Q: Why do the Sun and planets have magnetic fields?

Dana Longcope
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w/ liberal “borrowing” from Bagenal, Stanley, Christensen, Schrijver, Charbonneau, …

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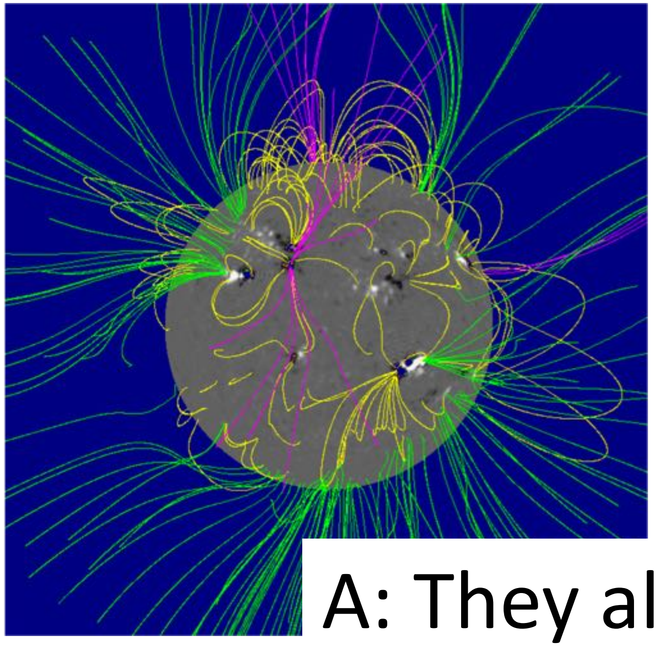
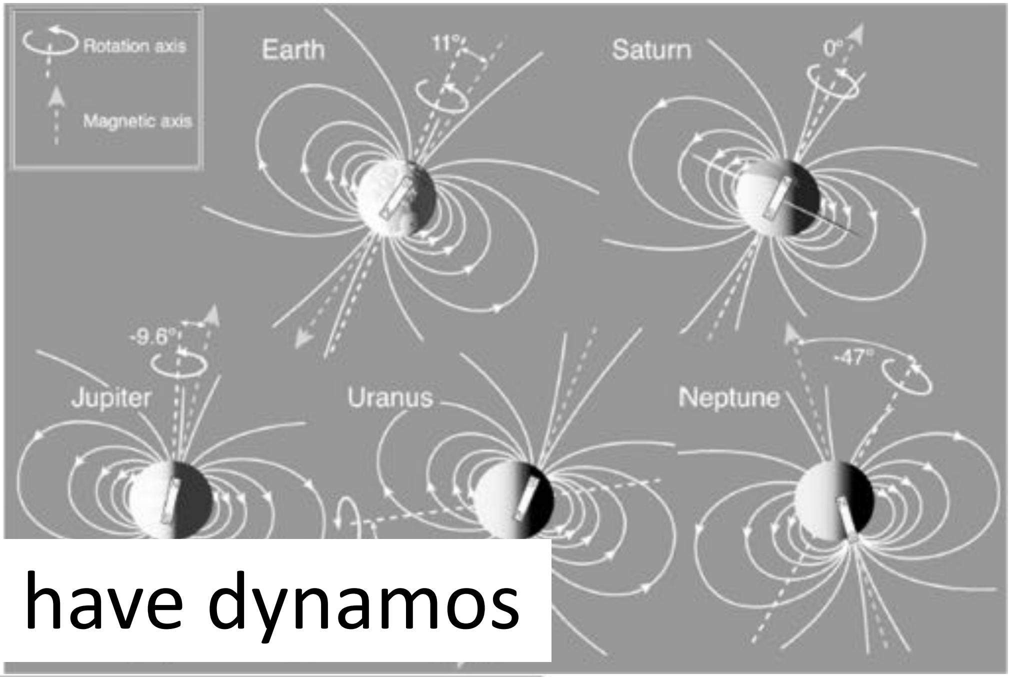
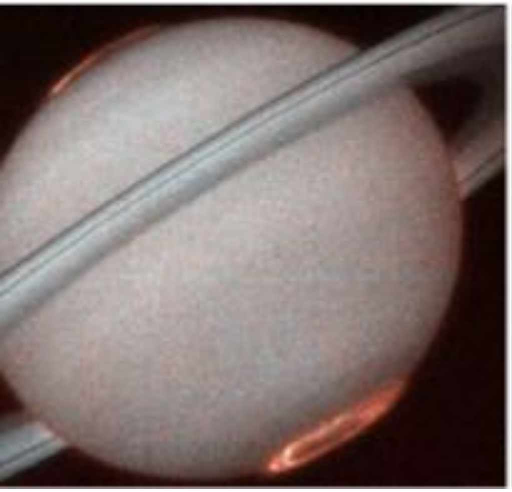
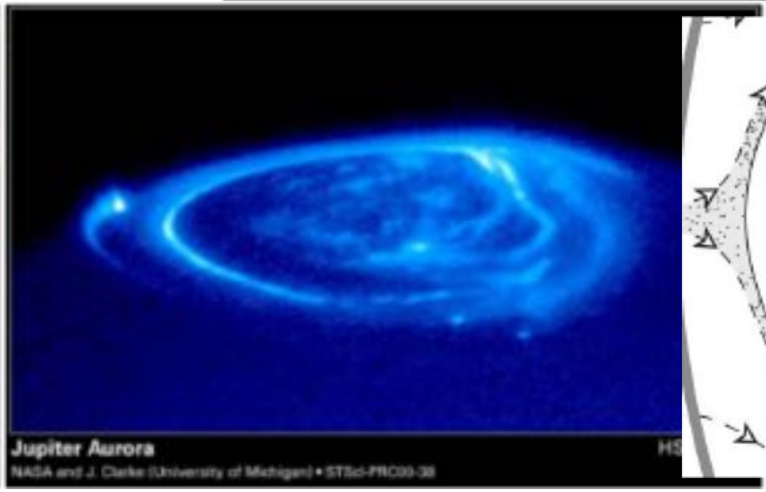
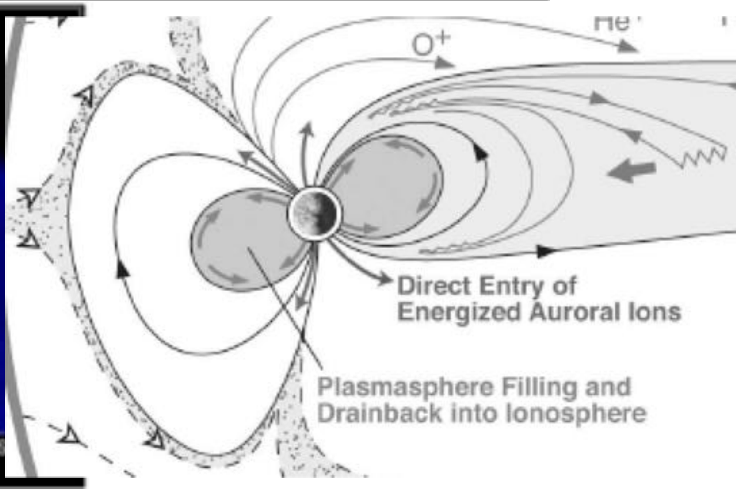




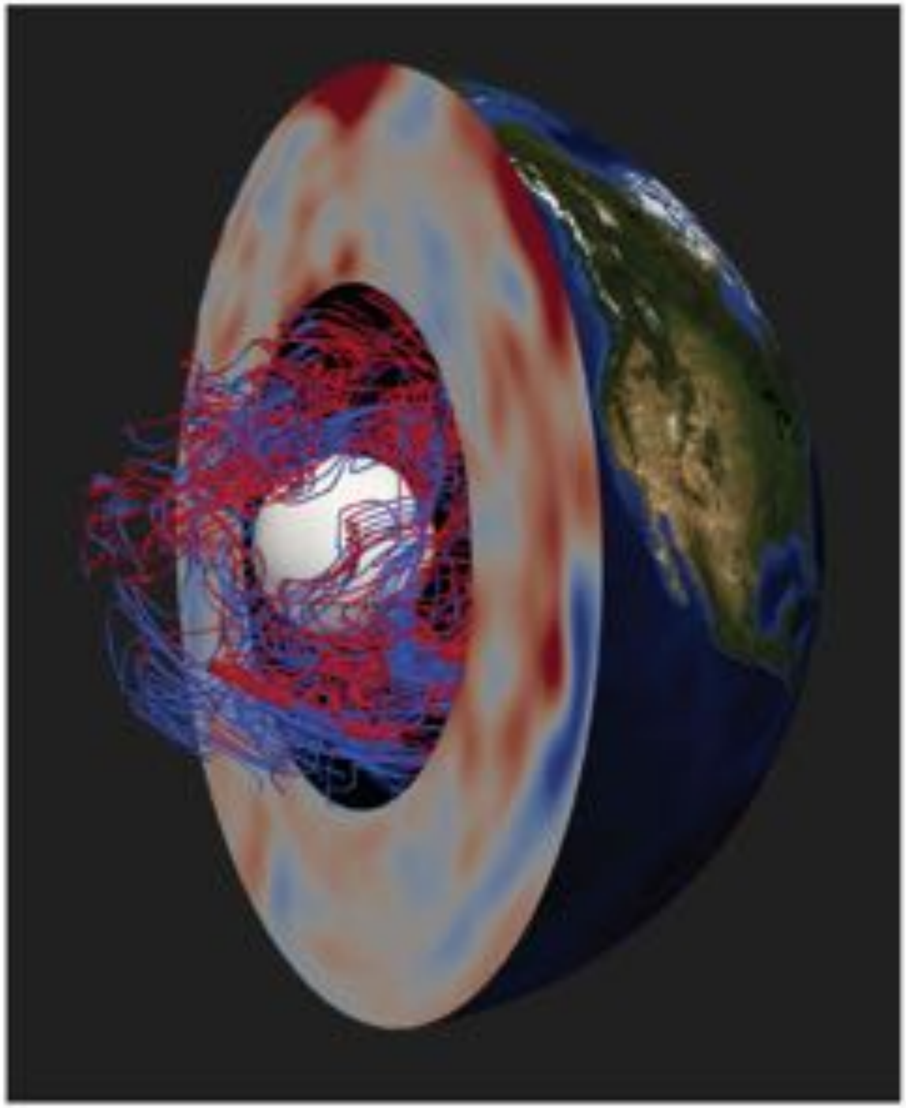
A: They all have dynamos







Dynamo Ingredients



From Stanley 2013

(1)electrically conducting fluid

- Plasma (stars)
- Liquid iron (terrestrial planets)
- Metallic hydrogen (gas giants)
- Ionized water (ice giants)

(2) fluid must have complex motions

- Complex turbulent flows
- Rotation: breaks mirror-symmetry not required, but needed for large-scale, organized fields

(3) motions must be vigorous enough

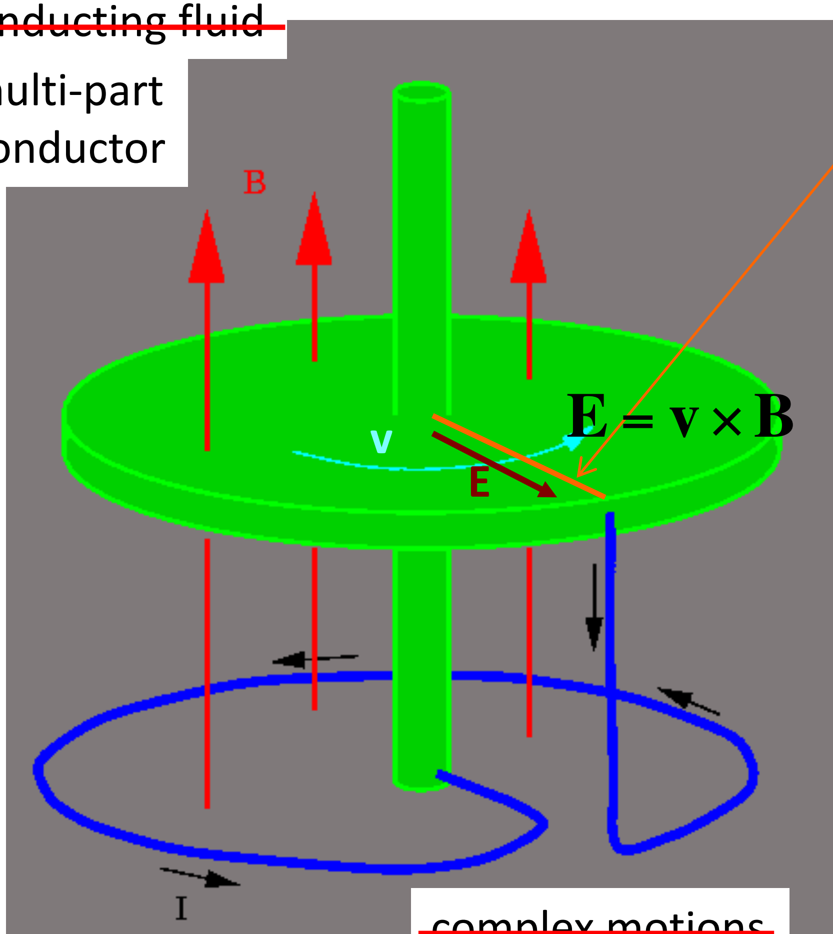
- Figure of merit: Magnetic Reynold's #

Rm = velocity × size × conductivity

A Toy w/ all ingredients

~~conducting fluid~~

multi-part conductor



~~complex motions~~

lack of mirror symmetry

differential motion of parts

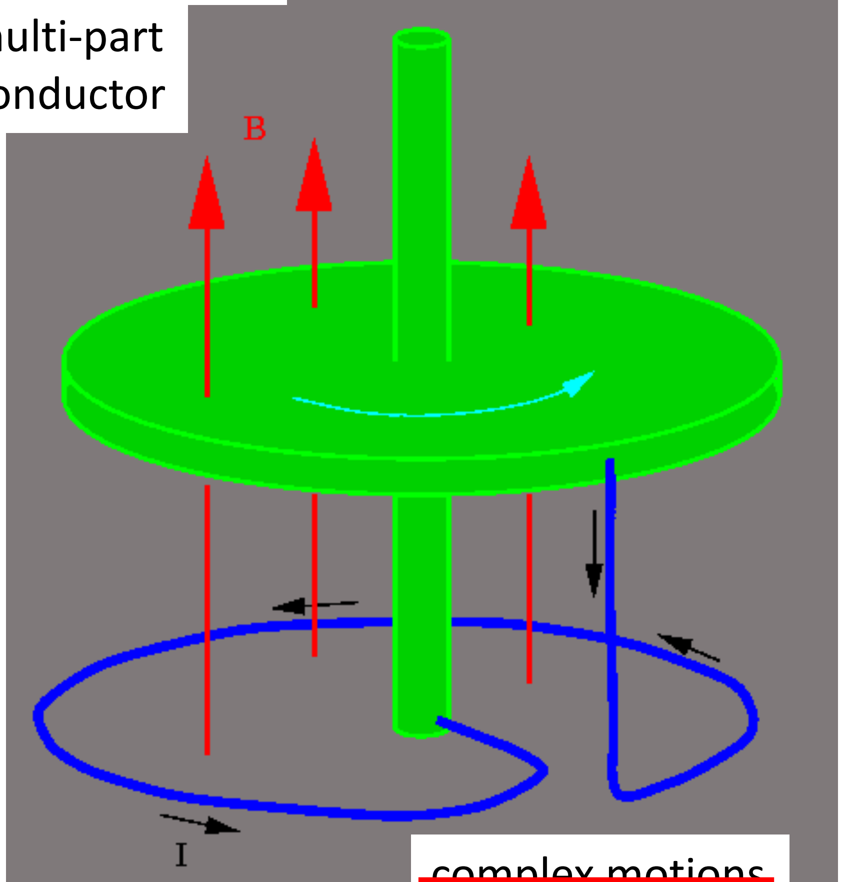
$$V_{\text{disk}} = \int_0^{\ell} vB\,dr = \int_0^{\ell} (\Omega r)\,B\,dr = \tfrac{1}{2\pi}\Omega\Phi_{\text{disk}}$$

- No magnets
- No batteries
- No (net) charge

A Toy w/ all ingredients

~~conducting fluid~~

multi-part conductor



~~complex motions~~

lack of mirror symmetry

differential motion of parts

$$V_{\text{disk}} = \int_0^{\ell} vB\,dr = \int_0^{\ell} (\Omega r) B\,dr = \tfrac{1}{2\pi}\Omega\Phi_{\text{disk}}$$

$$IR = \boxed{\frac{\Omega}{2\pi}\Phi_{\text{disk}}} - \boxed{L\frac{dI}{dt}} = \frac{\Omega}{2\pi}M_{w,d}I - L\frac{dI}{dt}$$

motional EMF — back EMF

$$\frac{dI}{dt} = \left(\boxed{\frac{\Omega}{2\pi}\frac{M_{w,d}}{L}} - \boxed{\frac{R}{L}}\right) I = \gamma I \qquad I(t) = I_0 e^{\gamma t}$$

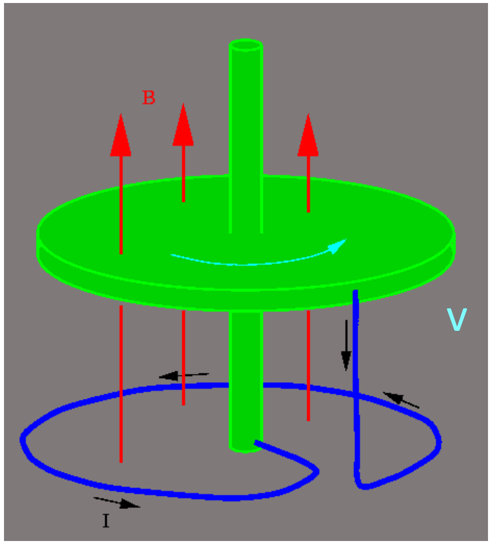
generation — dissipation

Growth: $\gamma > 0 \Leftrightarrow \Omega > 2\pi \dfrac{R}{M_{w,d}}$

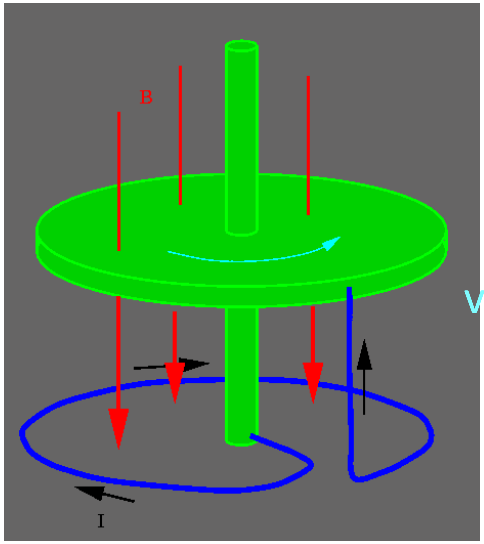
$$\frac{v}{\ell} > 2\pi \frac{1/\sigma\ell}{\mu_0 \ell} = \frac{2\pi}{\mu_0 \sigma \ell^2}$$

Growth: $Rm = \mu_0 \sigma \ell v \;>\; 2\pi$

Toy dynamo amplifies fields of either sign: two attracting states



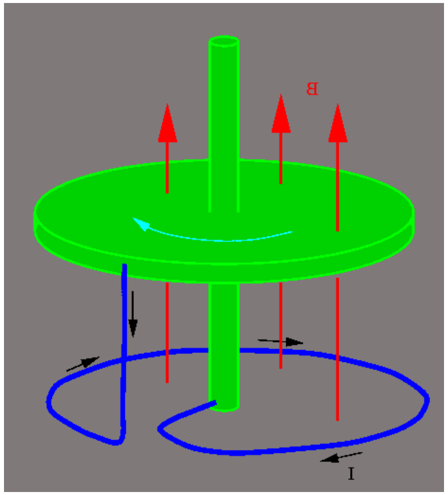
$I_0 > 0$



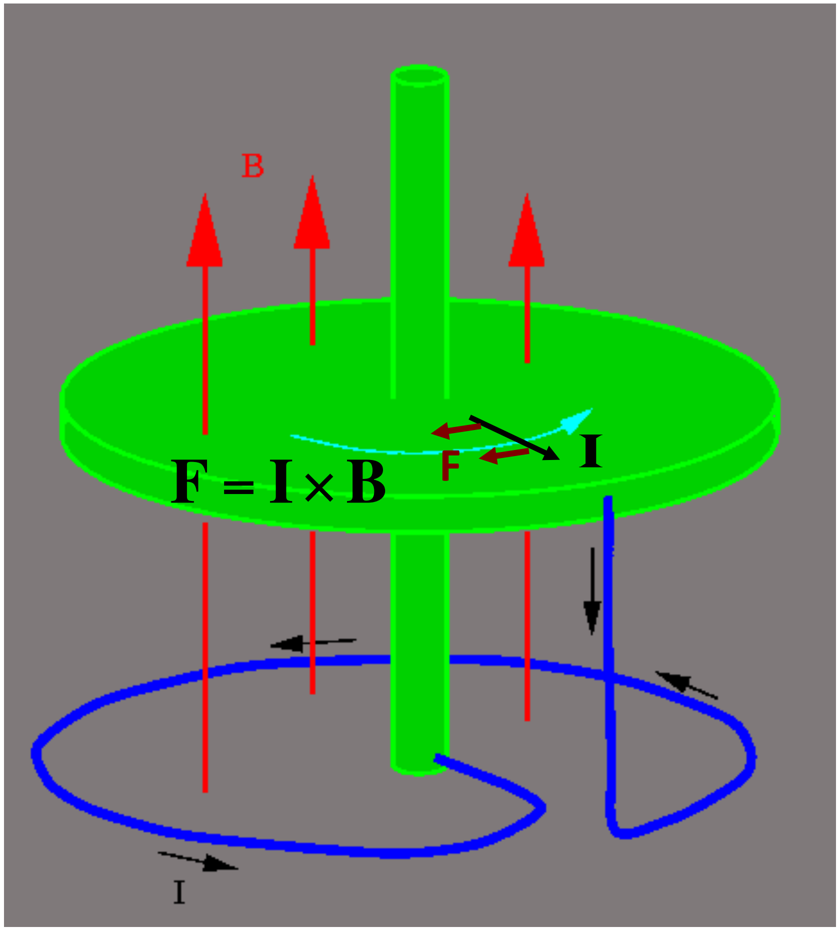
$I_0 < 0$

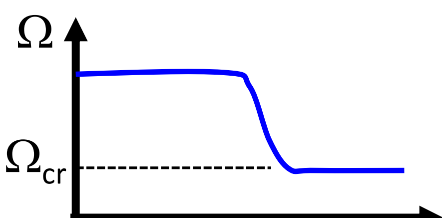
$$I(t) = I_0 e^{\gamma t}$$

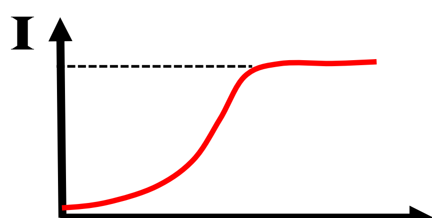
- Reverse velocity AND reflect in mirror ➔ still amplifies
- Do one and not the other ➔ no amplification



Will I grow forever?







Torque on disk carrying current:

$$\tau = \int_0^{\ell} Fr\,dr = \int_0^{\ell} IBr\,dr = \tfrac{1}{2\pi} I\Phi_{\text{disk}} = \frac{M_{w,d}}{2\pi} I^2$$

Power needed to turn disk:

$$P_\Omega = \Omega\tau = \frac{\Omega}{2\pi} M_{w,d} I^2$$

Subtracting Ohmic losses

$$P_\Omega - I^2 R = \left(\frac{\Omega}{2\pi} M_{w,d} - R\right) I^2 = \gamma L I^2$$

$$= \frac{d}{dt}\left(\tfrac{1}{2} L I^2\right)$$

Stored in energy of **B**

Reality: Conducting fluid – MHD

Effect of **B** on conducting fluid

$$\mathbf{J} = \frac{1}{\mu_0}\nabla\times\mathbf{B}$$

Fluid dynamics

$$\frac{\partial\rho}{\partial t} = -\nabla\cdot(\rho\mathbf{v})$$

$$\rho\frac{\partial\mathbf{v}}{\partial t} + \rho(\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla p + \rho\mathbf{g} + \nabla\cdot\overleftrightarrow{\sigma} + \boxed{\mathbf{J}\times\mathbf{B}}$$

Lorentz force

$$\rho c_v\left(\frac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla T\right) = -\tfrac{2}{3}T\nabla\cdot\mathbf{v} + \nabla\mathbf{v} : \overleftrightarrow{\sigma} + \dot{Q} + \boxed{\frac{1}{\sigma}|\mathbf{J}|^2}$$

Ohmic heat

Faraday's + Ohm's laws

$$\frac{\partial\mathbf{B}}{\partial t} = -\nabla\times\mathbf{E} = \nabla\times\left[\mathbf{v}\times\mathbf{B} - \frac{1}{\sigma}\mathbf{J}\right]$$

$$\nabla\times\left[-\frac{1}{\sigma}\mathbf{J}\right] = \nabla\times\left[-\frac{1}{\sigma\mu_0}\nabla\times\mathbf{B}\right] = \eta\nabla^2\mathbf{B} \qquad \eta = \frac{1}{\mu_0\sigma}$$

magnetic diffusivity

Reality: Conducting fluid – MHD

If **B** is weak: kinematic equations

Fluid dynamics

$$\begin{cases} \dfrac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v}) \\ \rho \dfrac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla p + \rho \mathbf{g} + \nabla \cdot \overleftrightarrow{\sigma} \\ \rho c_v \left(\dfrac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right) = -\tfrac{2}{3} T \nabla \cdot \mathbf{v} + \nabla \mathbf{v} : \overleftrightarrow{\sigma} + \dot{Q} \end{cases}$$

Traditional (neutral) fluid – solve first

Faraday's + Ohm's laws

$$\frac{\partial \mathbf{B}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{B} = (\mathbf{B} \cdot \nabla) \mathbf{v} - \mathbf{B} (\nabla \cdot \mathbf{v}) + \eta \nabla^2 \mathbf{B}$$

Linear equation **for B**(**x**,t) – solve w/ known **v**(**x**,t)

Dynamo action in MHD

$$\frac{D\mathbf{B}}{Dt} \qquad \mathbf{B}\cdot[\nabla\mathbf{v} - \mathbf{I}(\nabla\cdot\mathbf{v})] = \mathbf{B}\cdot\mathbf{M}$$

$$\underbrace{\frac{\partial\mathbf{B}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{B}}_{D\mathbf{B}/Dt} = \underbrace{(\mathbf{B}\cdot\nabla)\mathbf{v} - \mathbf{B}(\nabla\cdot\mathbf{v})}_{\mathbf{B}\cdot\mathbf{M}} + \eta\nabla^2\mathbf{B}$$

If **M** has a positive eigenvalue λ > 0

B can grow exponentially: **DYNAMO ACTION**

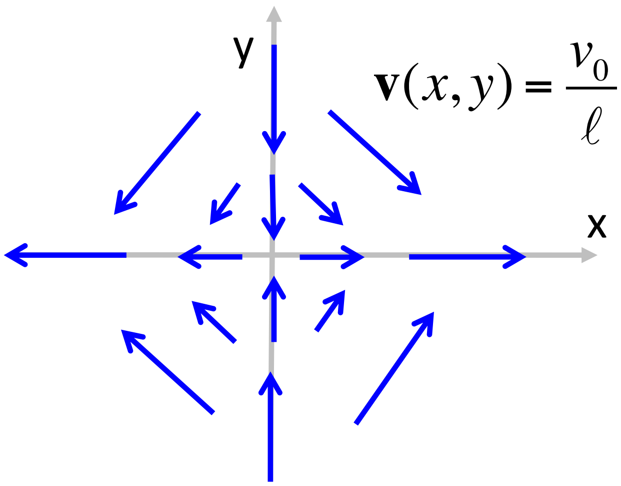
- **B** → -**B** : same e-vector ➔ same λ
- Reverse velocity AND reflect in mirror ➔ λ → λ
- Do one and not the other ➔ λ → -λ

λ → $\frac{v}{\ell}$, $\eta\nabla^2$ → $\frac{\eta}{\ell^2}$

$$\gamma \sim \frac{v}{\ell} - \frac{\eta}{\ell^2} = \frac{v}{\ell}\left(1 - \frac{\eta}{\ell v}\right)$$

Growth: $Rm = \frac{\ell v}{\eta} = \mu_0 \ell v \sigma > 1$

Q: What kind of flow has λ > 0?



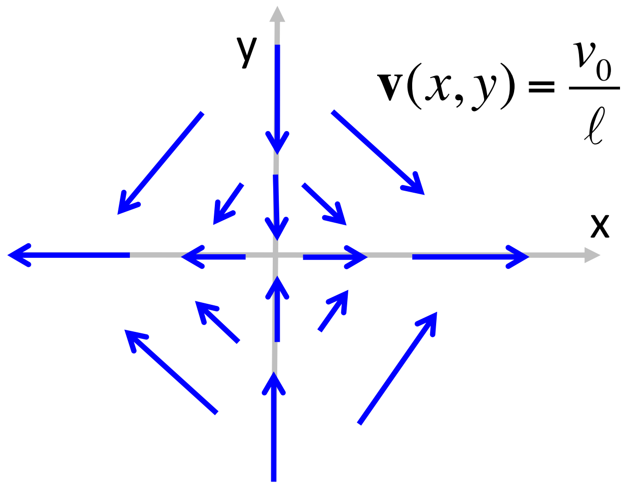
$$\mathbf{v}(x,y) = \frac{v_0}{\ell}\left(x\hat{\mathbf{x}} - y\hat{\mathbf{y}}\right)$$

$$\mathrm{M} = \begin{bmatrix} \partial v_x / \partial x & \partial v_x / \partial y \\ \partial v_y / \partial x & \partial v_y / \partial y \end{bmatrix} = \frac{v_0}{\ell}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\mathrm{M}\cdot\begin{bmatrix} B_0 \\ 0 \end{bmatrix} = \frac{v_0}{\ell}\cdot\begin{bmatrix} B_0 \\ 0 \end{bmatrix}$$

$$\lambda = +\frac{v_0}{\ell}$$

Q: What kind of flow has λ > 0?



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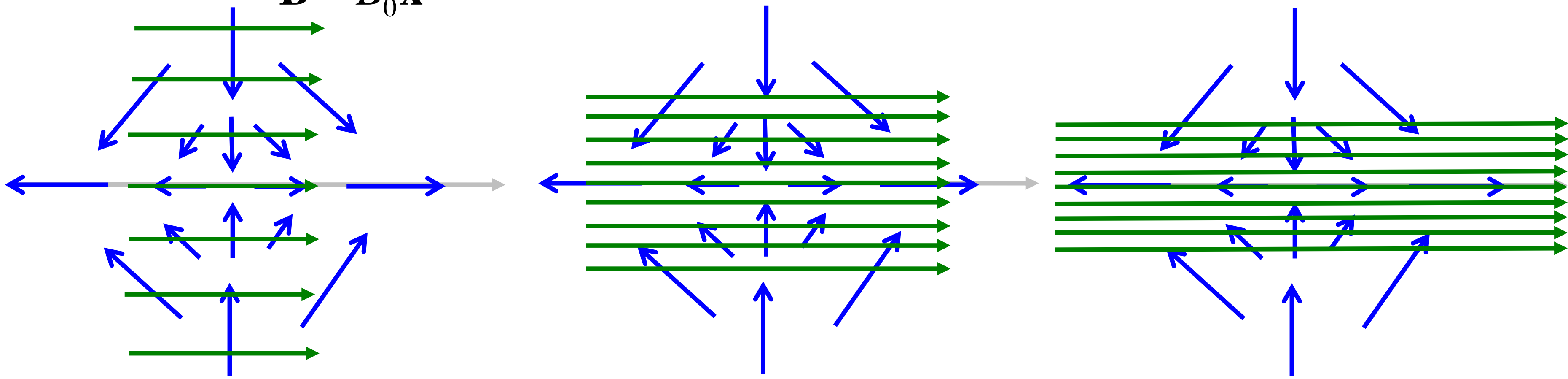
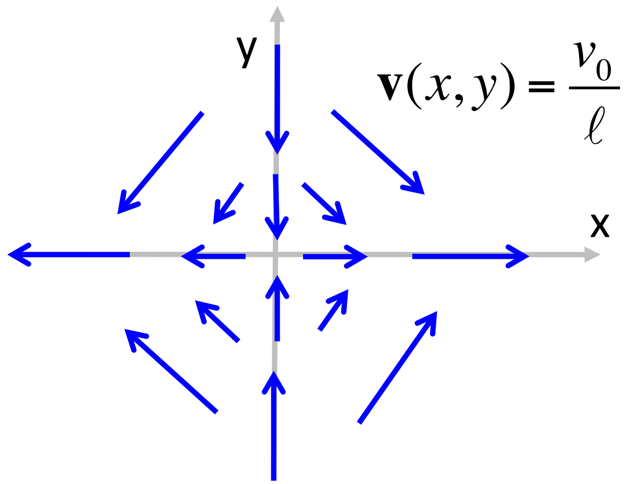
$$\mathbf{B} = B_0\hat{\mathbf{x}}$$



A: stretching flow

Q: What kind of flow has λ > 0?



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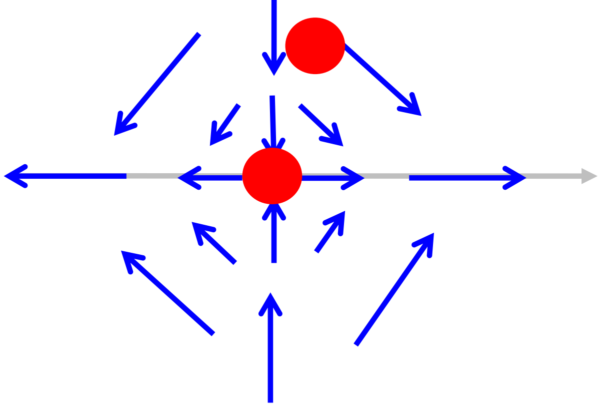
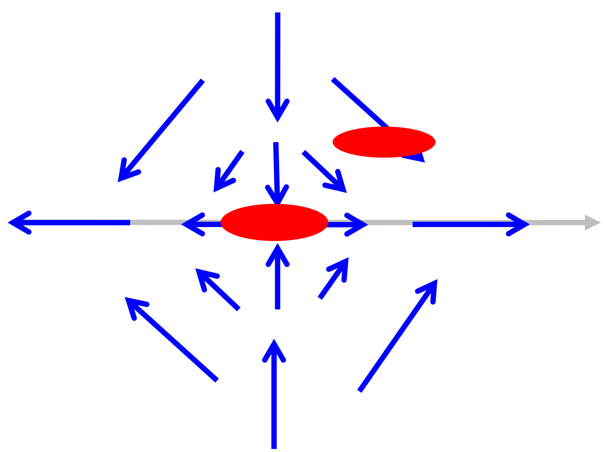
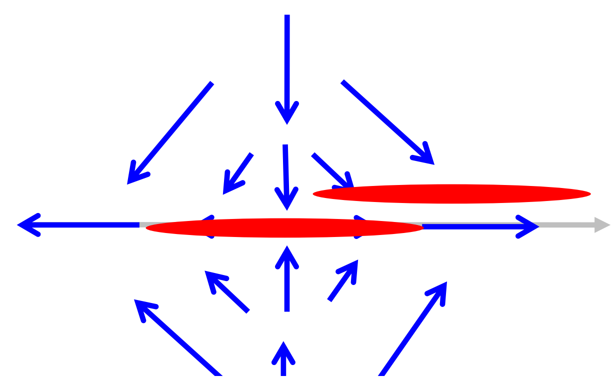






Aspect ratio growth
~ Lyapunov exponent

A: stretching flow

Q: What kind of flow has $\lambda > 0$?

- Turbulent flows have pos. Lyapunov exponent: $\lambda > 0$
  - tend to stretch balls into strands 
  - tend to amplify fields
- Conditions for turbulence:
  - driving: e.g. Rayleigh-Taylor instability
  - viscosity fights driving – must be small

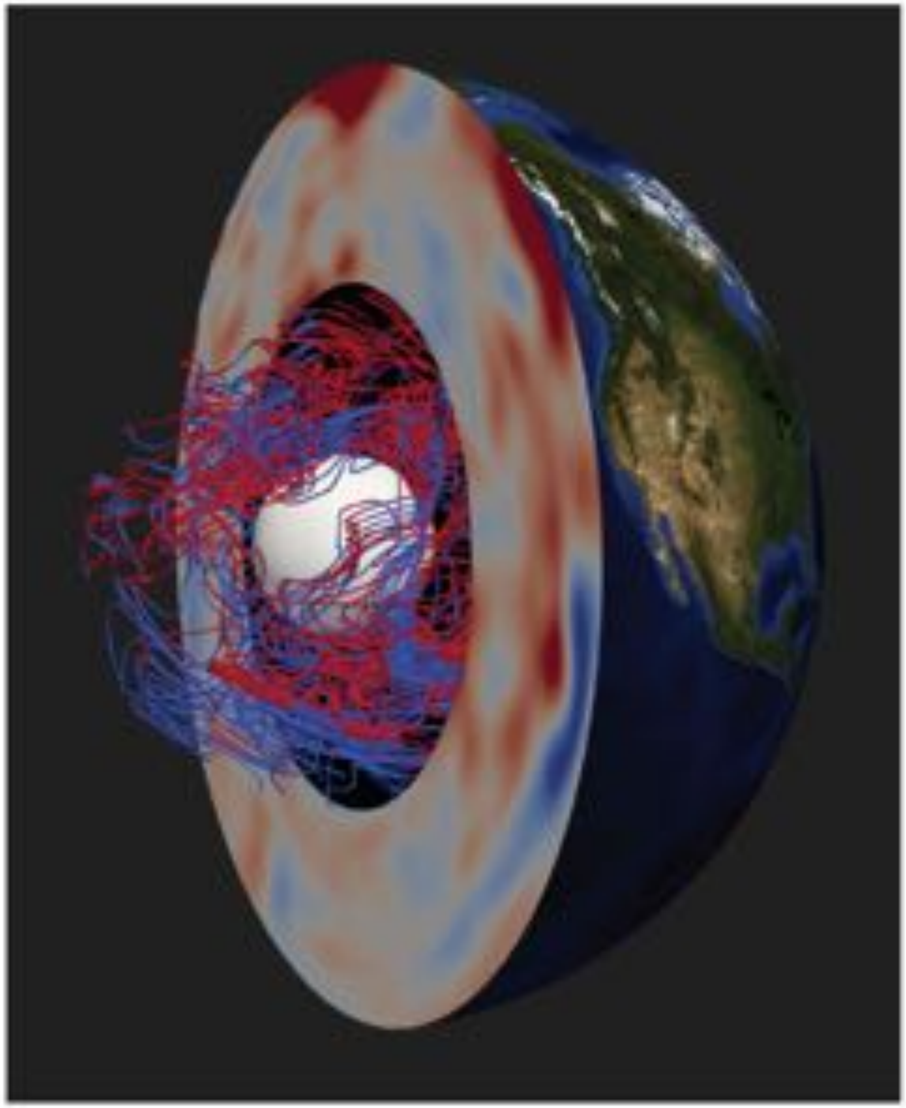
$$\mathrm{Re} = \frac{\ell v}{\upsilon} >> 1$$

- Rotation can organize turbulence:
  align stretching direction → azimuthal
  (toroidal) – known as Ω-effect
  – must be significant w.r.t. fluid motion

$$\mathrm{R}o = \frac{v}{\ell \Omega} << 1$$

	η [m²/s]	υ [m²/s]	L [m]	v [m/s]	Ω [rad/s]	Rm	Re	Ro
Sun (CZ)	1	10^{-2}	10^{8}	1	10^{-6}	10^{8}	10^{10}	10^{-2}
Earth (core)	1	10^{-5}	10^{6}	10^{-4}	10^{-4}	10^{2}	10^{7}	10^{-6}

Dynamo Ingredients



(1) electrically conducting fluid

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- Complex turbulent flows
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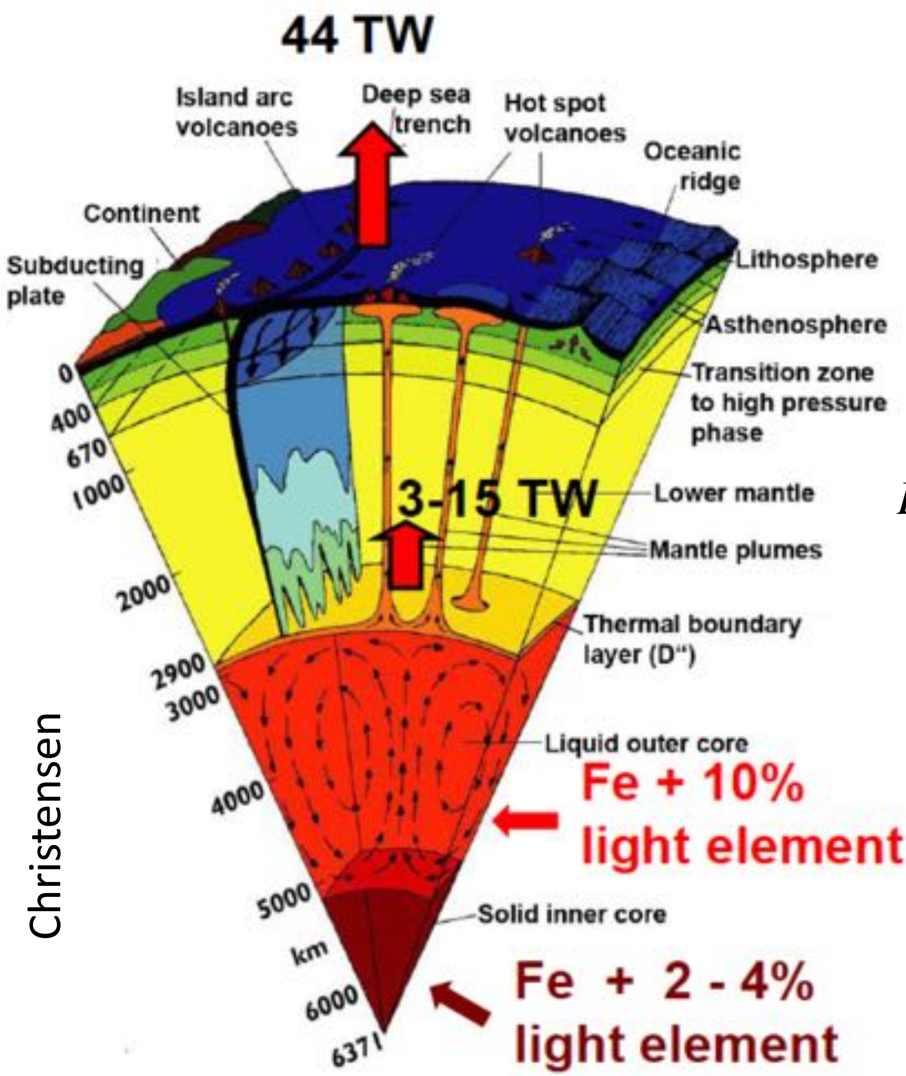
- Figure of merit: Magnetic Reynold's #

Rm = velocity × size × conductivity

From Stanley 2013

How this works for Earth



Christensen

Non-conducting mantle

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} = 0$$

$$\mathbf{B} = -\nabla \chi \qquad \nabla \cdot \mathbf{B} = -\nabla^2 \chi = 0$$

$$\chi(r,\theta,\phi) = \sum_{\ell,m} \tilde{g}_{\ell,m}\, Y_\ell^m(\theta,\phi) \left(\frac{R_\oplus}{r}\right)^{\ell+1}$$

$$B_r(r,\theta,\phi) = -\frac{\partial \chi}{\partial r} = \sum_{\ell,m} (\ell+1)\tilde{g}_{\ell,m}\, Y_\ell^m(\theta,\phi) \left(\frac{R_\oplus}{r}\right)^{\ell+2}$$

simplifies w/ increasing r

Turbulent conducting fluid:

DYNAMO

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \neq 0$$

Complex flows – complex field

A Spherical Harmonic Refresher

l = 1
dipole

$$Y_1^0(\theta,\varphi) \sim \cos\theta \qquad Y_1^{\pm 1}(\theta,\varphi) \sim \sin\theta \; e^{\pm i\varphi}$$

$$\tilde{g}_{1,\pm 1} \sim g_{1,1} \mp i h_{1,1} \qquad (g_{1,0}, g_{1,1}, h_{1,1}) \leftrightarrow \vec{\mu}$$

dipole moment

l = 2
quadrupole

$$Y_2^0(\theta,\varphi) \sim \tfrac{3}{4}\cos 2\theta + \tfrac{1}{4} \qquad Y_2^{\pm 2}(\theta,\varphi) \sim \sin\theta \, e^{\pm 2i\varphi}$$

$\ell = 2$

$$(g_{2,0}, g_{2,1}, h_{2,1}, g_{2,2}, h_{2,2}) \leftrightarrow \overleftrightarrow{Q}$$

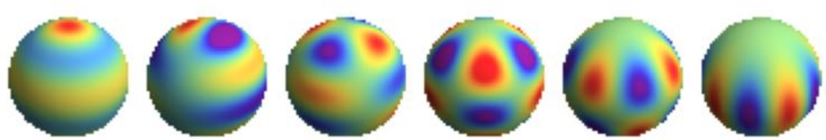
quadrupole tensor

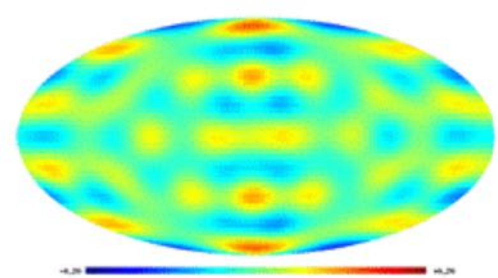
higher l:

$$Y_\ell^0(\theta,\varphi) \sim \cos \ell\theta + \cdots \qquad Y_\ell^{\pm \ell}(\theta,\varphi) \sim \sin\theta \, e^{\pm \ell i\varphi}$$

Finer scale: l periods around circle
More components: 2l +1 real coefficients

$\ell = 5$

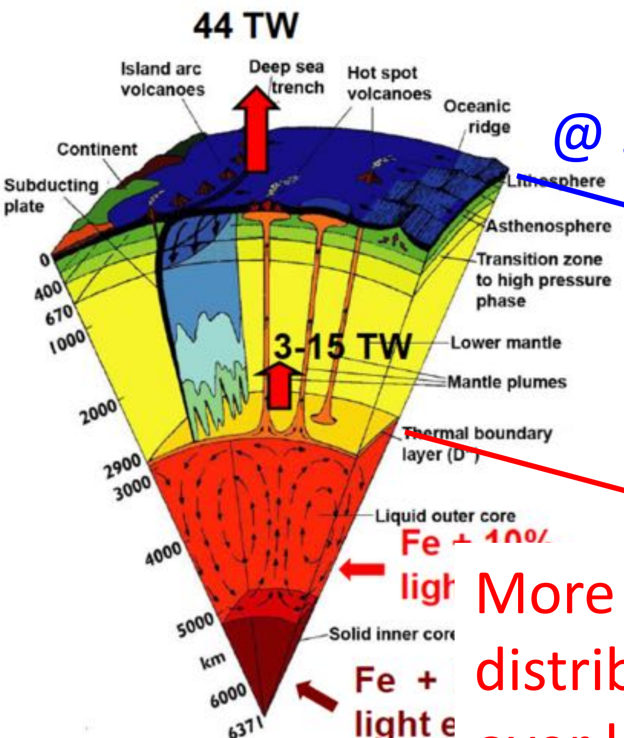


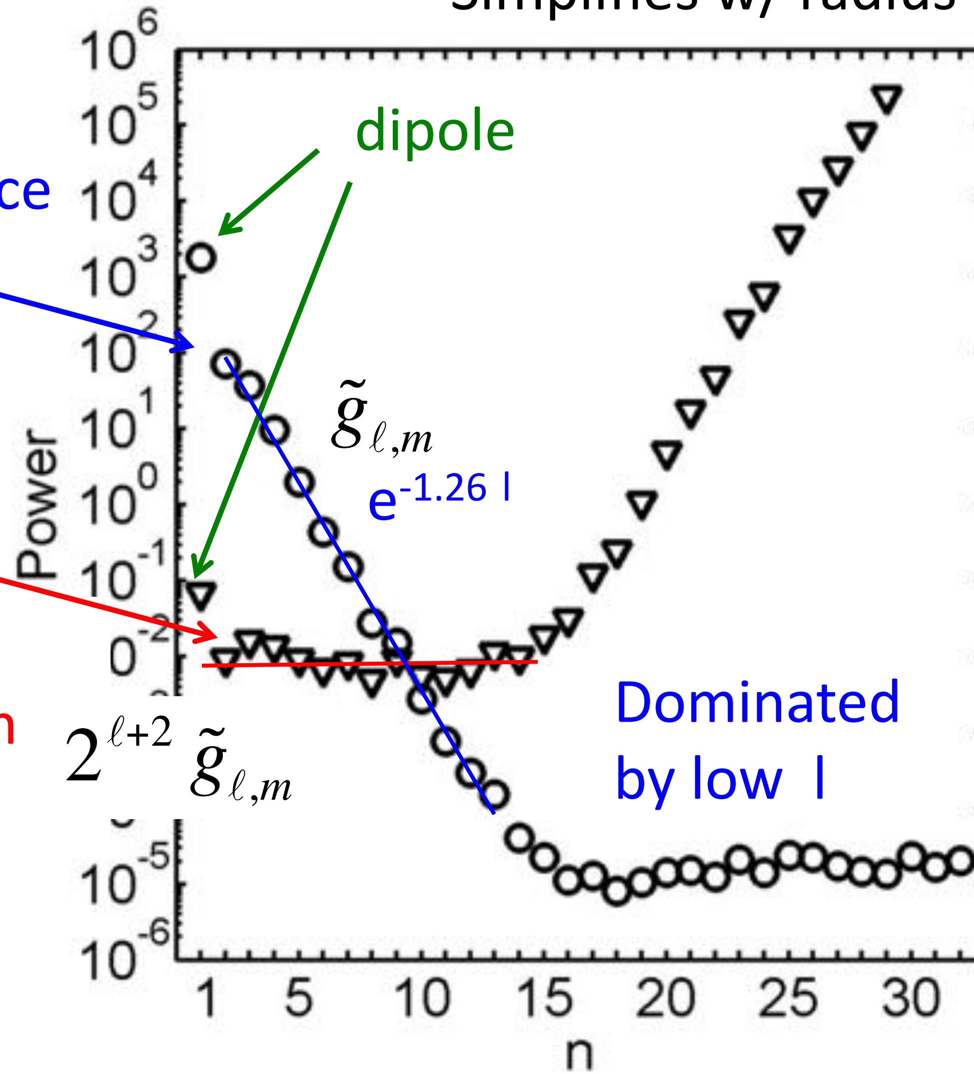


$\ell = 9$

$$B_r(r,\theta,\phi) = \sum_{\ell,m} (\ell+1)\tilde{g}_{\ell,m}\, Y_\ell^m(\theta,\phi) \left(\frac{R_\oplus}{r}\right)^{\ell+2}$$

Simplifies w/ radius





@ surface

More even distribution over l

@ core-mantle boundary

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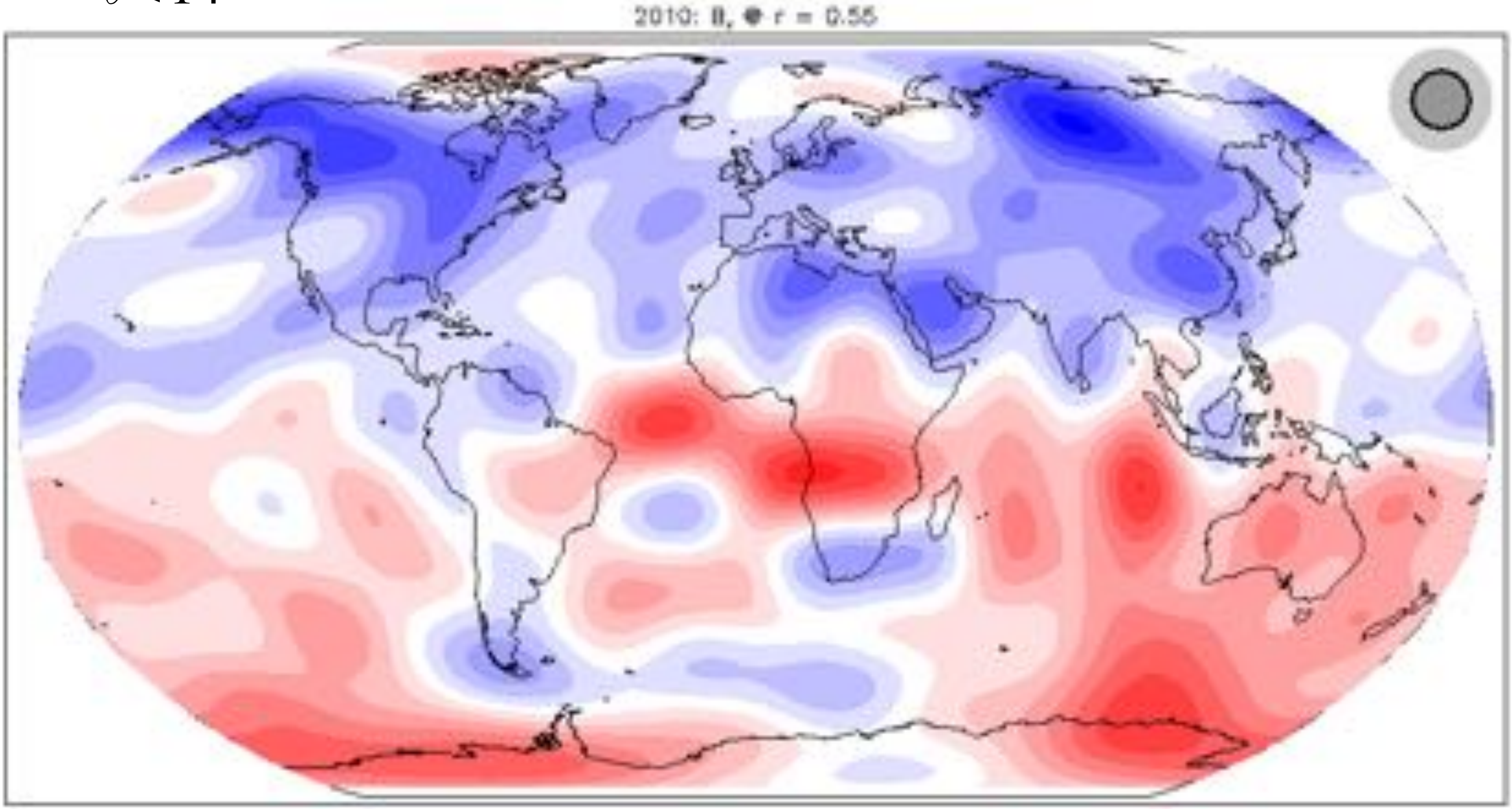
Observation: what B_r looks like today

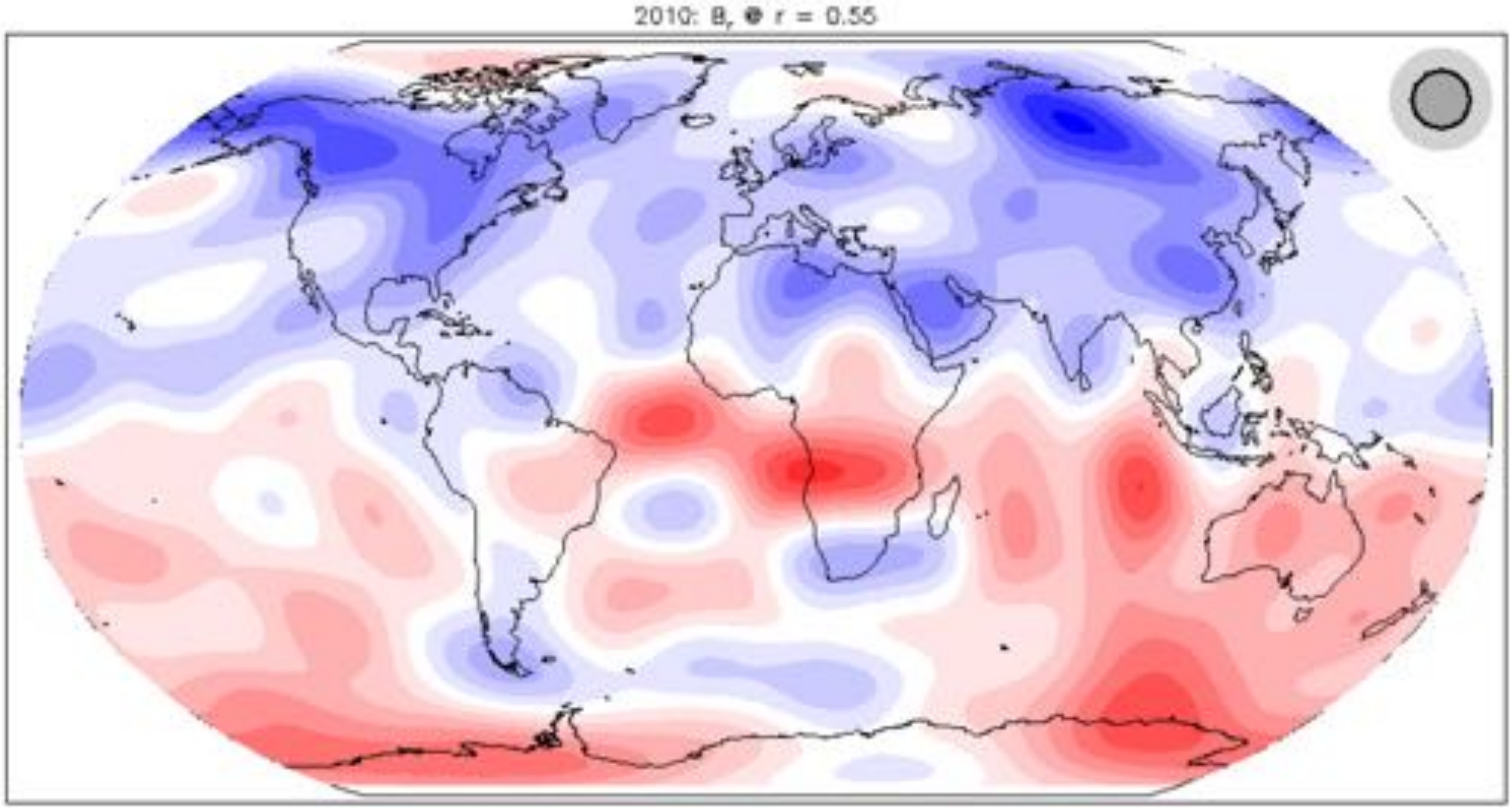
@ core-mantle boundary: lower boundary of potential region

$\ell < 14$



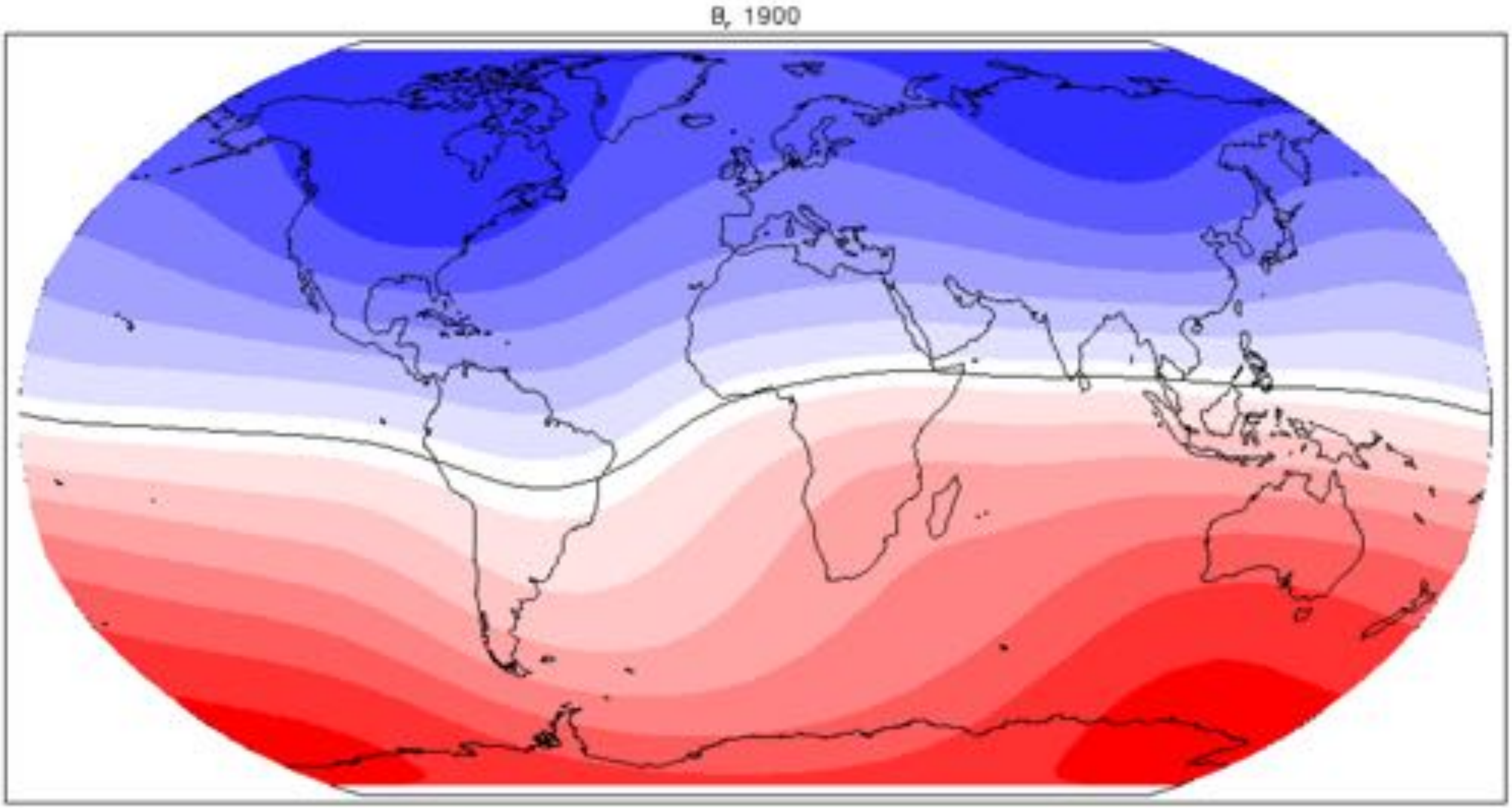
Simplifies w/ increasing r

$$B_r(r,\theta,\phi) = -\frac{\partial \chi}{\partial r} = \sum_{\ell,m} (\ell+1)\tilde{g}_{\ell,m}\, Y_\ell^m(\theta,\phi)\left(\frac{R_\oplus}{r}\right)^{\ell+2}$$



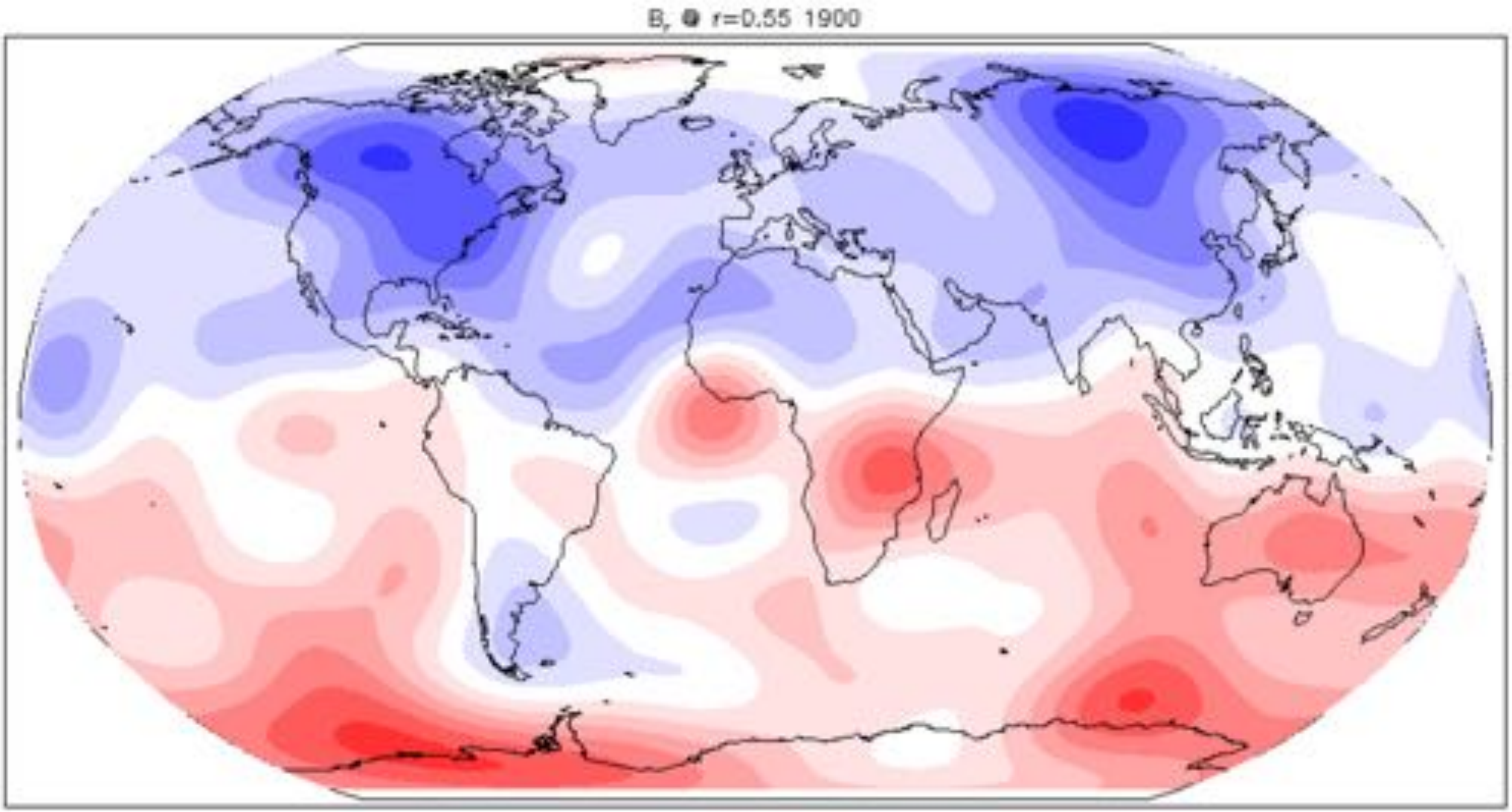
Evolution of field

@ surface for 100 years

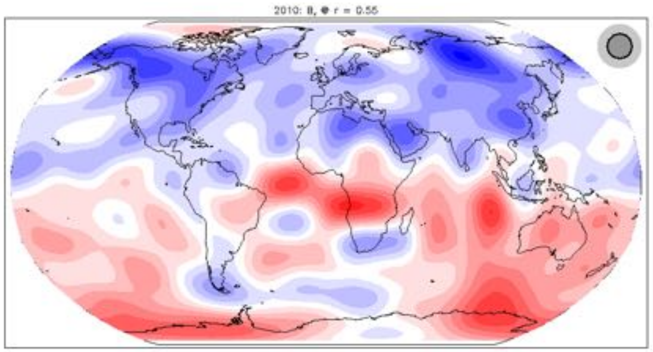


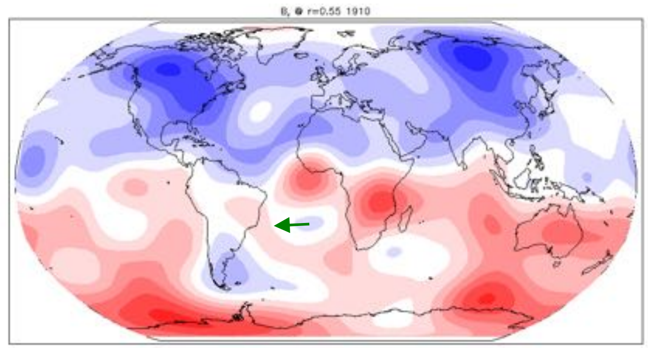
Evolution of field

@ core-mantle boundary for 100 years



Use evolution to infer fluid velocity

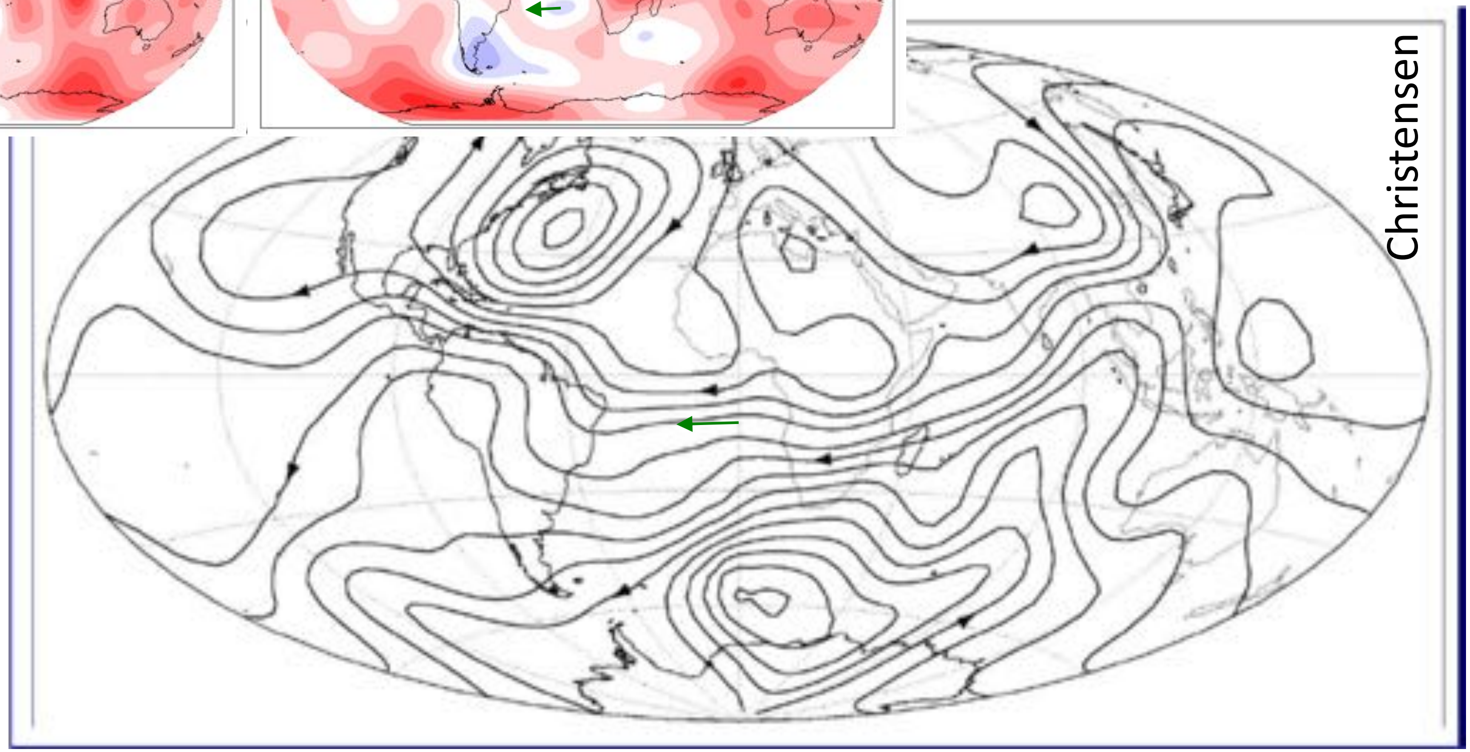




$v \sim 3 \times 10^{-4}$ m/s

➔ Δx = 1,000 km

➔ in 100 years



Subsonic flow,
Ignore
stratification

$$\nabla \cdot \mathbf{v} = 0$$

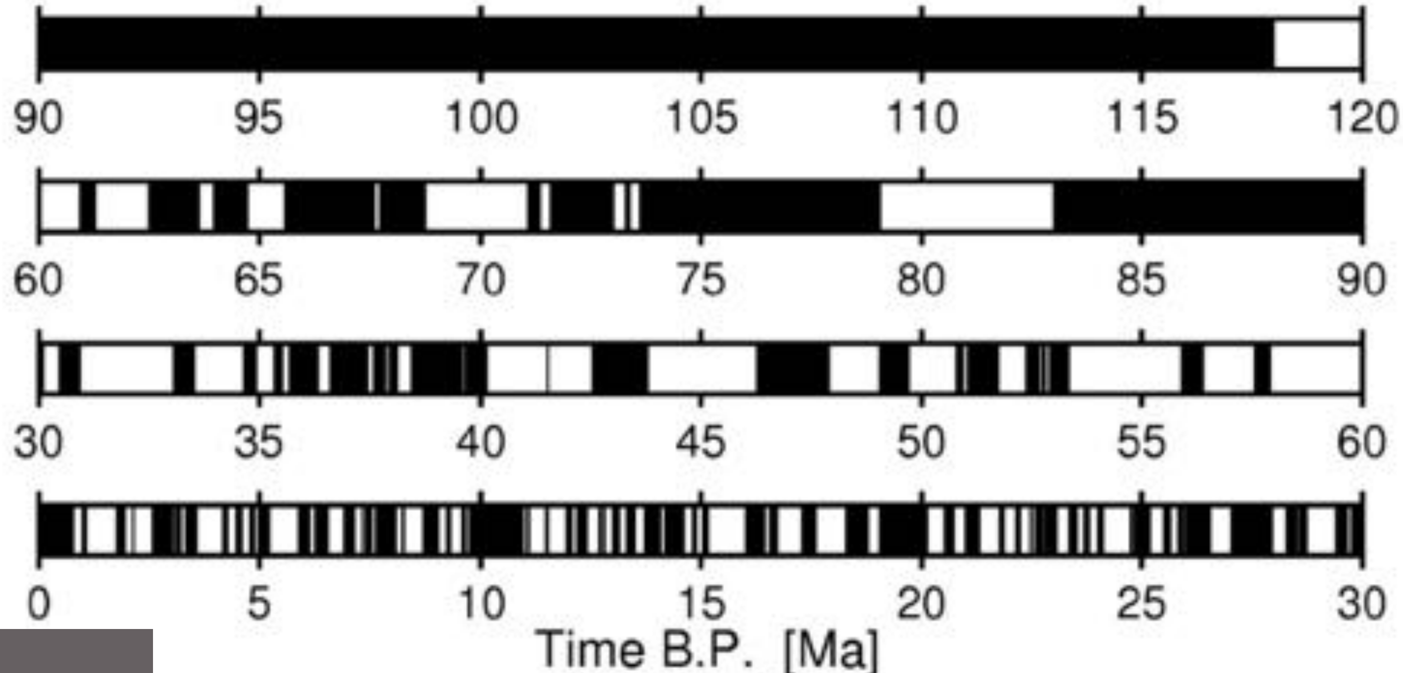
Ignore diffusion

$$\frac{\partial \mathbf{B}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{B} = (\mathbf{B} \cdot \nabla)\mathbf{v} - \mathbf{B}(\nabla \cdot \mathbf{v}) + \eta \nabla^2 \mathbf{B}$$

known

$$\frac{\partial B_r}{\partial t} + \nabla \cdot (\mathbf{v} B_r) = 0$$

Longer-term evolution



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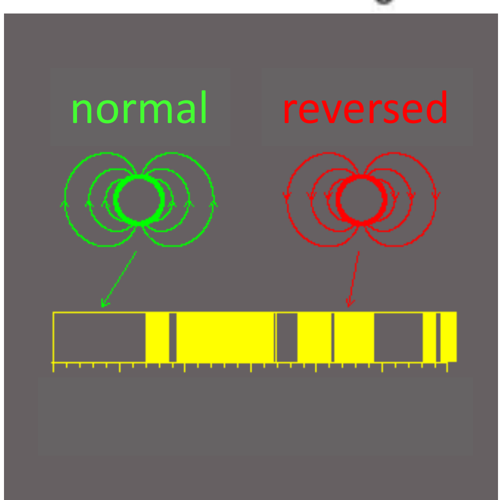
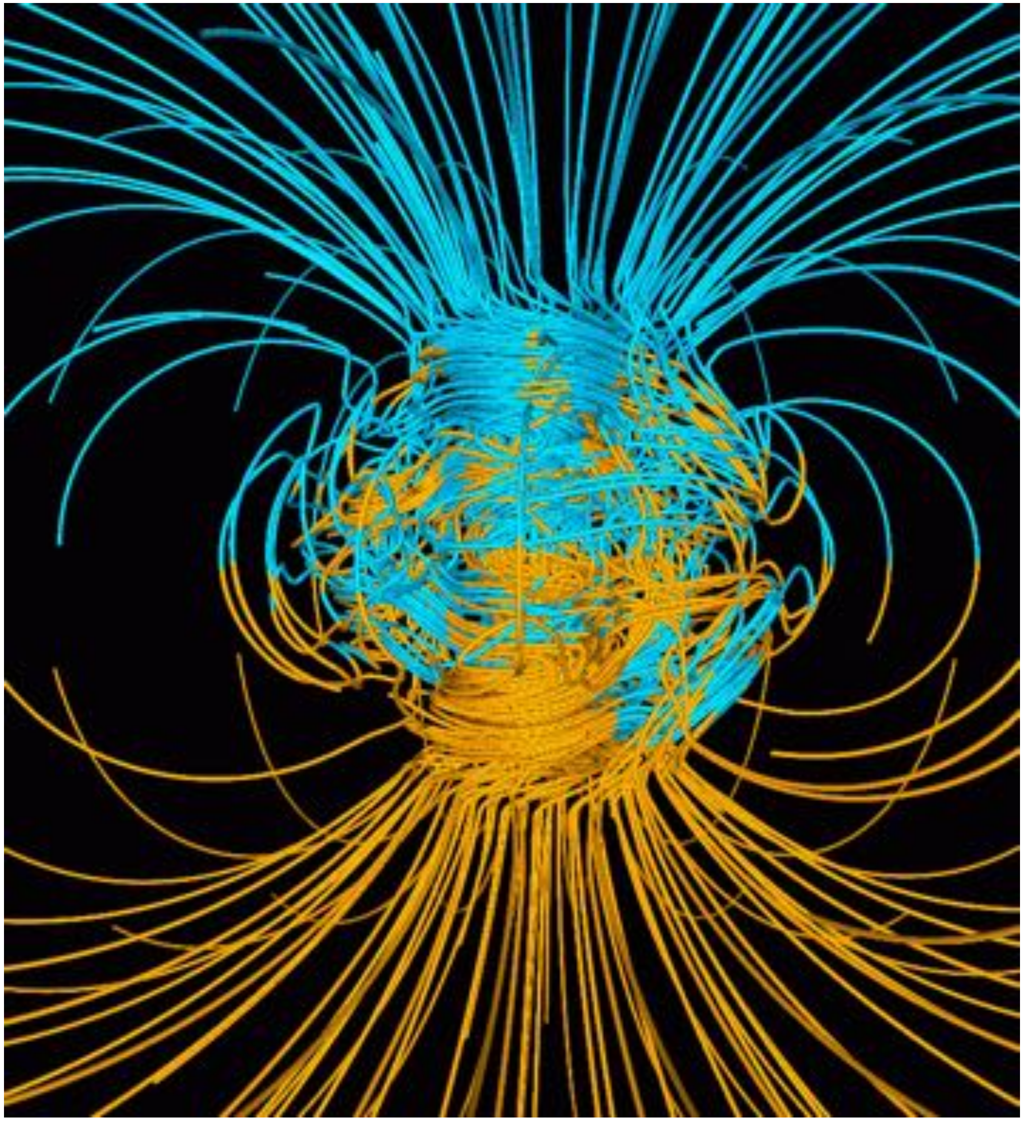


Like toy dynamo, Earth works in 2 modes. Flips between them seemingly at random

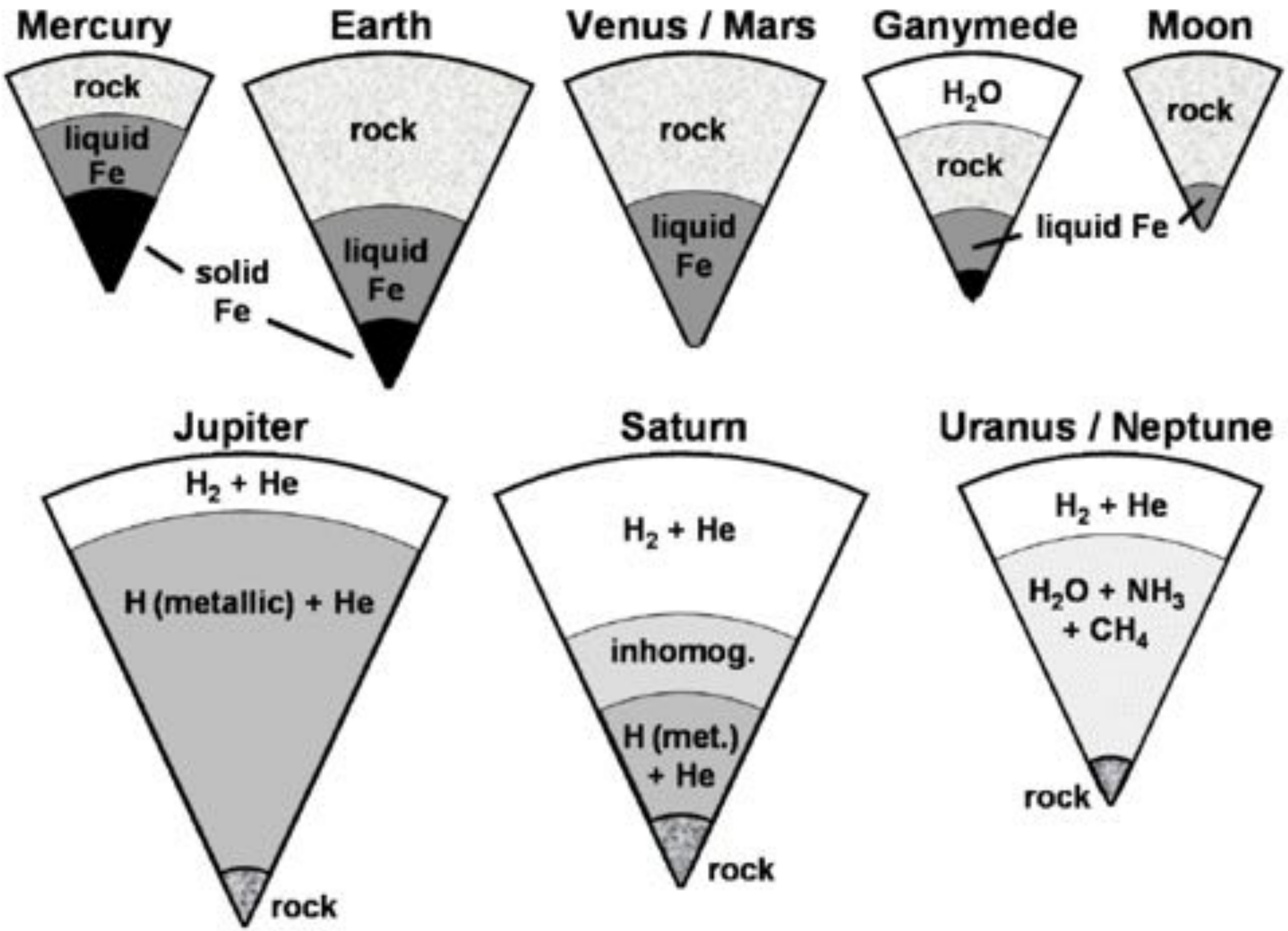
Model geodynamo

http://www.es.ucsc.edu/~glatz/geodynamo.html



- Glatzmaier & Roberts 1995
- Numerical solution of MHD
- Toroidal structure inside convecting core

Other planets



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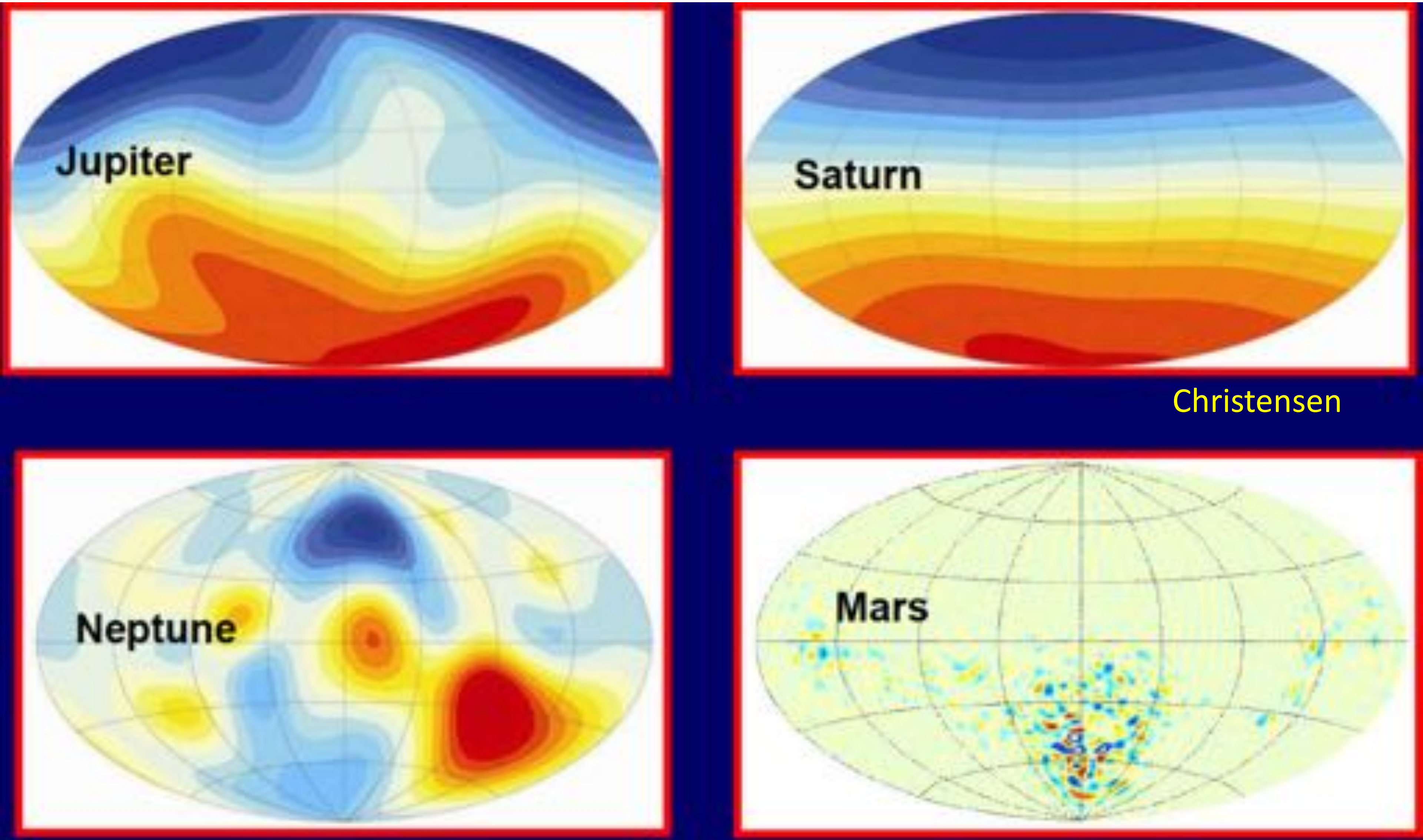
Other planets

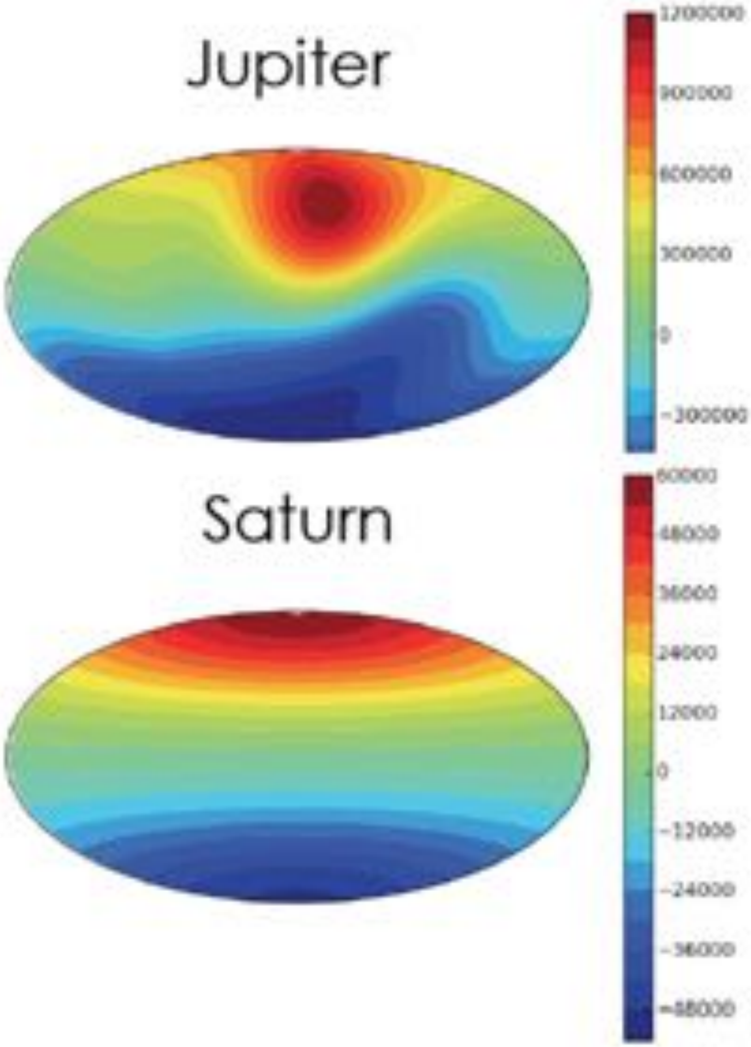
Christensen

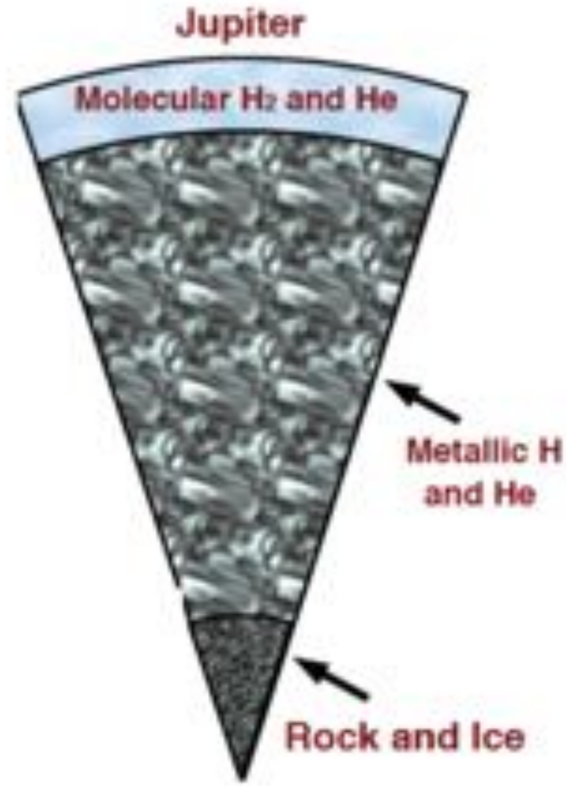
Planet	Dynamo	R_c/R_p	B_s [μT]	Dip. tilt	$\frac{\text{Quadr}}{\text{Dipole}}$
Mercury	Yes (?)	0.75	0.35	<5° ?	0.1-0.5
Venus	No	0.55			
Earth	Yes	0.55	44	10.4°	0.04
Moon	No	0.2 ?			
Mars	No, but in past	0.5			
Jupiter	Yes	0.84	640	9.4°	0.10
Saturn	Yes	0.6	31	0°	0.02
Uranus	Yes	0.75	48	59°	1.3
Neptune	Yes	0.75	47	45°	2.7
Ganymede	Yes	0.3 ?	1.0	< 5° ?	?

What that means



Gas Giants



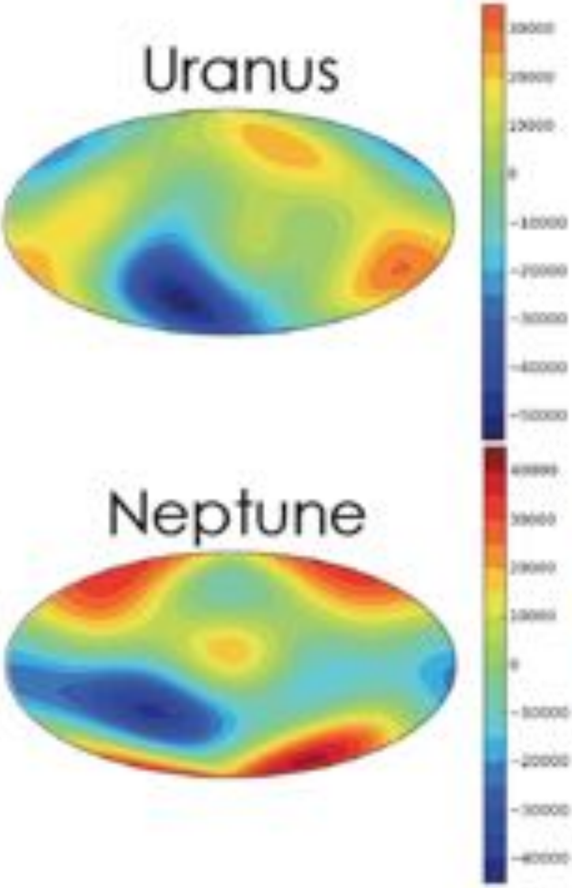


Stanley

- large pressure range → radially variable properties can be important

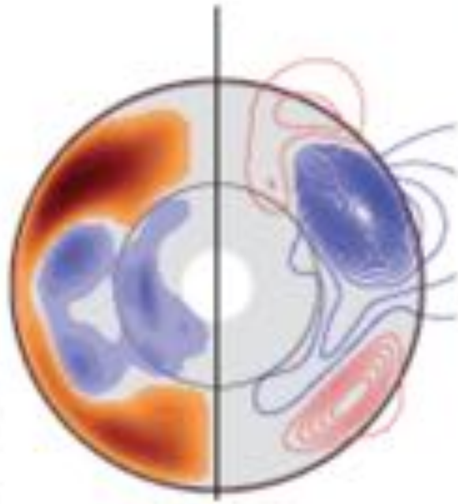
ICE GIANTS

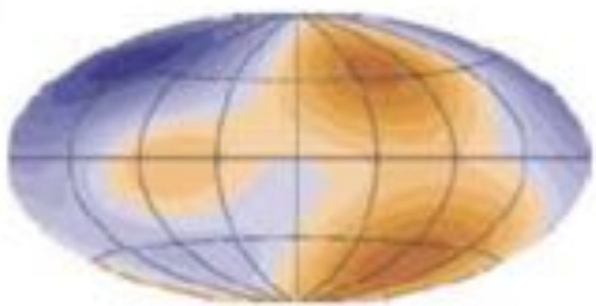
Stanley

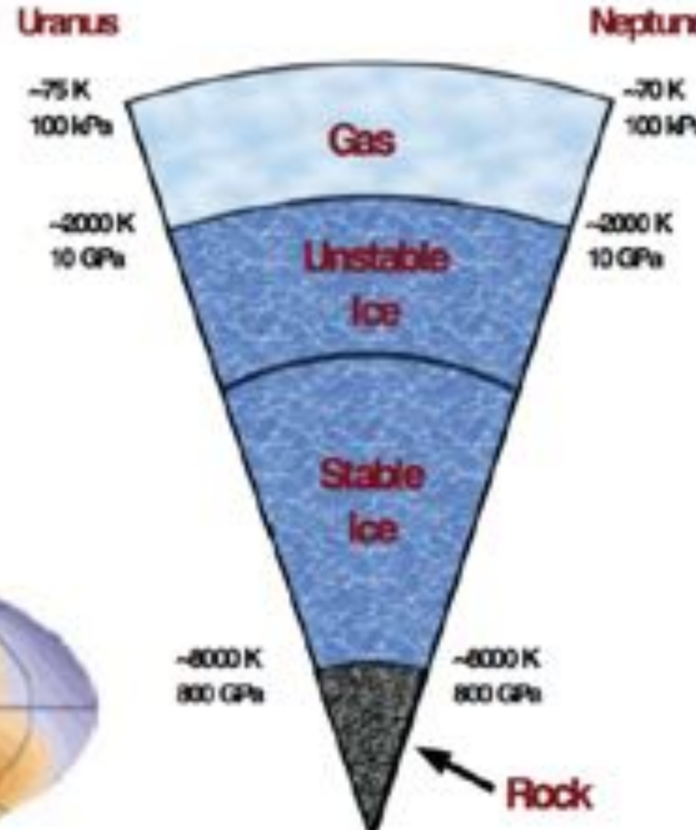


Stanley & Bloxham 2004, 2006

- work in a geometry suggested by low heat flow observations

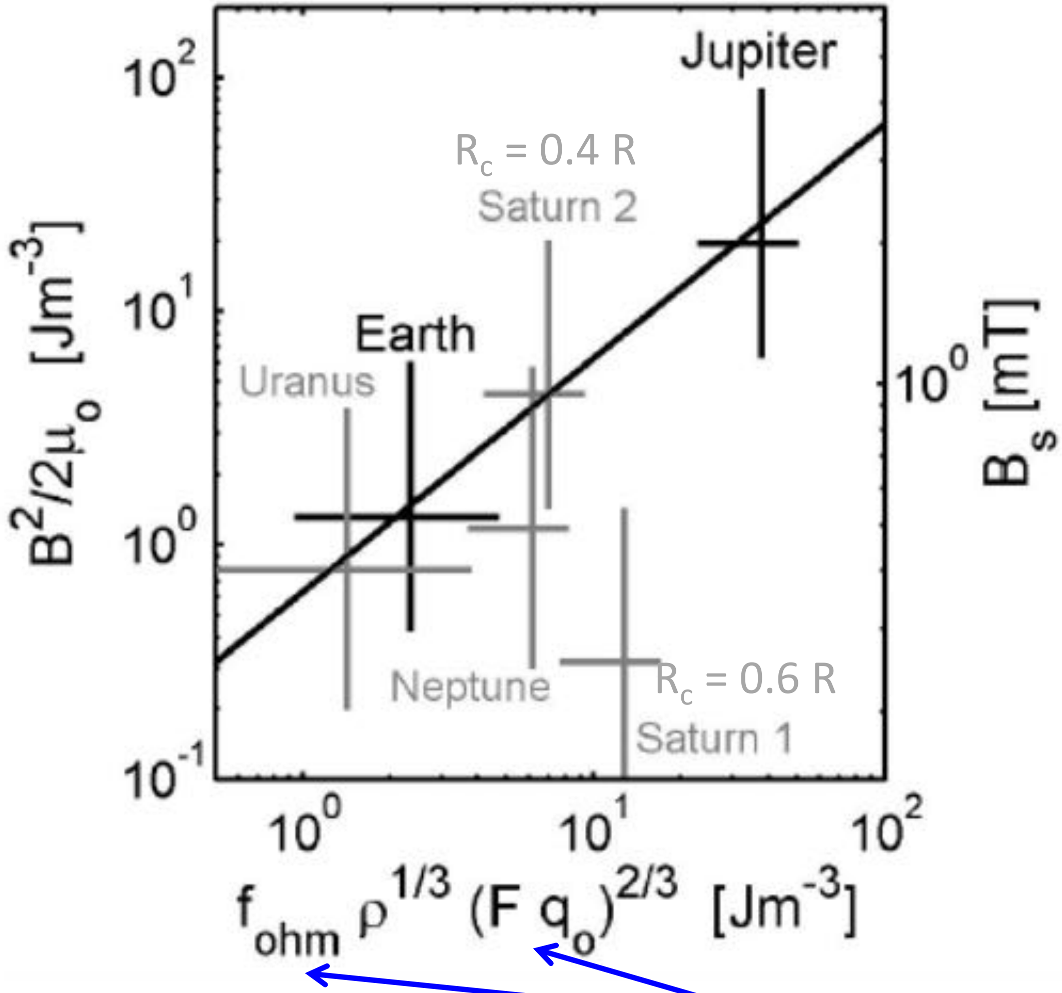






Level of saturation

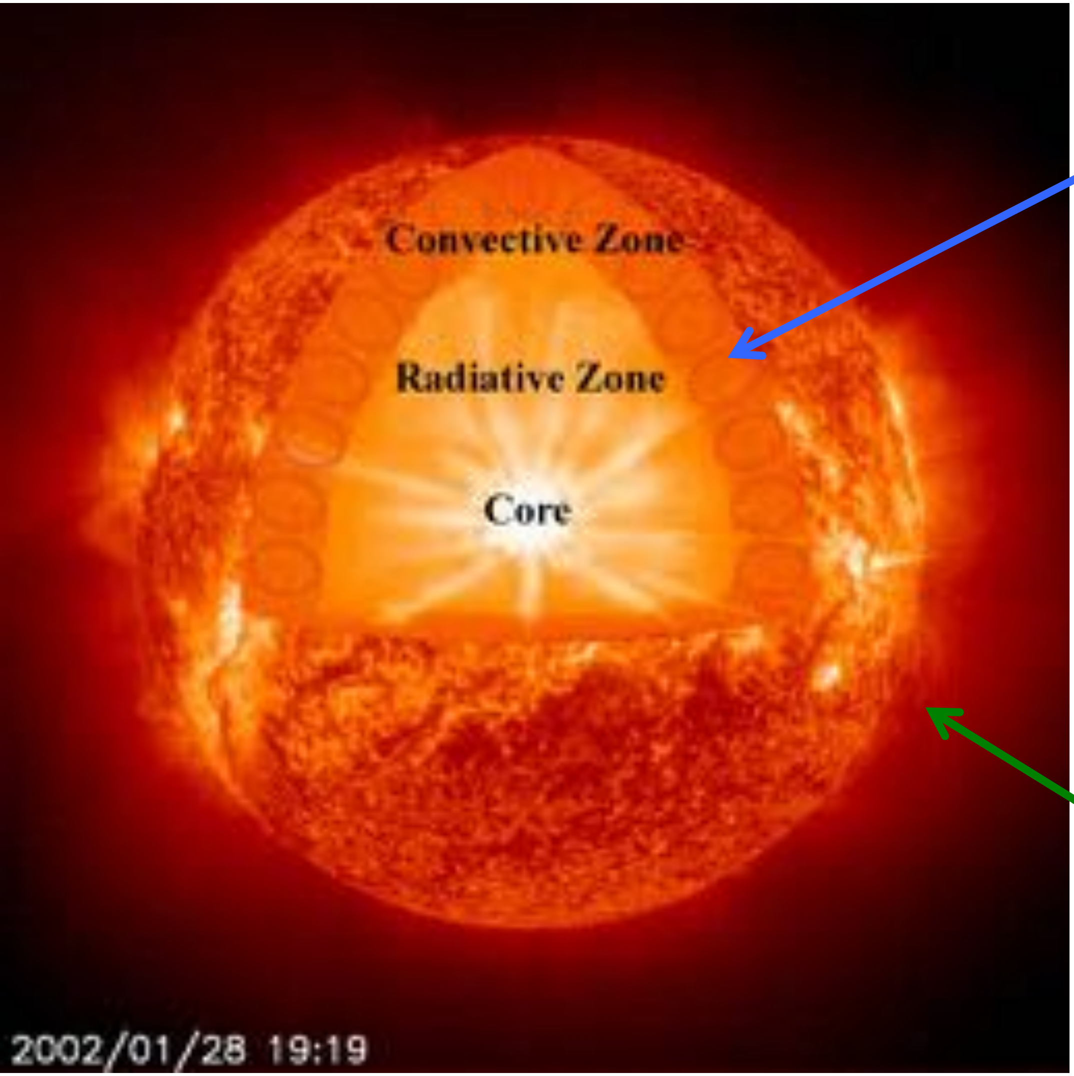
from Christensen



dimensionless factors

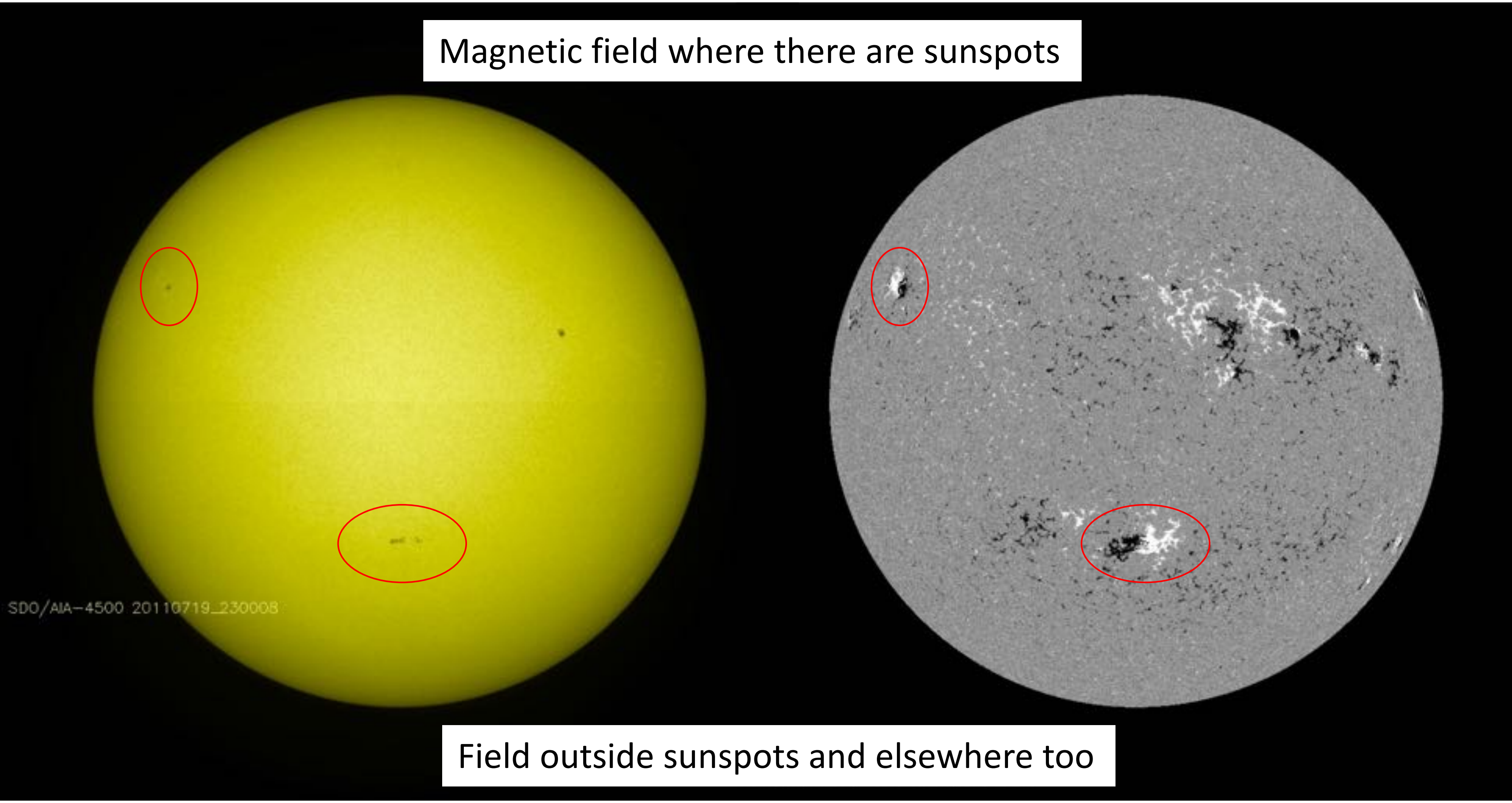
B saturates
(exp growth ends)
when driving power – thermal conduction q_0 – balances Ohmic dissipation

How it works in the Sun



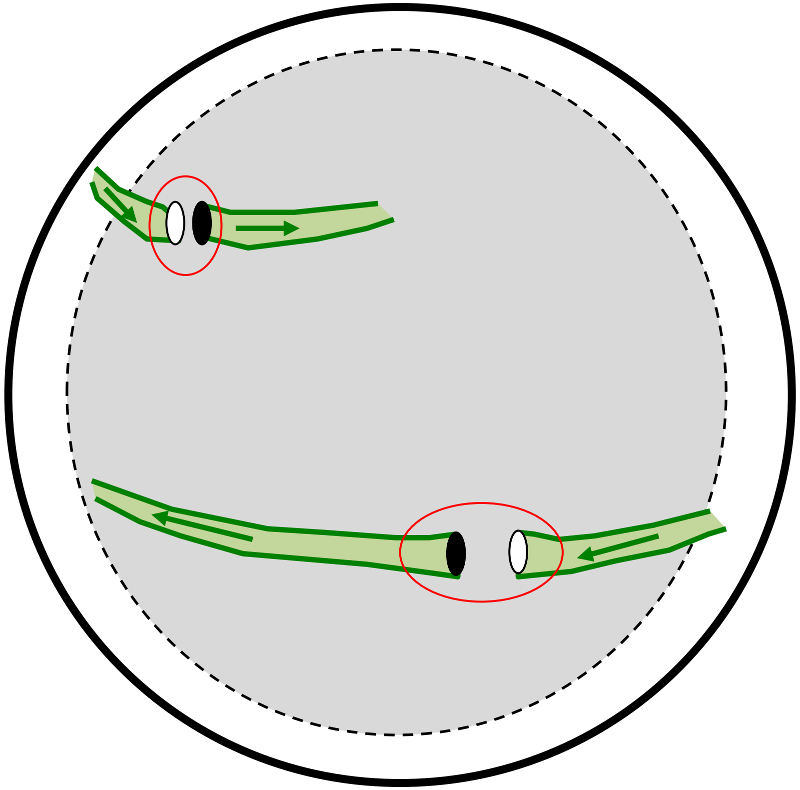
- Entire Star: H/He plasma
- Convection Zone (CZ)
  - Outer 200,000 km
  - Turbulence:
    $Re = 10^{10}$
  - Thermally driven
  - Good conductor
    $Rm = 10^{8}$
  - Rotation effective
    $Ro = 10^{-2}$
- Corona – conductive but tenuous:
  J smaller (~0?)

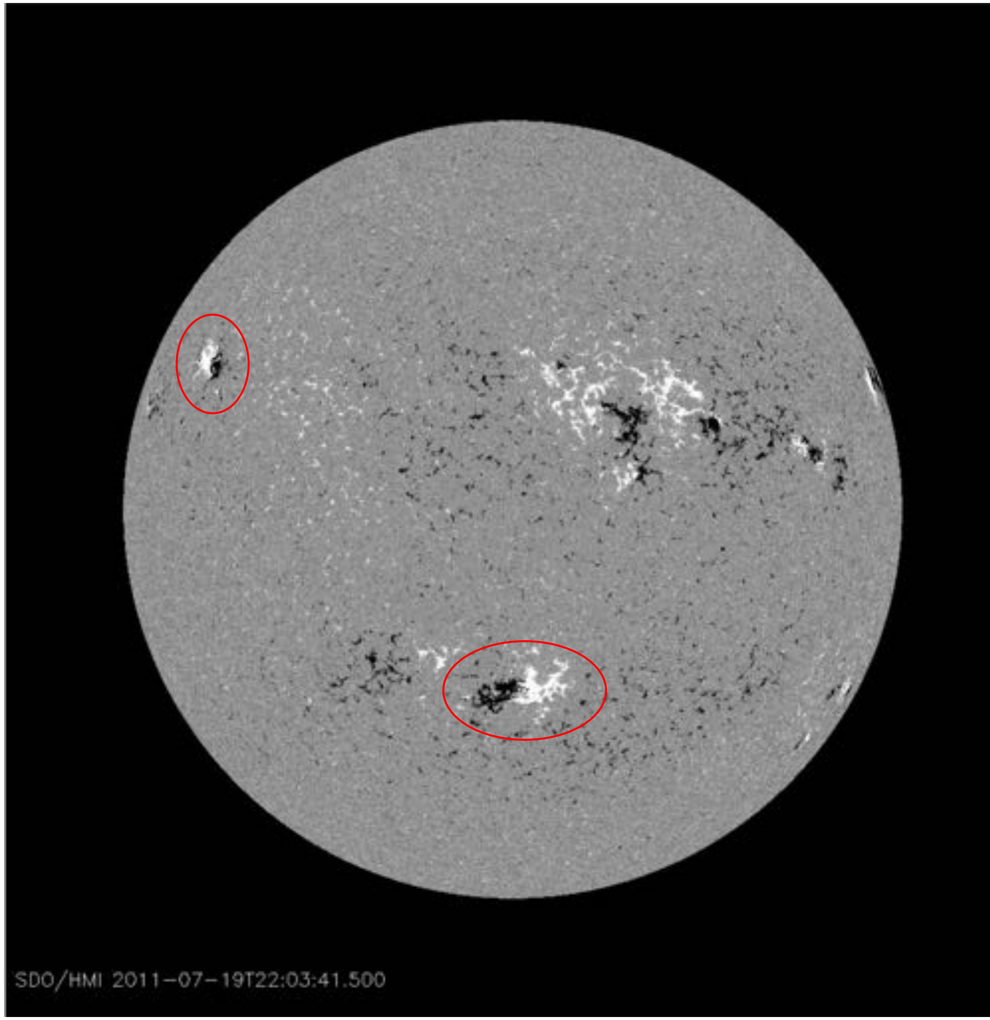
Evidence of the dynamo



Evidence of the dynamo

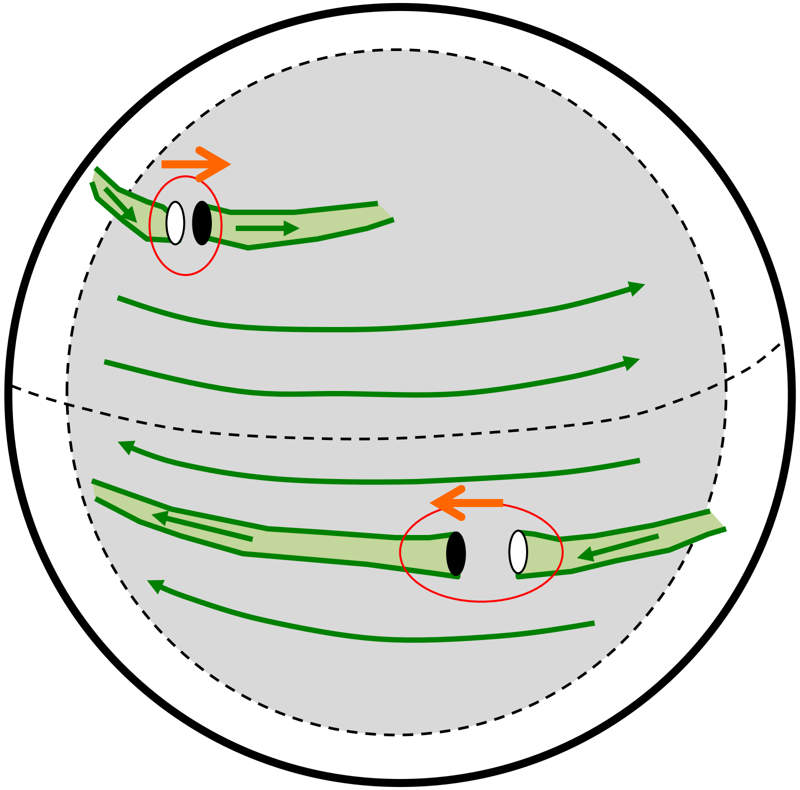
Field is **fibril**

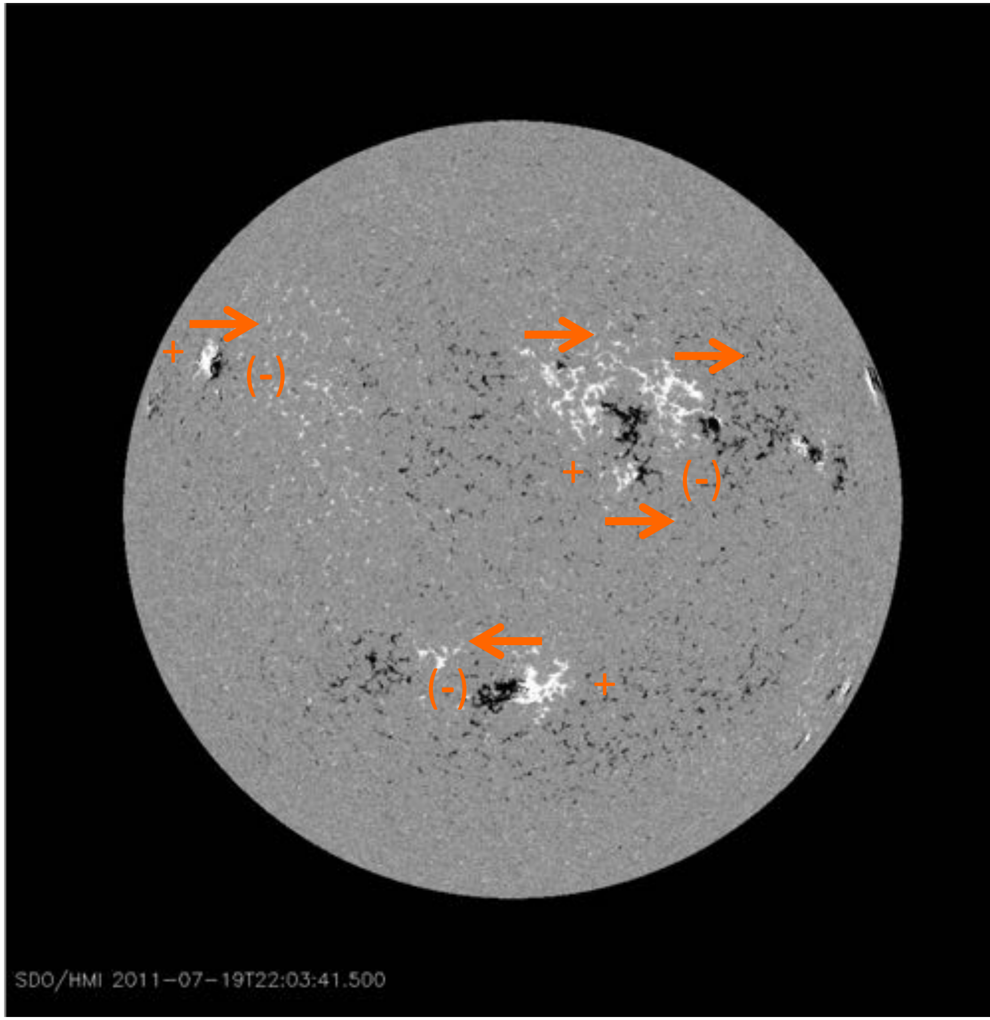


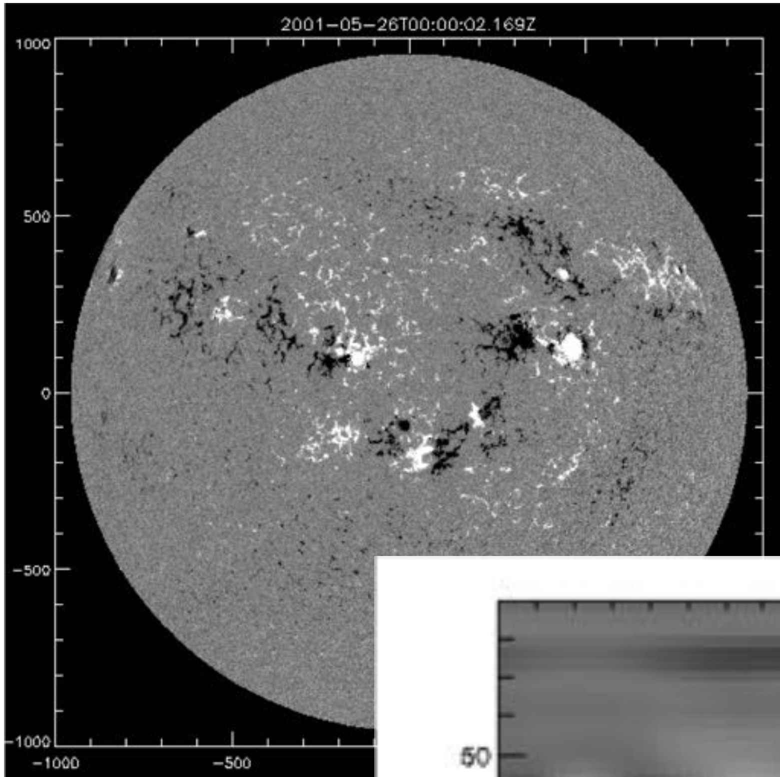


Evidence of the dynamo

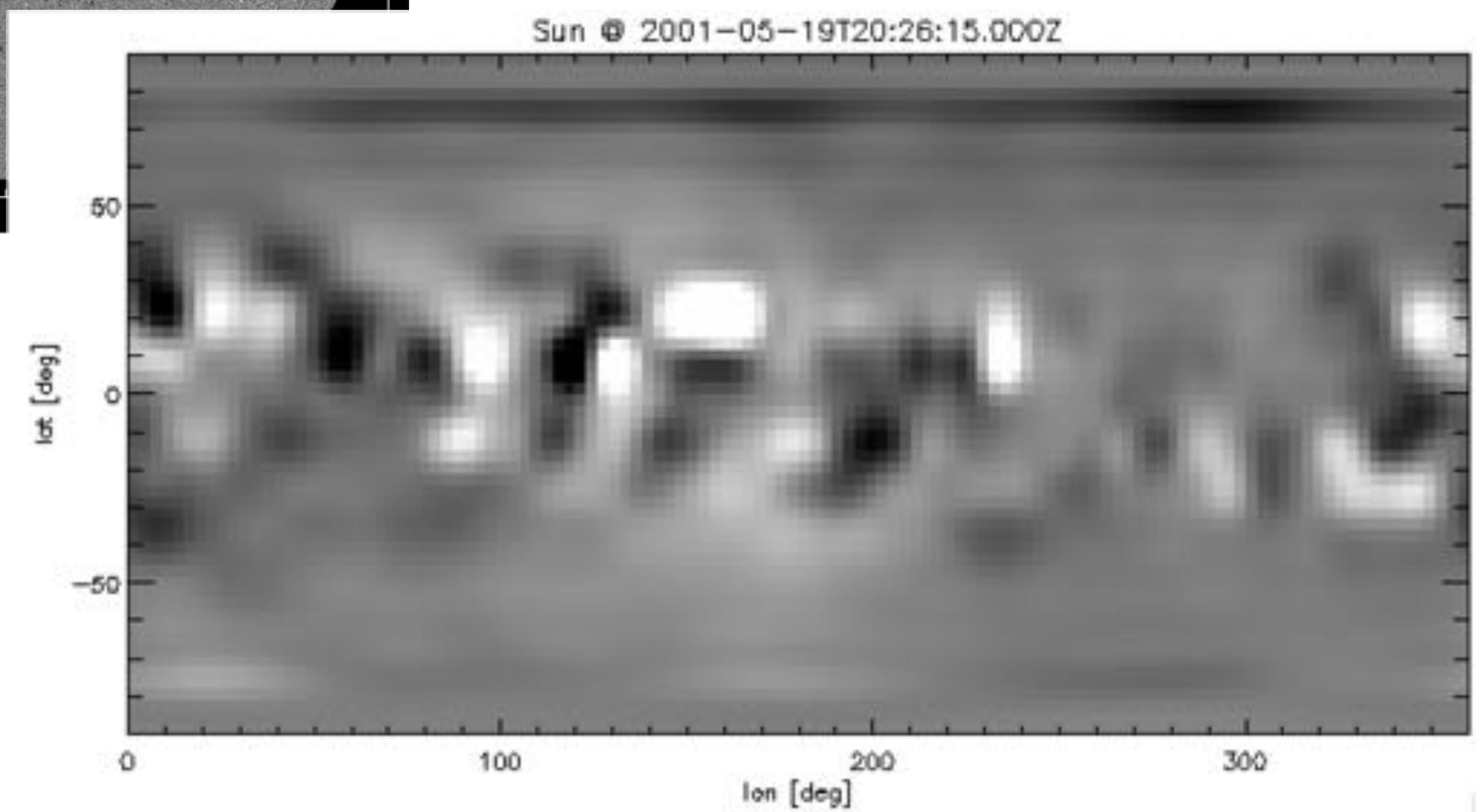
Field orientation: mostly toroidal

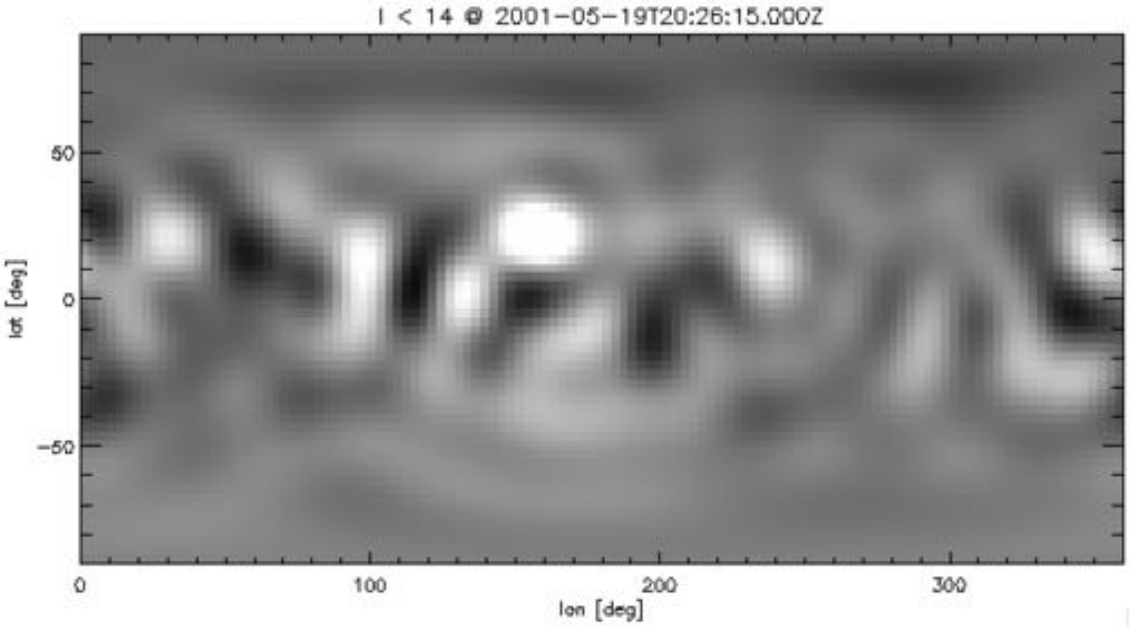


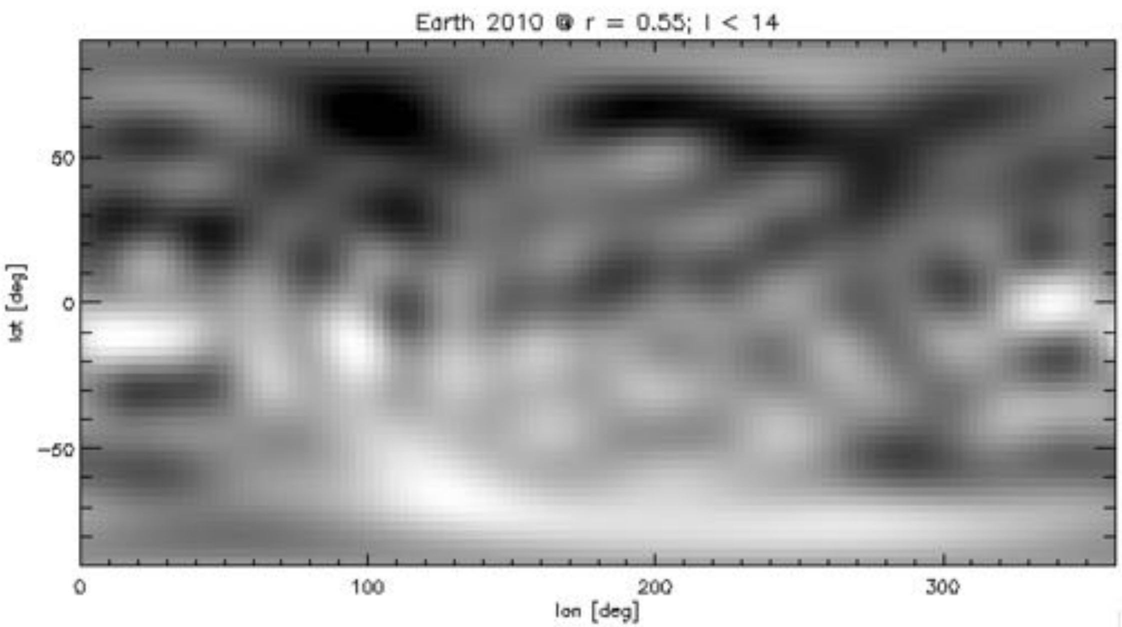




Synoptic plot: unwrapped view built up over time



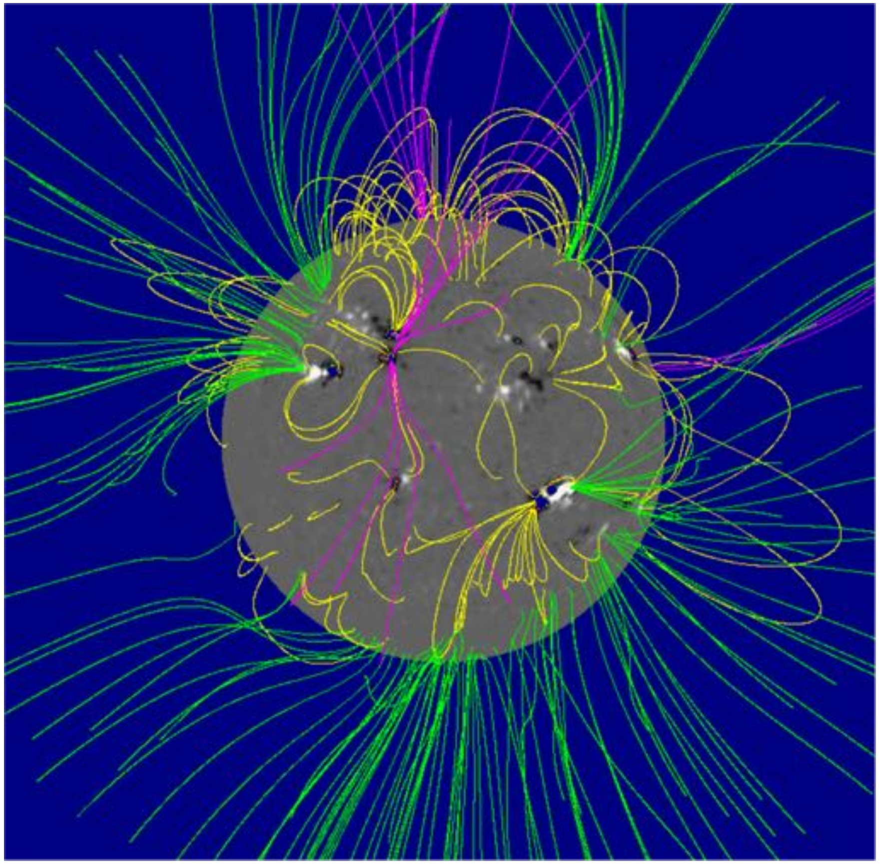




$Rm = 10^8$
$Ro = 10^{-2}$

Dynamo comparison: Sun vs. Earth

$Rm = 10^2$
$Ro = 10^{-6}$



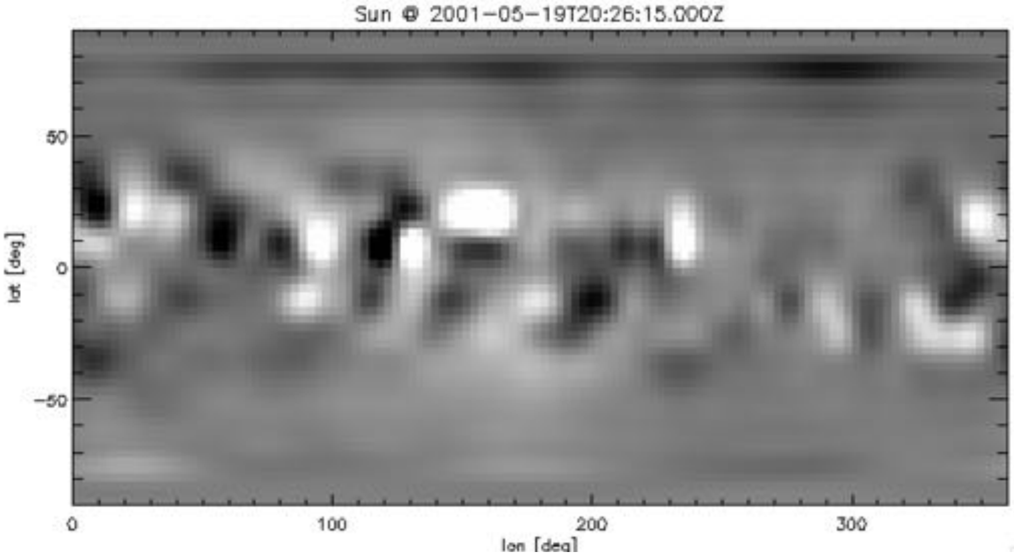
Assume corona has small (negligible) current:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} = 0$$

$$\mathbf{B} = -\nabla \chi$$

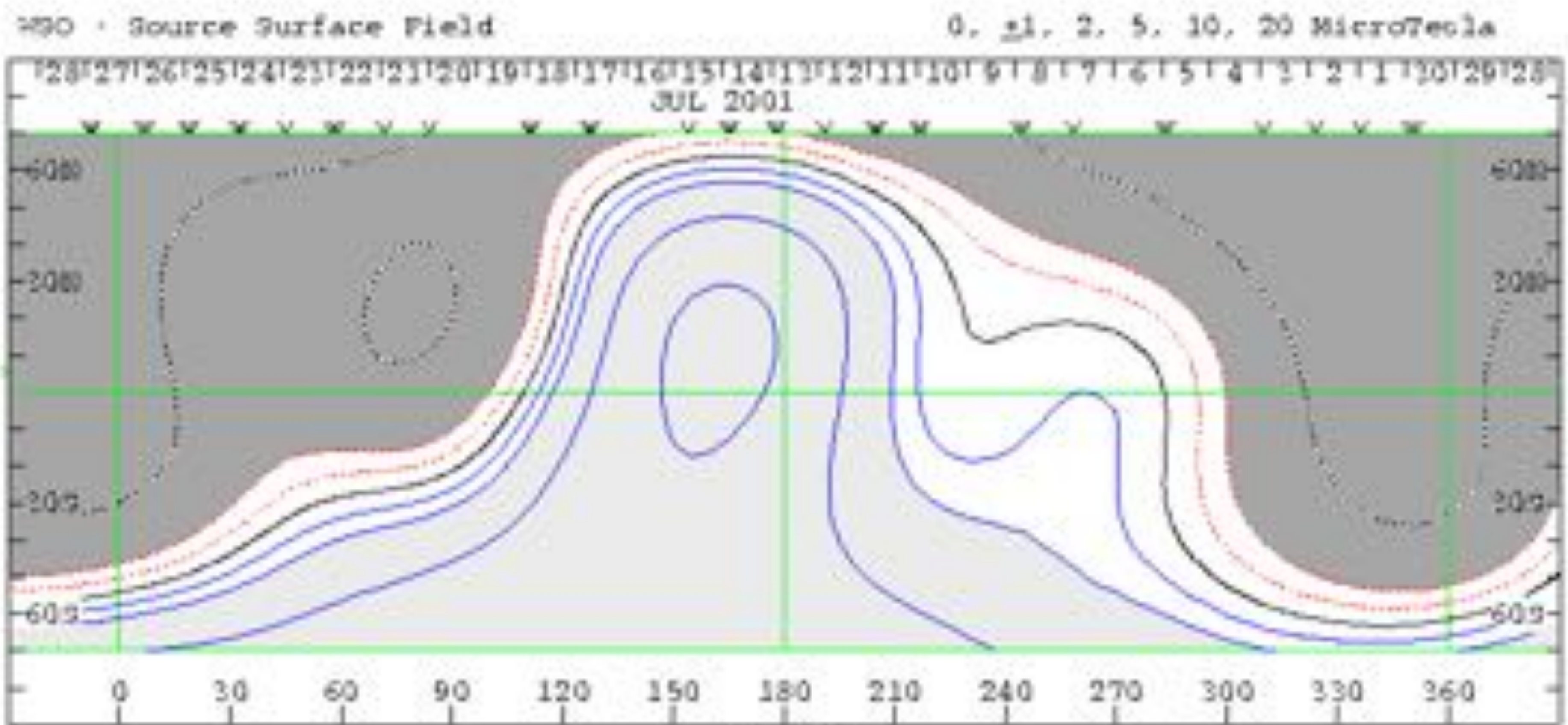
$$\nabla \cdot \mathbf{B} = -\nabla^2 \chi = 0$$

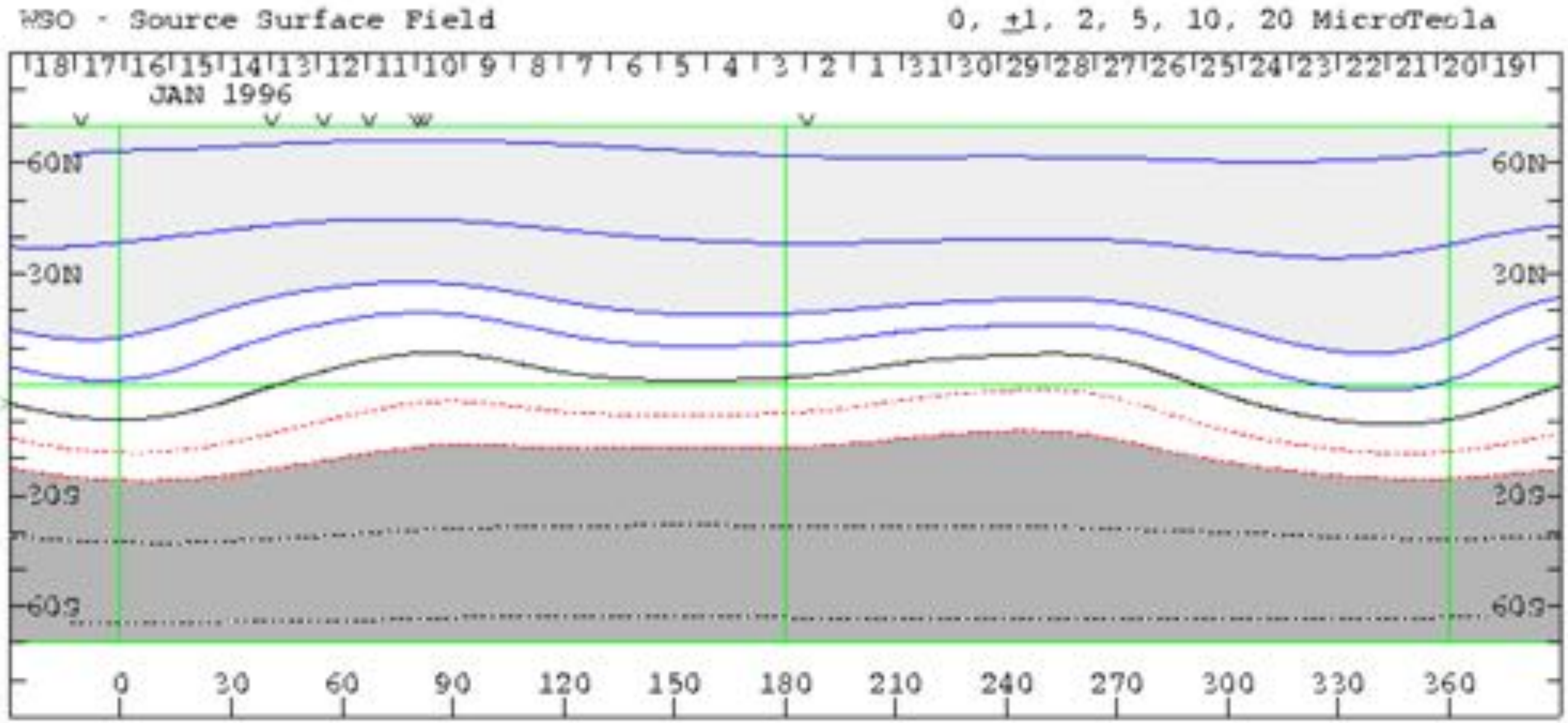
$$\chi(r,\theta,\phi) = \sum_{\ell,m} \tilde{g}_{\ell,m}\, Y_\ell^m(\theta,\phi) \left(\frac{R_\oplus}{r}\right)^{\ell+1}$$



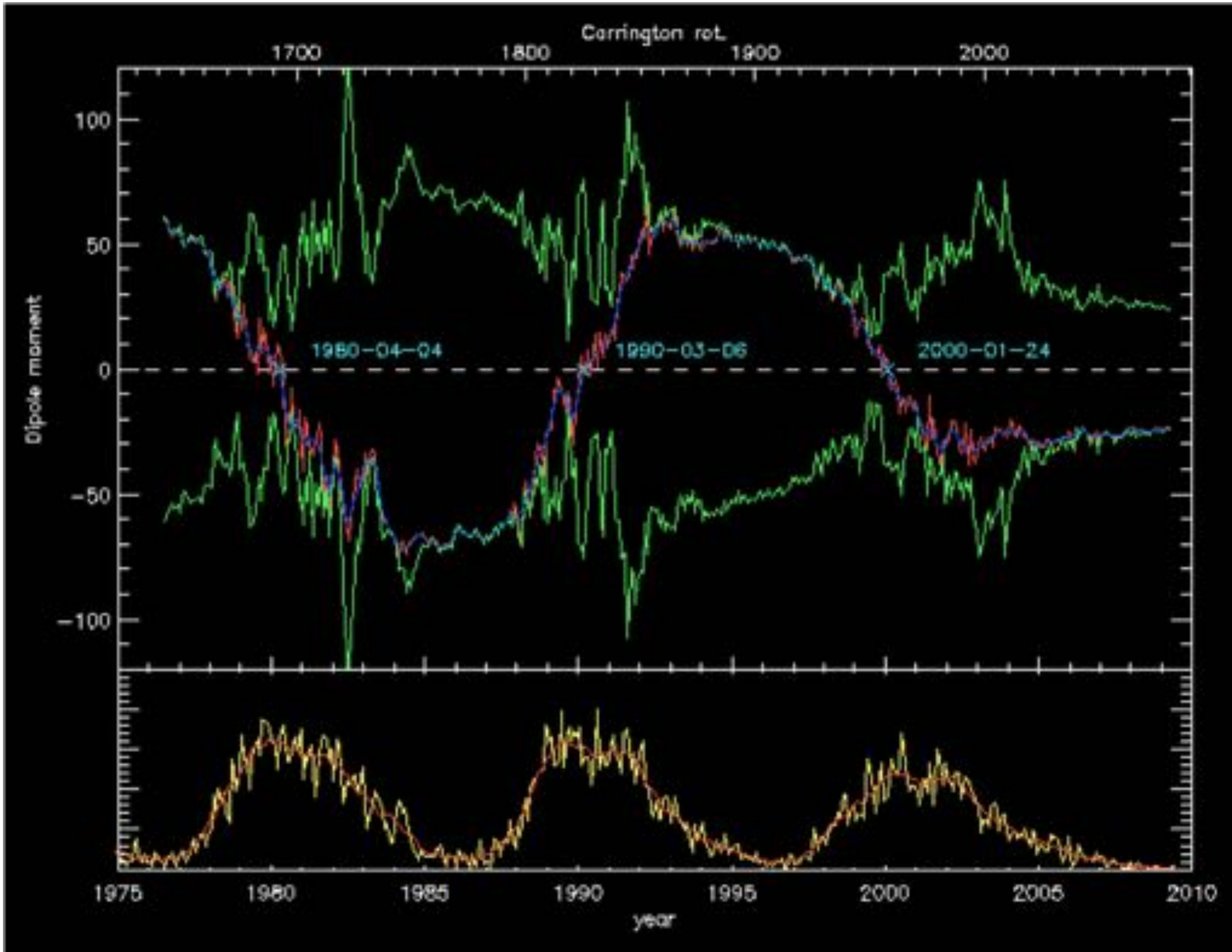
$r = R_\odot$

$r = 2.5\ R_\odot$

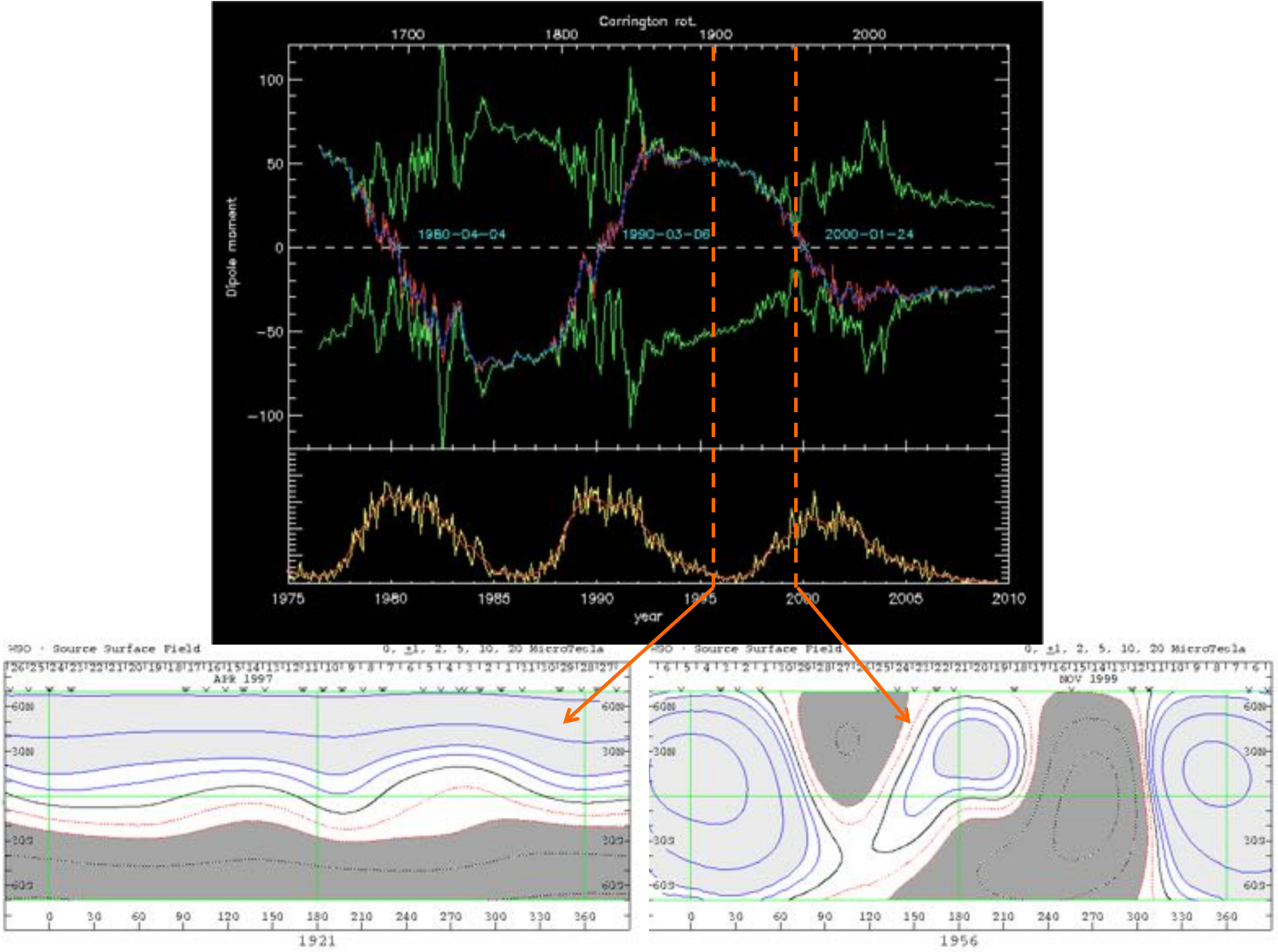



WSO - Source Surface Field
0, ±1, 2, 5, 10, 20 MicroTesla
18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 31 30 29 28 27 26 25 24 23 22 21 20 19
JAN 1996
60N
30N
20S
60S
0
30
60
90
120
150
180
210
240
270
300
330
360

1904

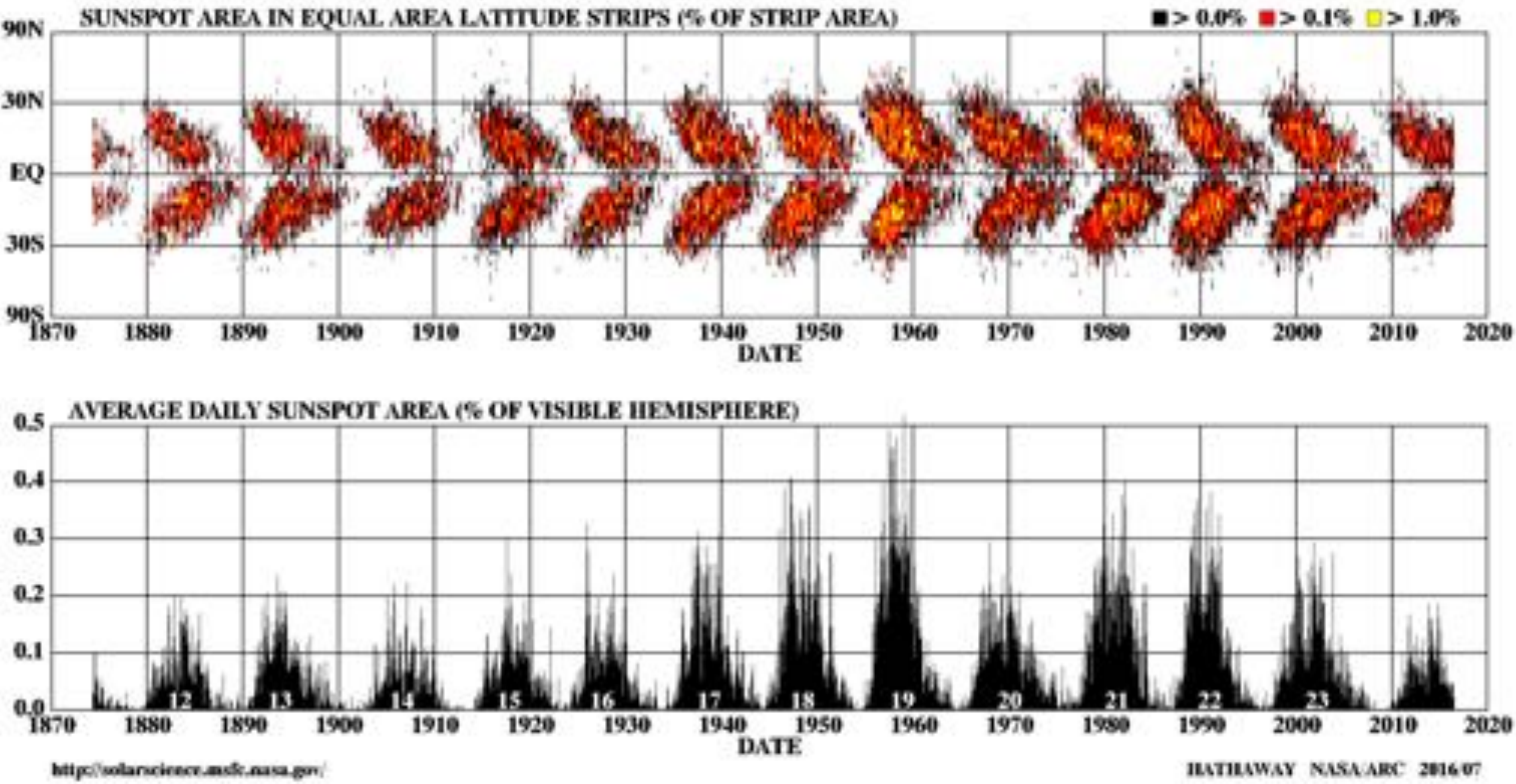


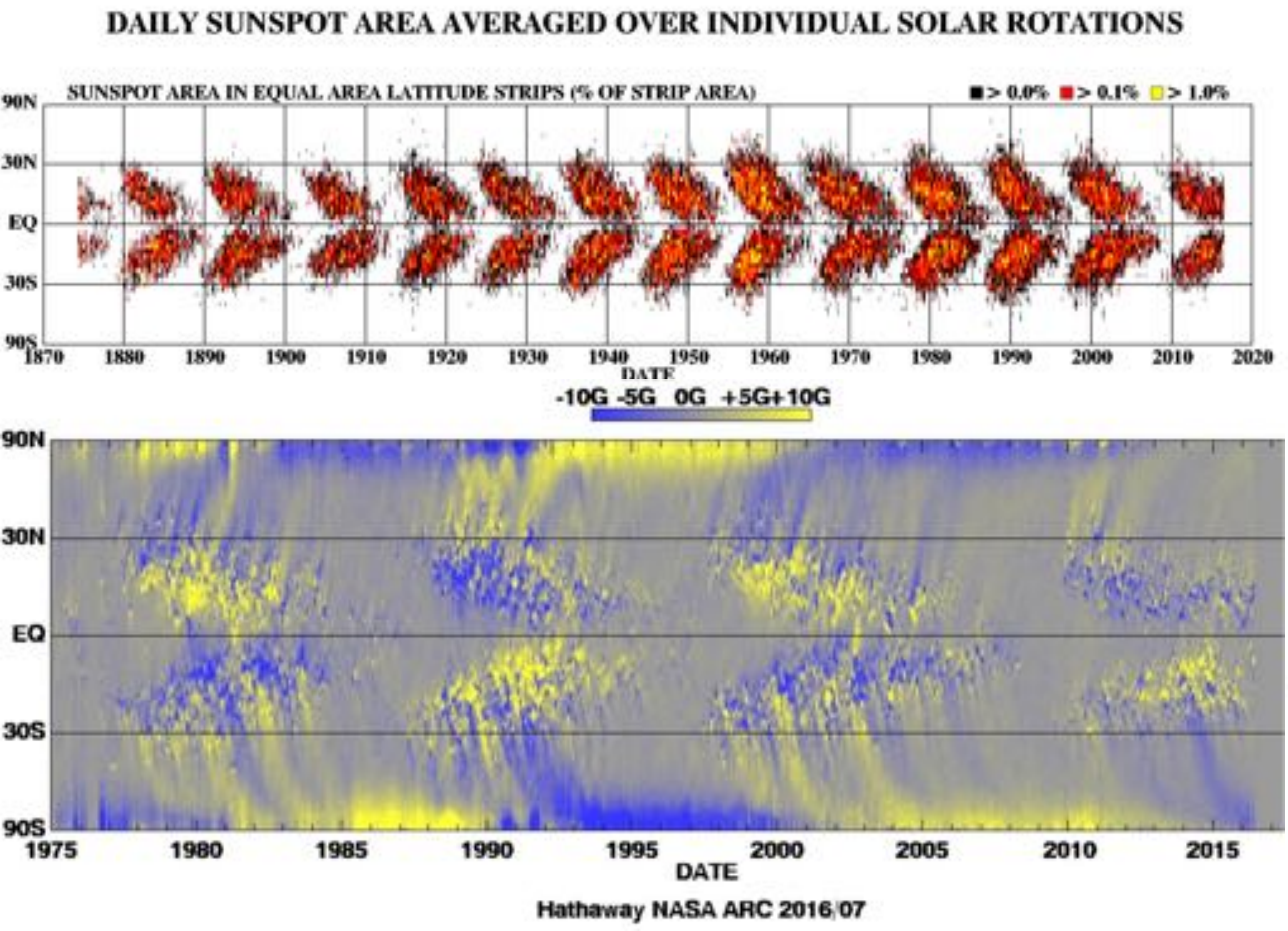
Carrington rot.
1700
1800
1900
2000
100
50
0
-50
-100
Dipole moment
1980-04-04
1990-03-06
2000-01-24
1975
1980
1985
1990
1995
2000
2005
2010
year



Carrington rot.
1700
1800
1900
2000
100
50
0
−50
−100
Dipole moment
1980-04-04
1990-03-06
2000-01-24
1975
1980
1985
1990
1995
2000
2005
2010
year
WSO · Source Surface Field
0, ±1, 2, 5, 10, 20 MicroTesla
APR 1997
1921
NOV 1999
1956
0
30
60
90
120
150
180
210
240
270
300
330
360

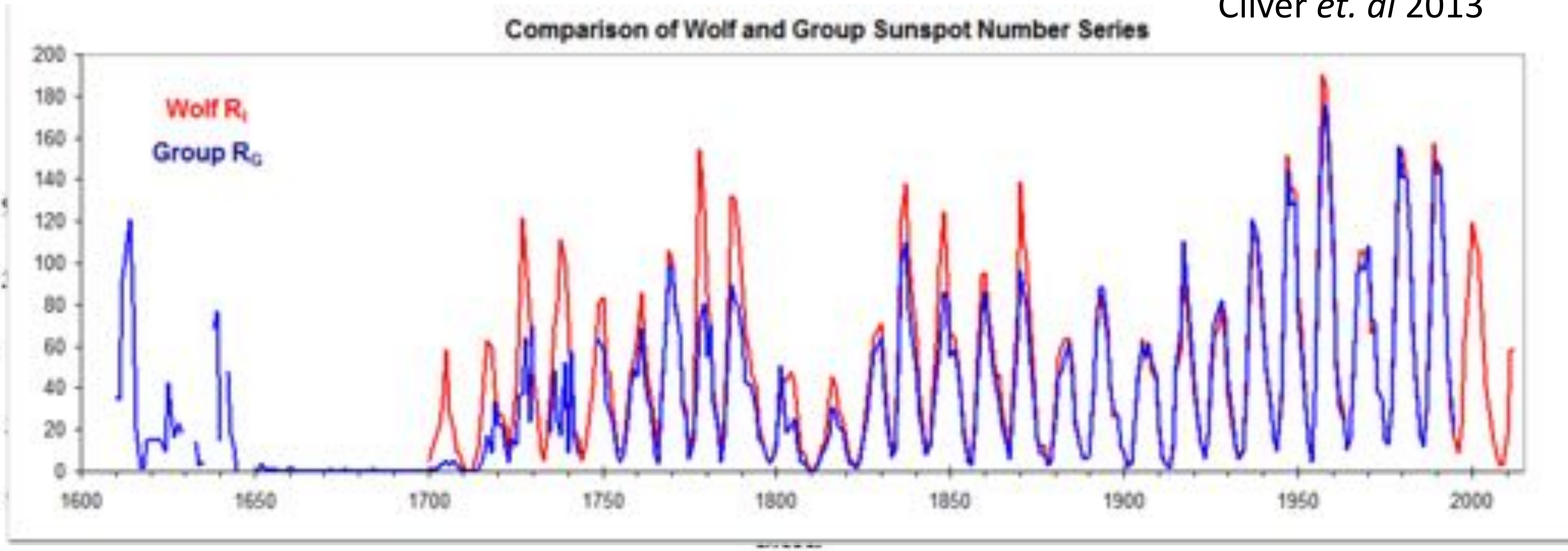
DAILY SUNSPOT AREA AVERAGED OVER INDIVIDUAL SOLAR ROTATIONS

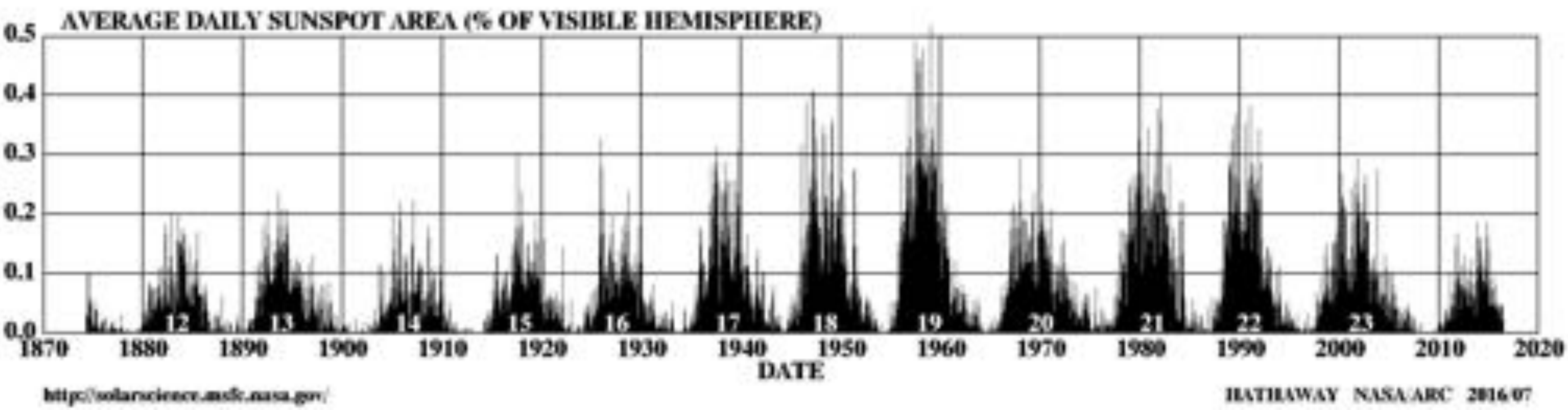




DAILY SUNSPOT AREA AVERAGED OVER INDIVIDUAL SOLAR ROTATIONS
SUNSPOT AREA IN EQUAL AREA LATITUDE STRIPS (% OF STRIP AREA)
> 0.0% > 0.1% > 1.0%
90N
30N
EQ
30S
90S
1870 1880 1890 1900 1910 1920 1930 1940 1950 1960 1970 1980 1990 2000 2010 2020
DATE
-10G -5G 0G +5G +10G
90N
30N
EQ
30S
90S
1975 1980 1985 1990 1995 2000 2005 2010 2015
DATE
Hathaway NASA ARC 2016/07

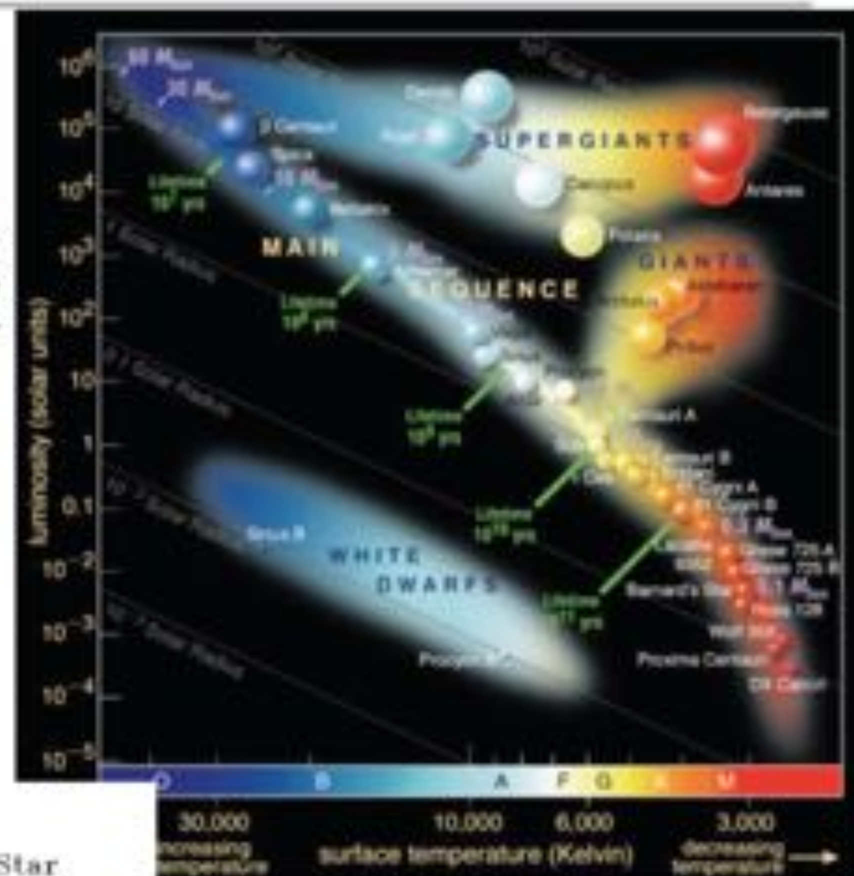
Cliver *et. al* 2013



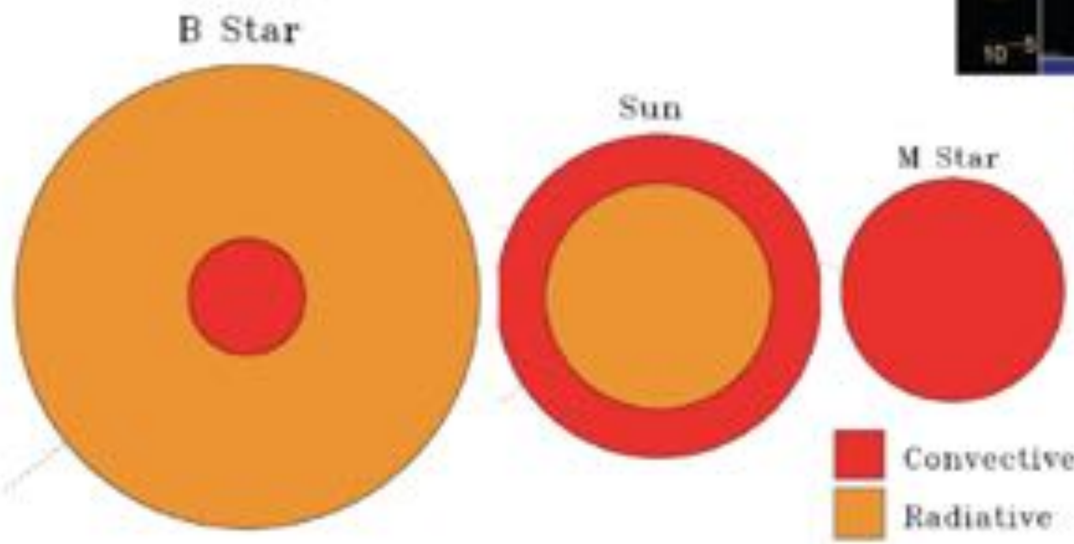


STELLAR MAGNETIC FIELDS

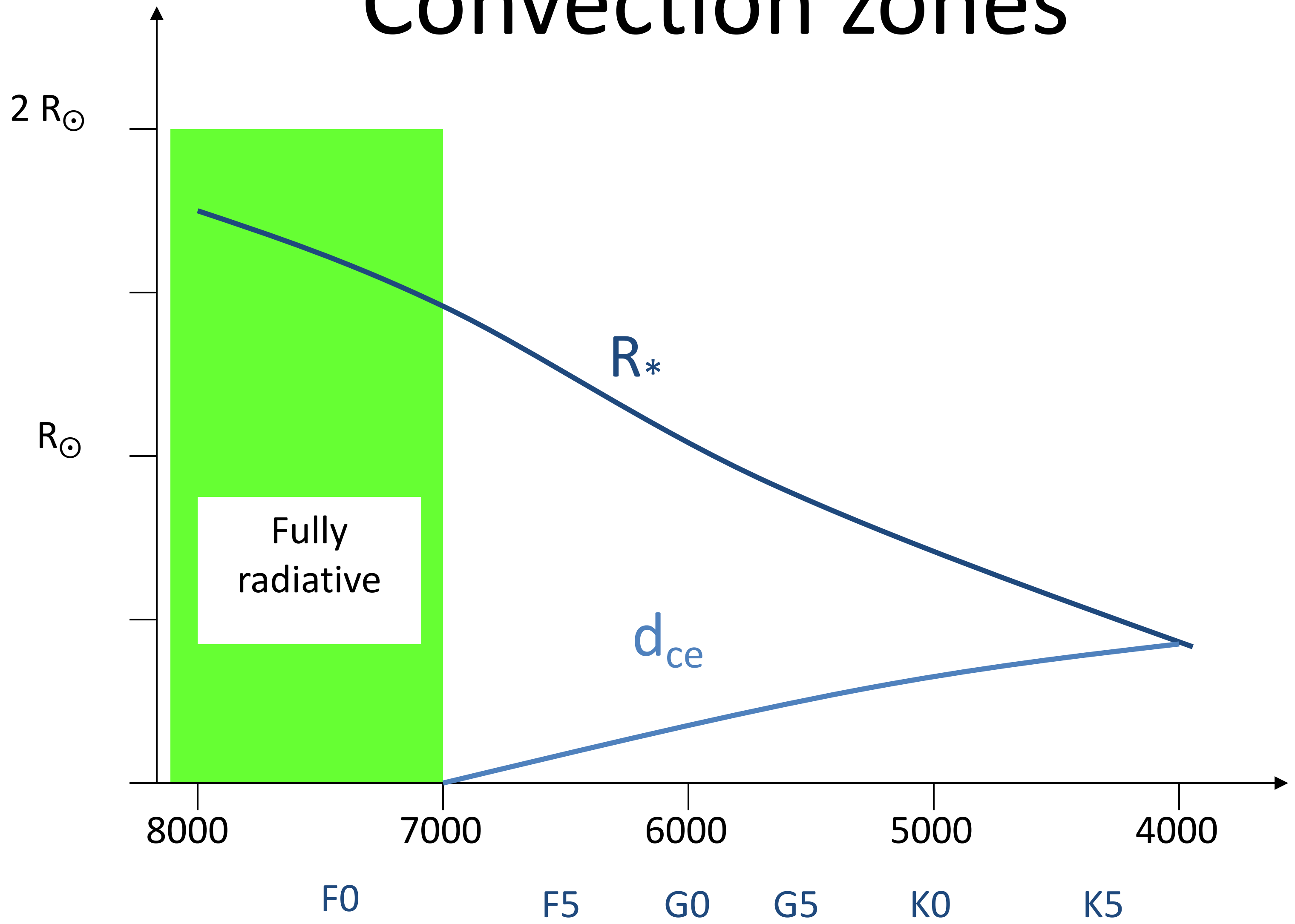
- correlations between stellar types and magnetic field properties, probably due to geometry of convection zones
- stars with outer convection zones (late-type stars) have observed magnetic fields whose strength tends to increase with their angular velocity
- Cyclic variations are known to exist only for spectral types between G0 and K7).



Stanley



Convection zones

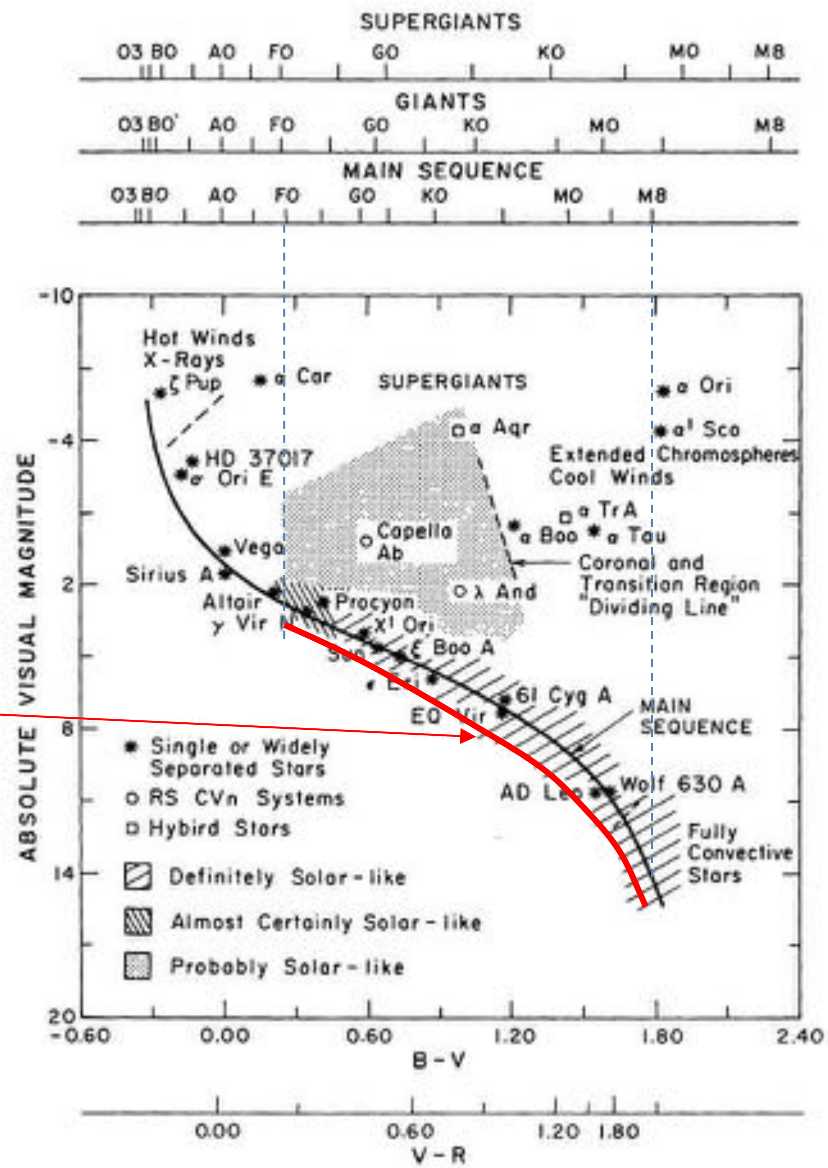


Other stars

Evidence of magnetic activity

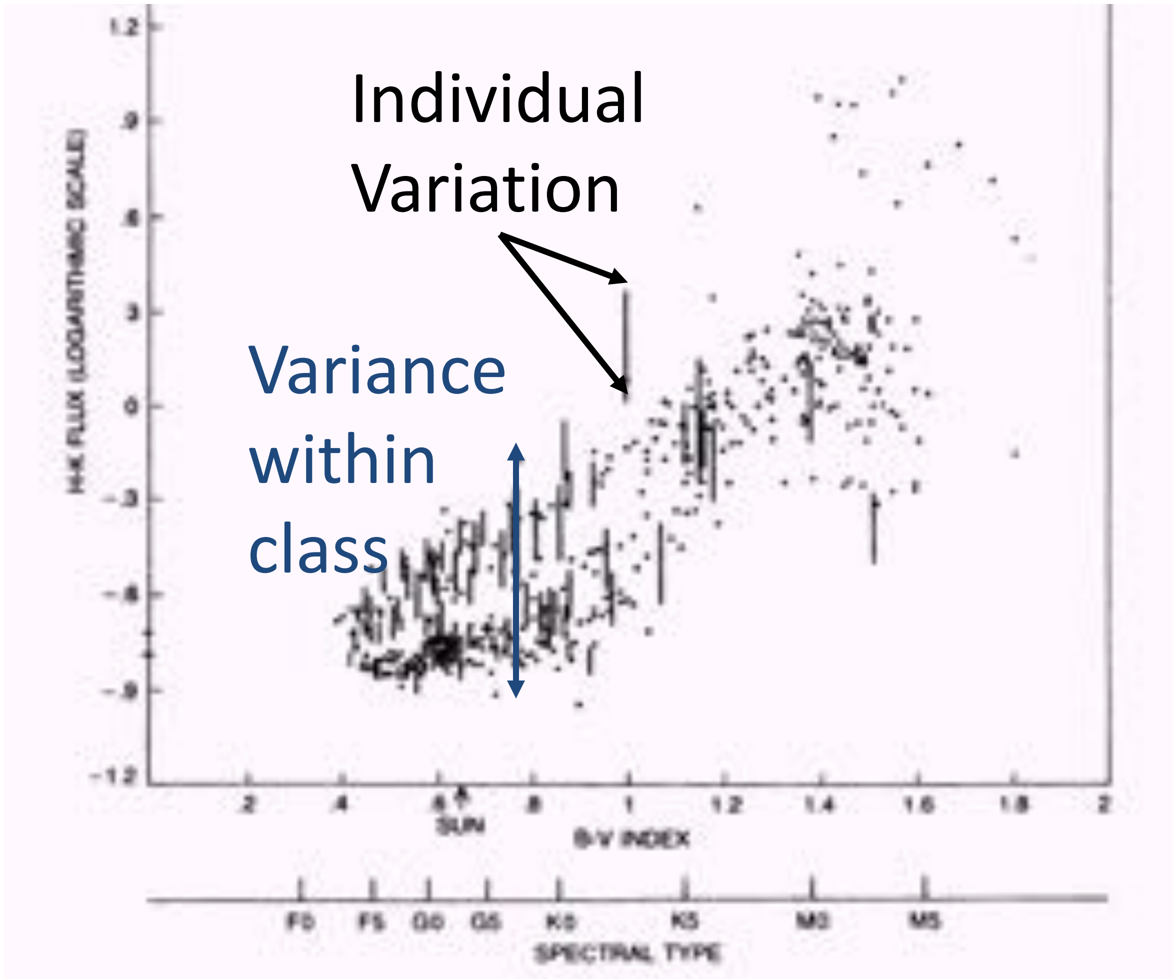
Activity on main sequence: types **F → M**

B-V > 0.4



(From Linsky 1985)

Explaining Activity Levels



The Dynamo Number

Parker's Dynamo #

$$N_D = \frac{\alpha_{\rm dyn} \Omega' d^4}{\eta^2}$$

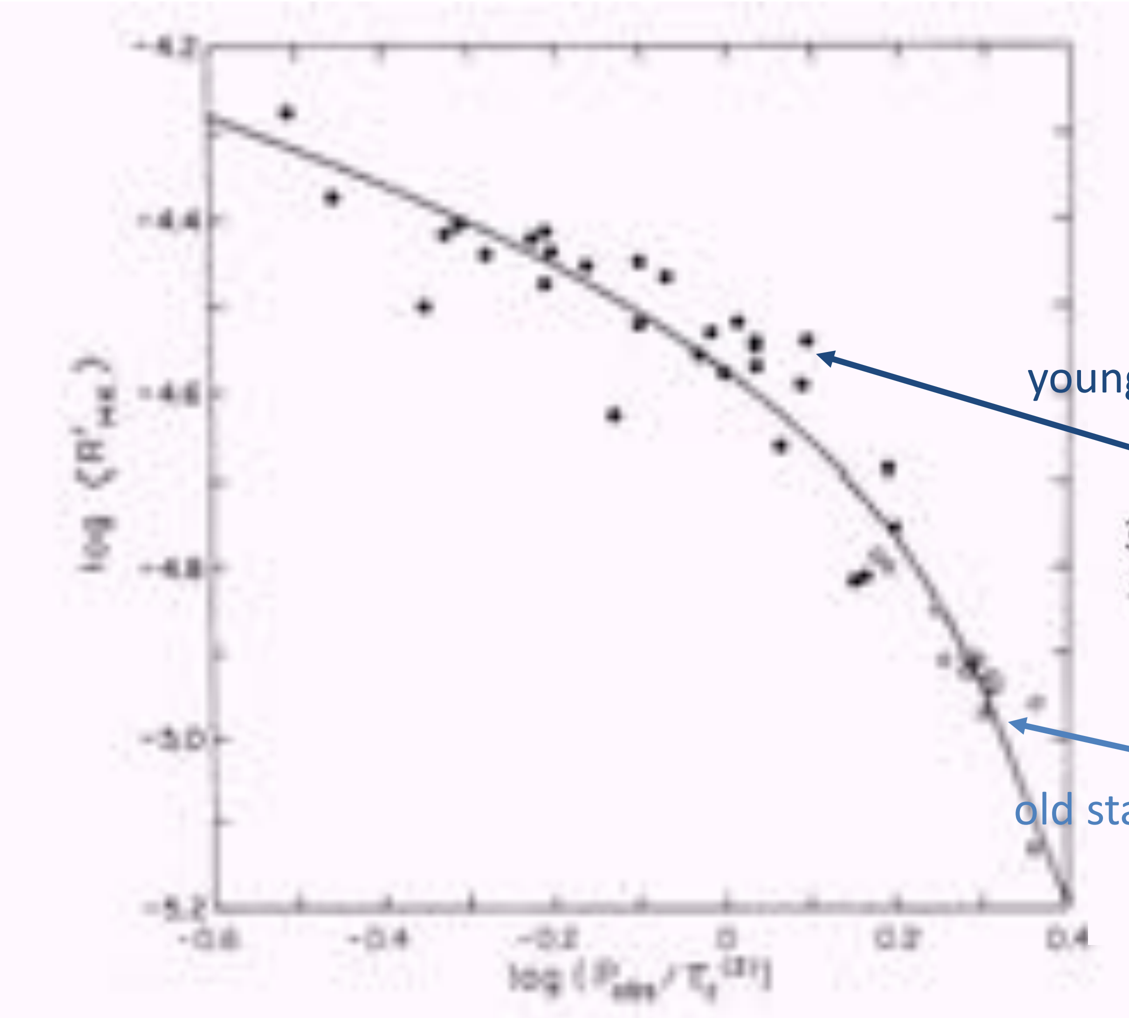
Dynamo is linear inst-ability for $N_D > N_{crit}$

Dynamo α-effect: $\alpha_{\rm dyn} \equiv \tau \langle \mathbf{v} \cdot (\nabla \times \mathbf{v}) \rangle \sim \Omega d$

$$\eta = \eta_{\rm turb} \sim \frac{d^2}{\tau_c} \qquad \Omega' = \frac{d\Omega}{dr} \sim \frac{\Omega}{d}$$

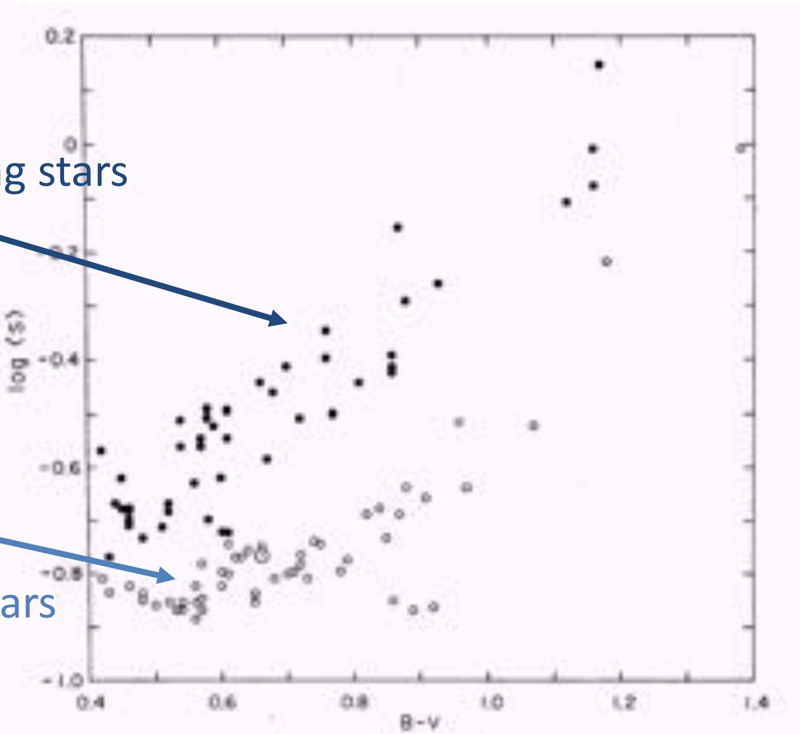
$$N_D \sim (\Omega \tau_c)^2 \sim (P_{\rm obs} / \tau_c)^{-2} = Ro^{-2}$$

Activity vs. Rossby Number



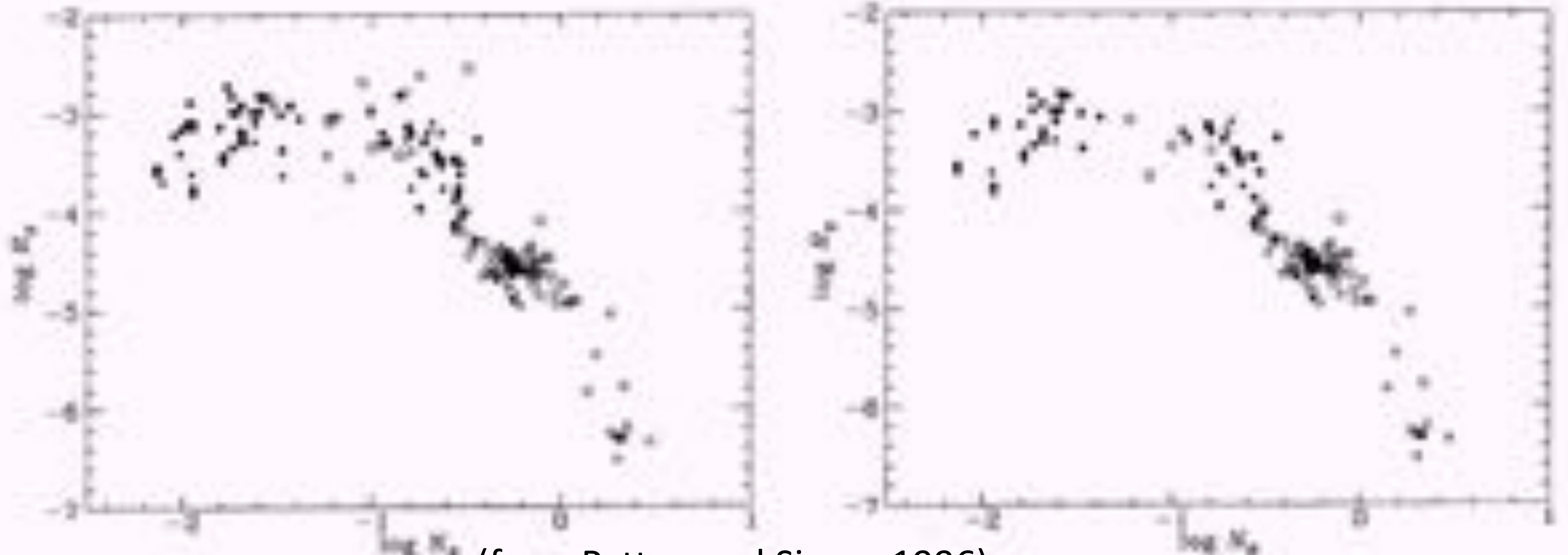
41 Local stars

P_{obs} from S(t)



(from Noyes et al. 1984)

Activity vs. Rossby Number



(from Patten and Simon 1996)

- Stars in open cluster 2391 (30My old)
- R_X from ROSAT observations
- Rotation periods P_{obs} from optical photometry
- $N_R = Ro = P_{obs}/t_c$

Summary

- Magnetic fields – all from dynamos
  - Conducting fluid
  - Complex motions w/ enough *umph*
- Create complex fields
- Fields evolve in time – reverse occasionally
- Differences from different parameters:

  Rm, Re, Ro