}{B} \times \nabla \underset{\sim}{B}$

$$W = \frac{1}{2} m v^2$$

IN general, the guiding center drift velocity is given by

$$\underset{\sim}{b} = \frac{\underset{\sim}{B}}{B}$$

$$\underset{\sim}{v}_{G.C.} = \left[v_{\parallel} + \frac{c W_{\perp}}{2qB} \underset{\sim}{b} \cdot (\nabla \times \underset{\sim}{b}) \right] \underset{\sim}{b} + \frac{c W_{\perp}}{2qB} \underset{\sim}{b} \times \nabla B + \frac{c W_{\parallel}}{qB} \underset{\sim}{b} \times (\underset{\sim}{b} \cdot \nabla) \underset{\sim}{b}$$

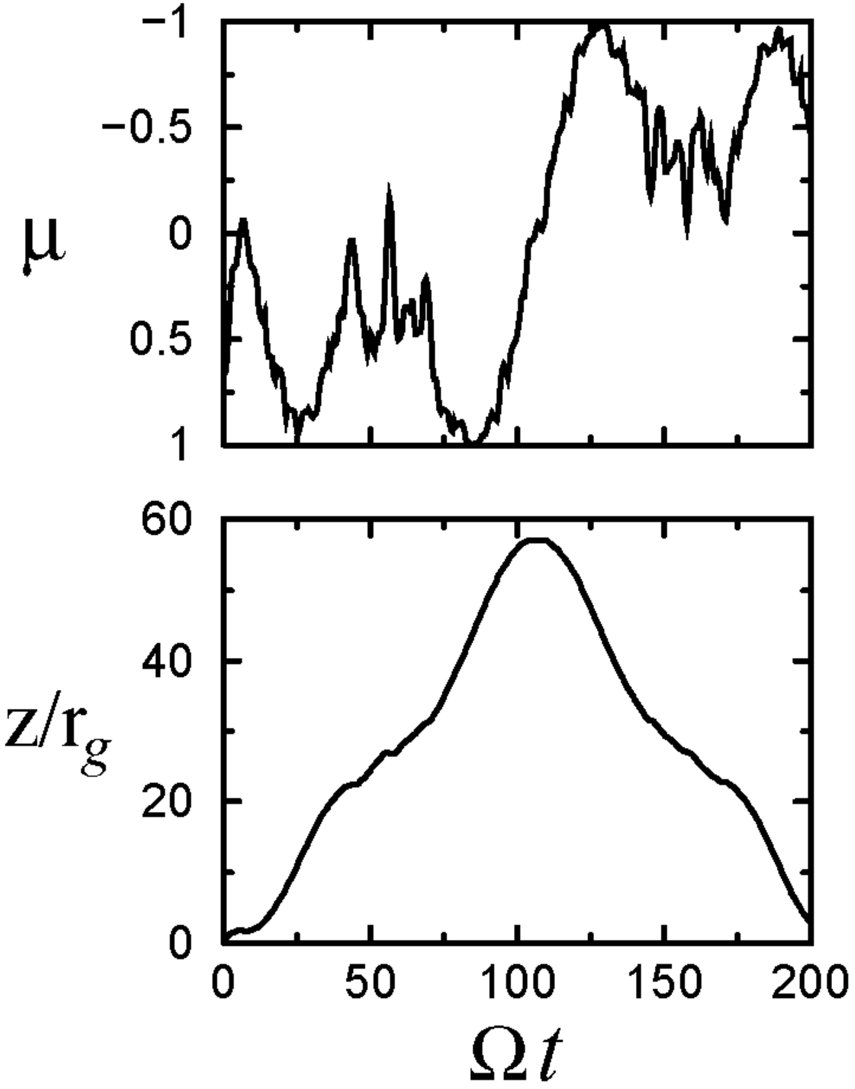
if we average this over an isotropic dist. of particles we find $\quad \underset{\sim}{v}_{drift} = \frac{c m v^2}{q} \nabla \times \left(\underset{\sim}{B} / B^2 \right)$

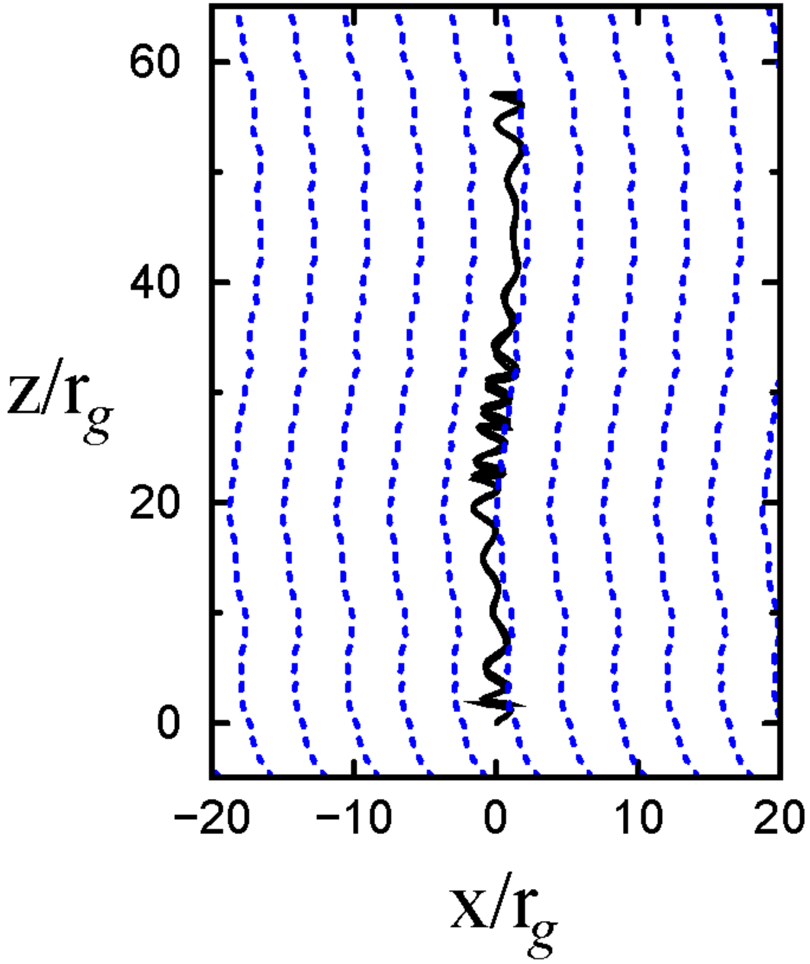
Varying E and B fields
Case 2: Scale of variation ≈ gyroradius of particles

what if $r_g \sim L$?

- A "resonance" can occur such that the particles pitch angle is reversed. This is much like a "scattering" event in scattering theory. The resonance condition is $kw\mu=\Omega$, where k is the wavenumber of the fluctuation, w is the particle speed, μ is the cosine of the pitch angle, and Ω is the particle cyclotron frequency.

A charged particle moving in a turbulent magnetic field (numerical integration)

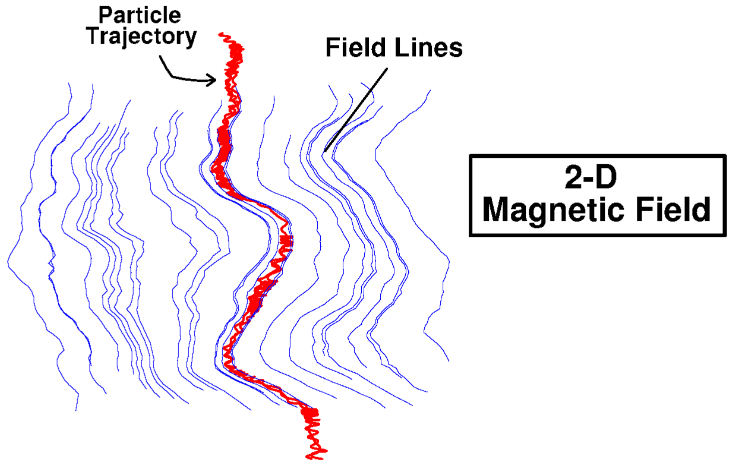


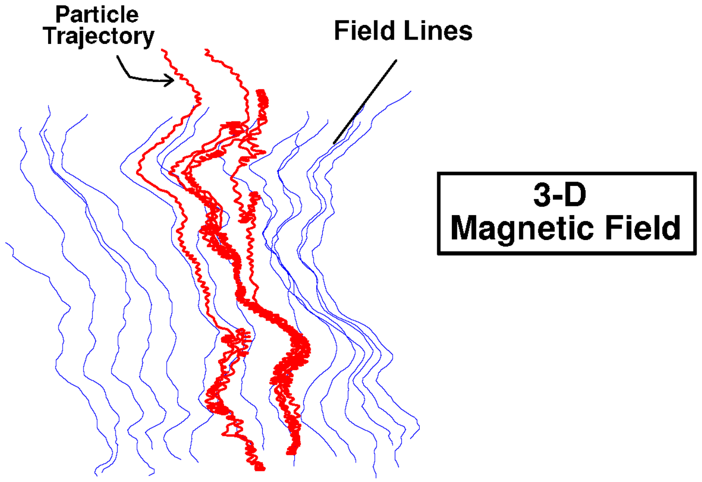


Restrictions on particle motions imposed by artificially limiting the dimensionality of the fields

- Charged particles are strictly tied to magnetic lines of force in 1 and 2D electric and magnetic fields
  - This can be proven rigorously and follows directly from the equations of motion

- This is an artificial and unphysical constraint on charged-particle motion!
  - Be aware!





Spatial Diffusion in 1 D

- If there are enough "scattering" events then the resulting motion is similar to that of a random walk in space. This type of motion is commonly described as "diffusive." The distribution of a collection of particles is said to be "isotropic" i.e. the distribution function is independent of pitch angle and phase angle. Its evolution can be described with a diffusion equation, which, in 1D is given by:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial}\left(\kappa \frac{\partial f}{\partial x}\right)$$

κ is the "diffusion coefficient" and is related to the magnetic-field fluctuations for the case of charged particles moving within a turbulent magnetic field (see Jokipii, ApJ, 1966). It is related to the mean-free path of scattering by:

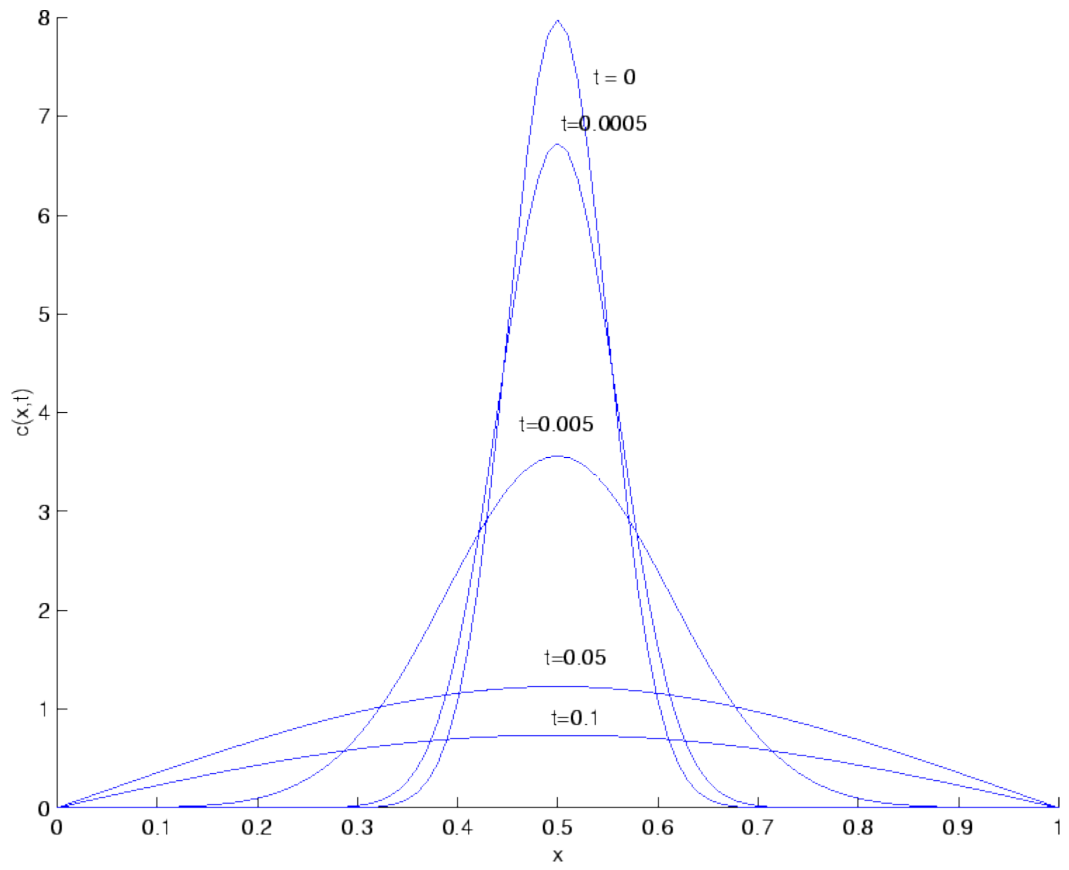
$$\kappa = (1/3)\, w\, \lambda_k$$

- Consider an "impulsive" release of particles at the origin. That is, $f(0,t)=\delta(t)$. The solution to the 1D diffusion equation for this situation is given by:

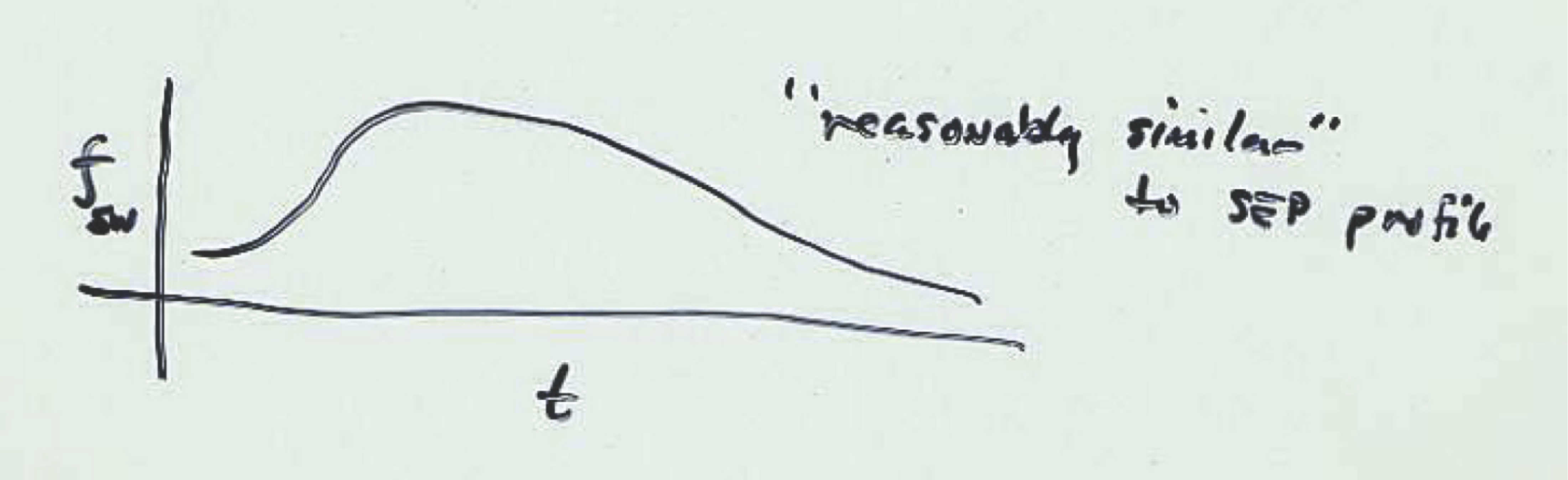
$$f(x;t) = \frac{N_0}{4\pi\cdot t}\exp\left(-\frac{x^2}{4\cdot t}\right)$$

- Where N_0 is the number of particles

- The solution for f as a function of x for various times resembles that shown at right



The distribution as a function of time at a particular spatial location away from the source



- This resembles many large solar-energetic particle events

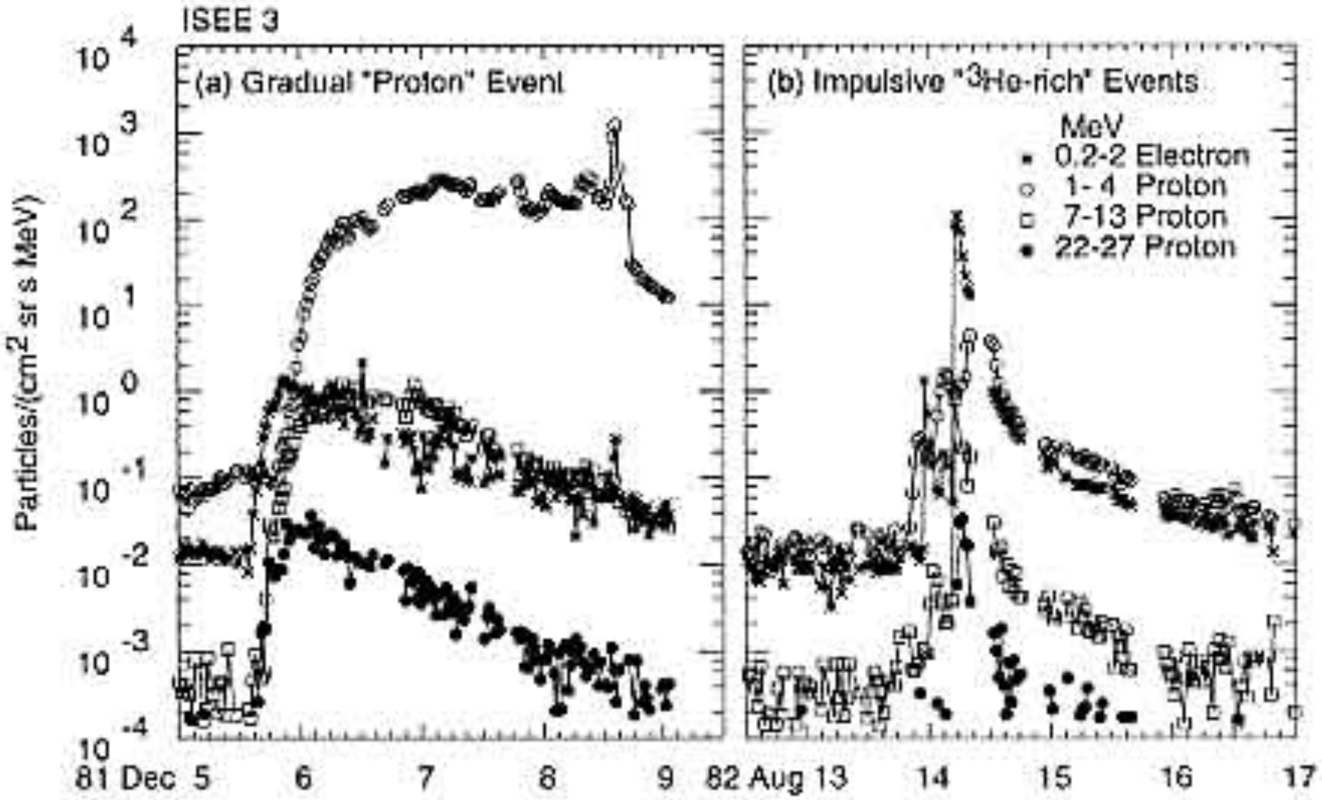
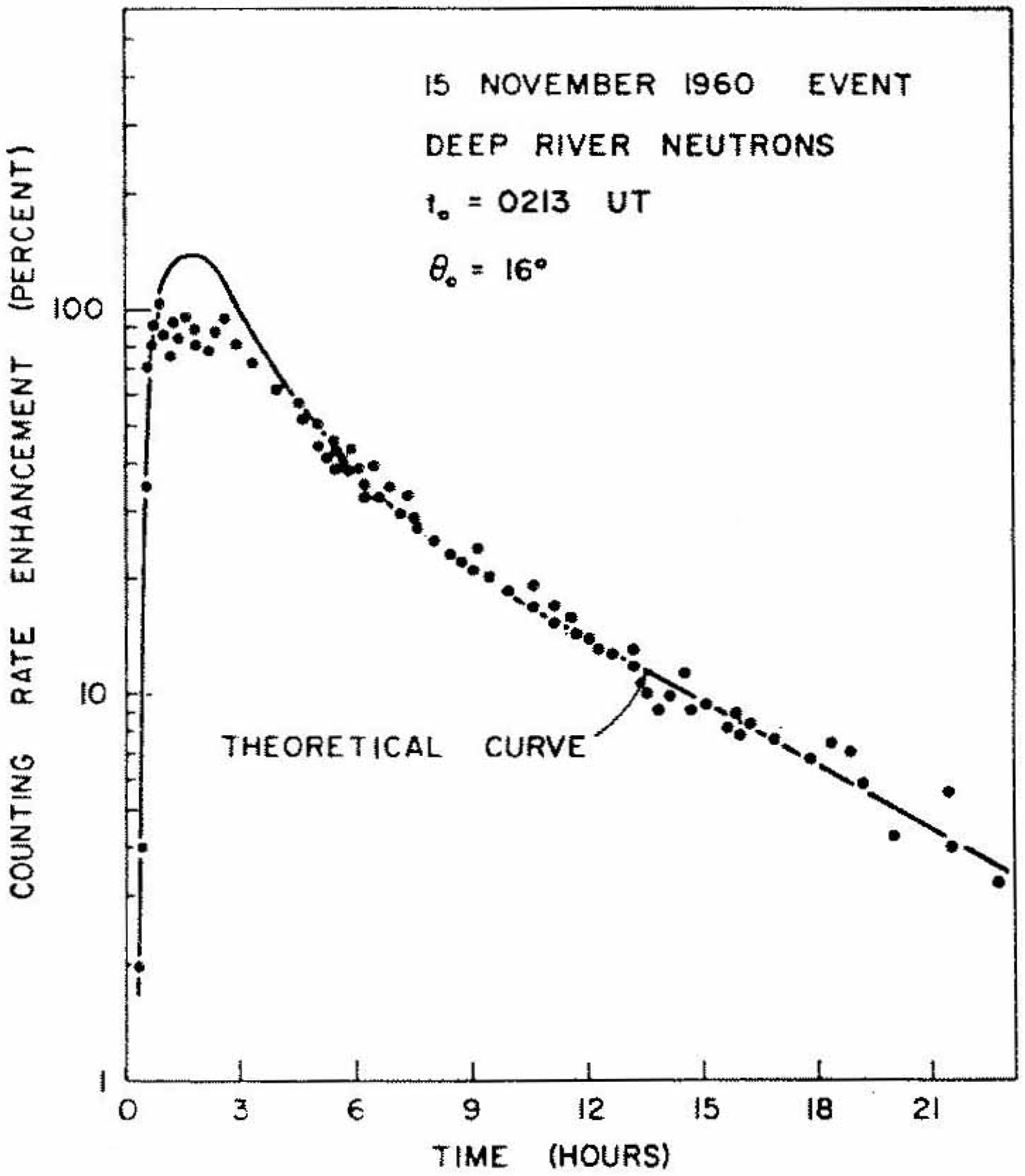


Figure 2.2. Intensity-time profiles of electrons and protons in 'pure' (a) gradual and (b) impulsive SEP events. The gradual event is a disappearing-filament event with a CME but no impulsive flare. The impulsive events come from a series of flares with no CMEs.



Theoretical fit, using equation 122, to the Deep River neutron monitor data for the November 15, 1960, event. θ_0 is the angle between the flare and the foot of the average magnetic field line passing through the point of observation [*Burlaga*, 1967].

Diffusion in multi-dimensions

- The 2D diffusion equation is given by (ignoring crossed terms)

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x}\left(\kappa_{xx}\frac{\partial f}{\partial x}\right) + \frac{\partial}{\partial y}\left(\kappa_{yy}\frac{\partial f}{\partial y}\right)$$

- Or, in terms of coordinates along and across a mean magnetic field

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x_{\parallel}}\left(\kappa_{\parallel}\frac{\partial f}{\partial x_{\parallel}}\right) + \frac{\partial}{\partial x_{\perp}}\left(\kappa_{\perp}\frac{\partial f}{\partial x_{\perp}}\right)$$

In addition to diffusion, there are other important collective effects on charged particles: they include

- Advection

$$U \cdot \nabla f$$

(arises because the "scattering centers" are moving with the bulk plasma flow)

- Energy Change

$$\frac{p}{3} \nabla \cdot U \frac{\partial f}{\partial p}$$

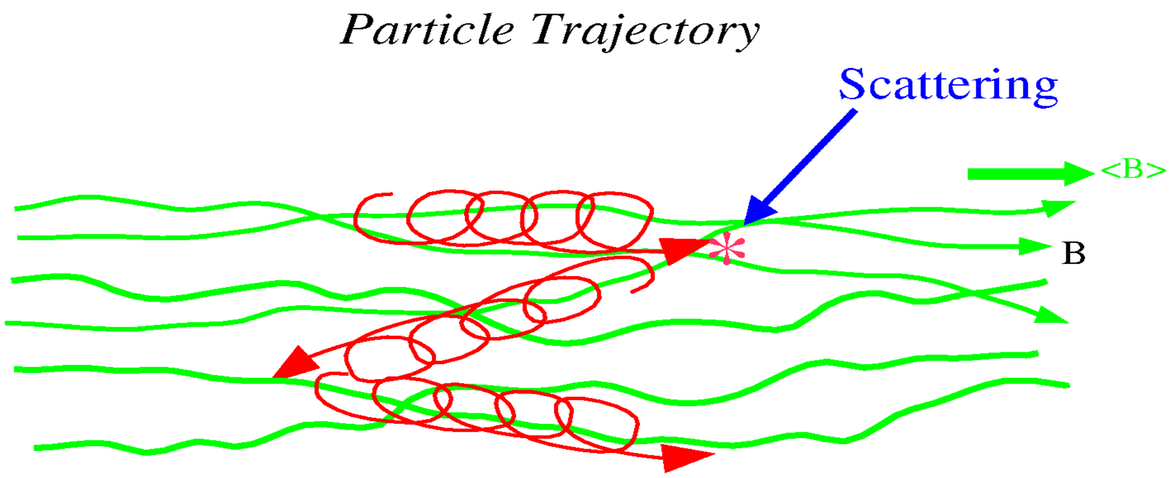
(arises because of scattering in converging or diverging flows)

Parker's energetic-particle transport equation

$$\frac{\partial f}{\partial t} = -V_{w,i}\frac{\partial f}{\partial x_i} + \frac{\partial}{\partial x_i}\kappa_{ij}\frac{\partial f}{\partial x_j} - V_{D,i}\frac{\partial f}{\partial x_i} + \frac{1}{3}\frac{\partial V_{w,i}}{\partial x_i}\frac{\partial f}{\partial \ln p} + Q$$

advection ***diffusion*** ***drift*** ***energy change***

Basic Physics of Cosmic-Ray Diffusion



Diffusion coefficients are related to the magnetic field power spectrum, as has been discussed by many authors: *quasi-linear theory*

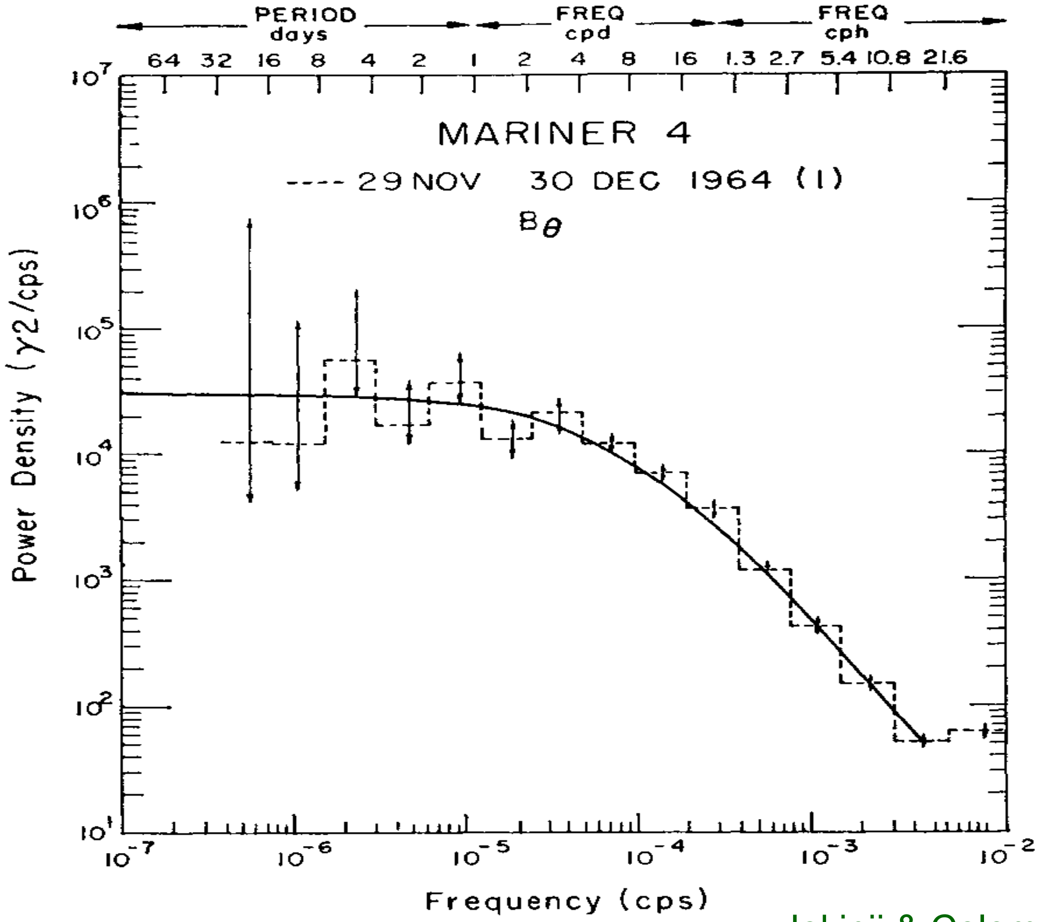
- For example, the rate of scattering depends on the power at the scale of the particle gyroradius (i.e. a resonant condition)

$$D_{1\,1} = \frac{¼}{4}(1 \; i \; {}^{1\;2}) - {}_{0}\frac{k_r P\,(k_r)}{B_0^2} \qquad k_r = \frac{-\;\;0}{w\, j^{1}\; j}$$

$$\cdot_{k} = \frac{w^2}{4} \angle_{0}^{1} \frac{(1 \; i \; {}^{1\;2})^2 d^{1}}{D_{1\,1}}$$

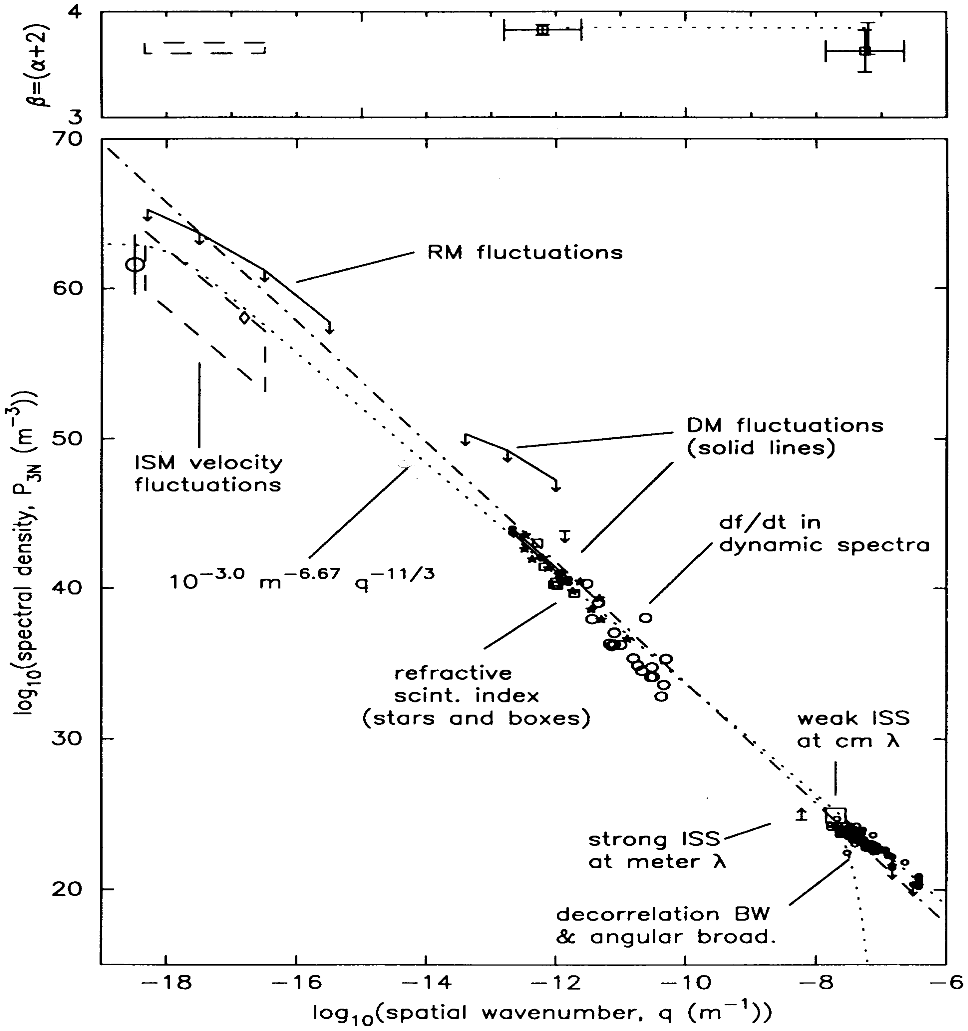
Power spectra in space

Interplanetary magnetic-field turbulence



Jokipii & Coleman, 1968

Interstellar density turbulence



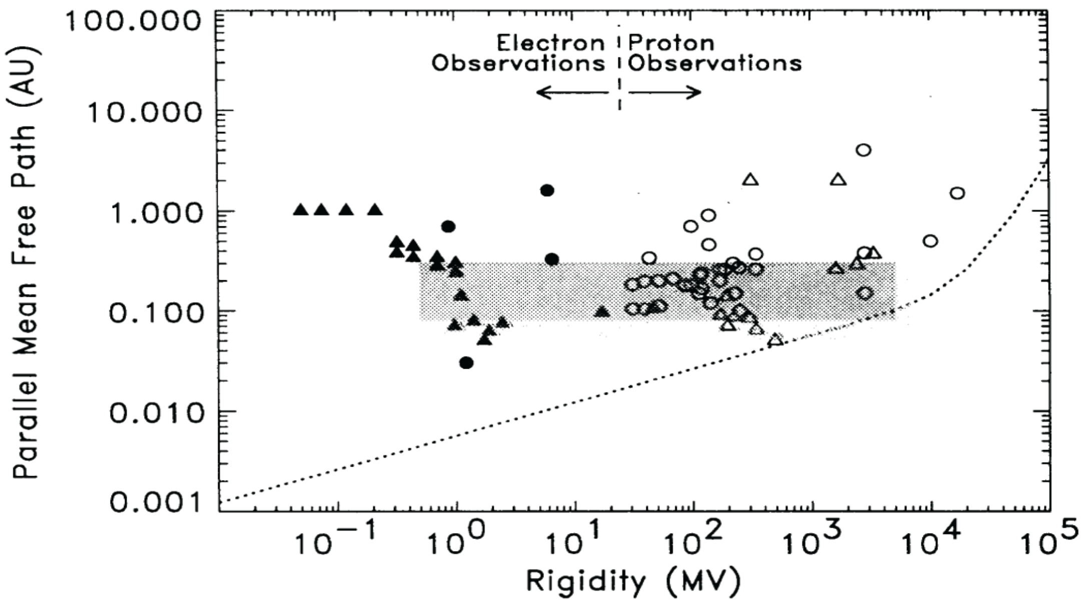
Armstrong et al, 1995

Parallel Diffusion

- *G*enerally, the observed λ is larger than the prediction from QLT using standard slab turbulence (1D)
- *Bieber et al.* (1994) argued that IMF turbulence has a significant contribution from turbulence that does not effectively scatter particles (so-called "composite" turbulence which is a combination of slab + 2D)

The "Palmer consensus"

(Palmer, 1982)



Bieber et al., 1994

Perpendicular diffusion

- The transport of particles normal to the magnetic field has been more difficult to understand.
    - QLT: · ? / P (0)
    - Numerical simulations do not agree with any of the existing theories

- The simulations generally show that the ratio is independent of energy = · k