Solar-Wind Structure and Turbulence

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Outline

I. 3D Structure of the Solar Wind

II. Introduction to Turbulence

III. Measurements of Solar-Wind Turbulence

IV. Magnetohydrodynamic (MHD) Turbulence

V. Reflection-Driven MHD Turbulence and the Origin of the Solar Wind.

Collaborators

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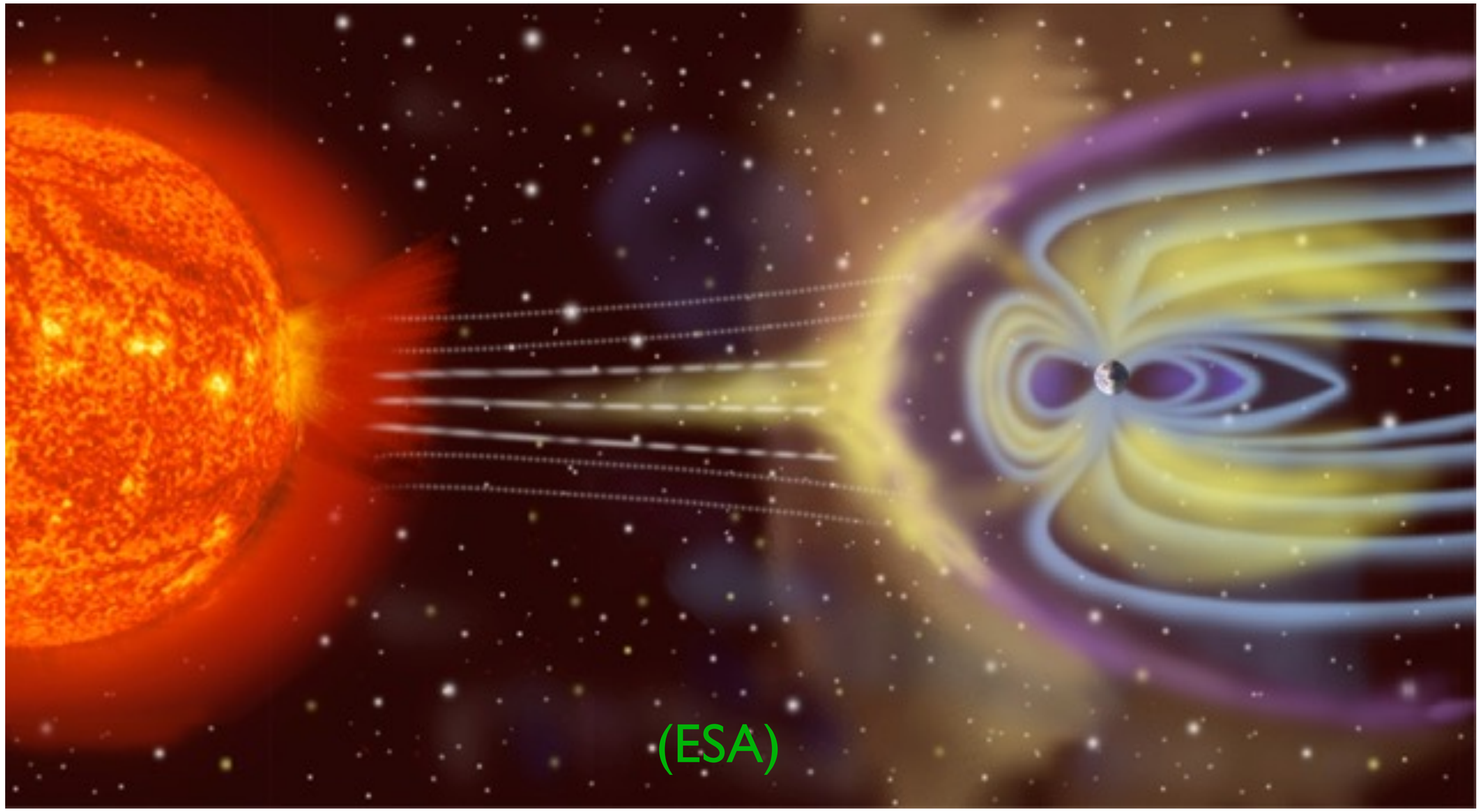
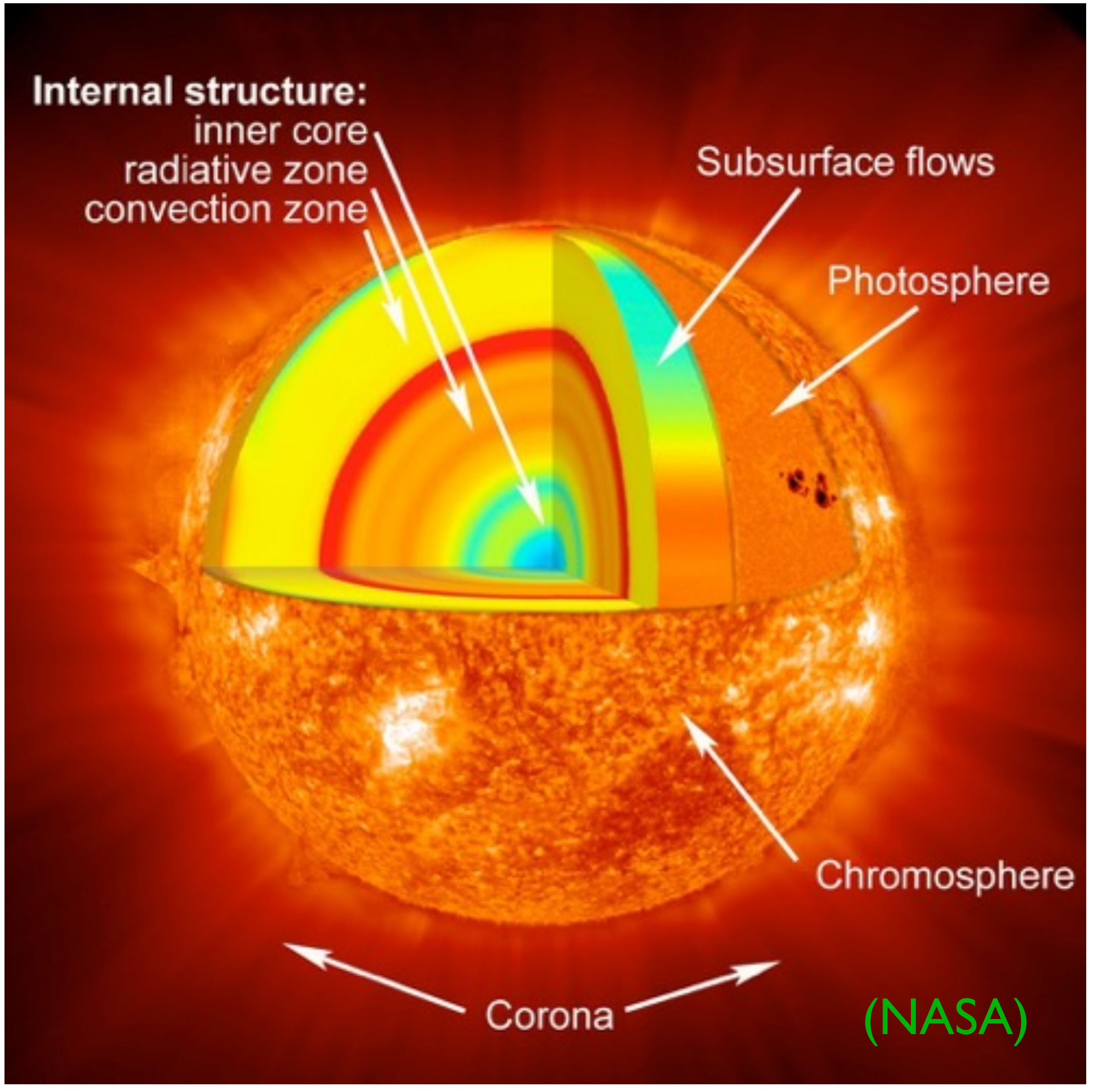
What is the Solar Wind?



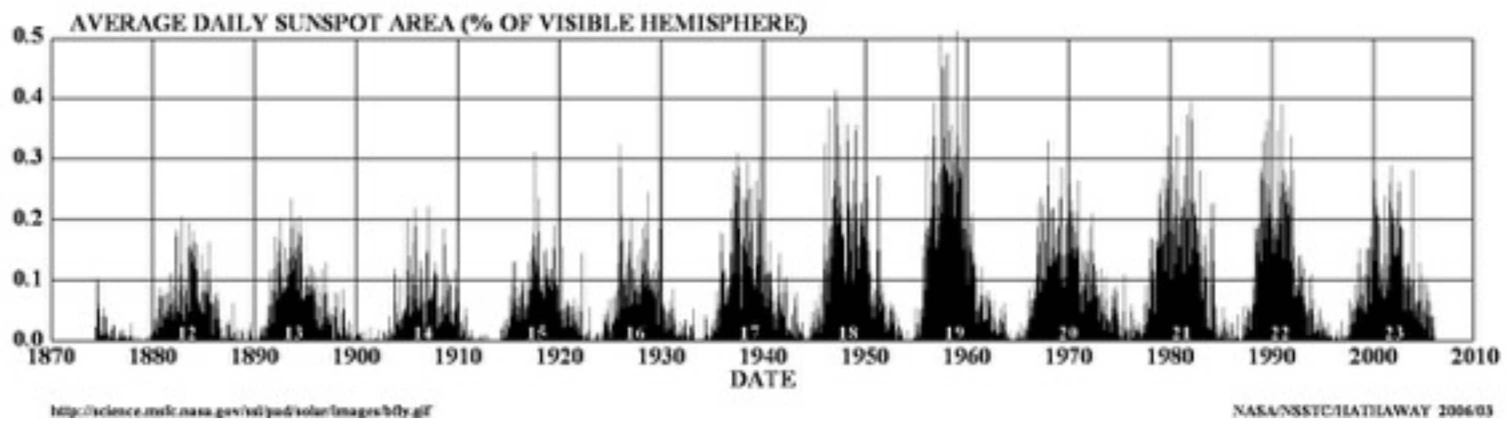
- Quasi-continuous, radial outflow of particles from the Sun
- Fast (300 - 800 km/s), hot and dilute (10^5 K, 5 cm^{-3} at 1 AU)
- Plasma: behaves like a fluid, but it generates and is in turn influenced by electromagnetic fields

The Solar Wind in Relation to the Sun

- it is the extension of the solar atmosphere
- represents about 10^{-6} of the energy output of the Sun
- mass loss rate is about $10^{-14}\ M_{sun}\ yr^{-1}$



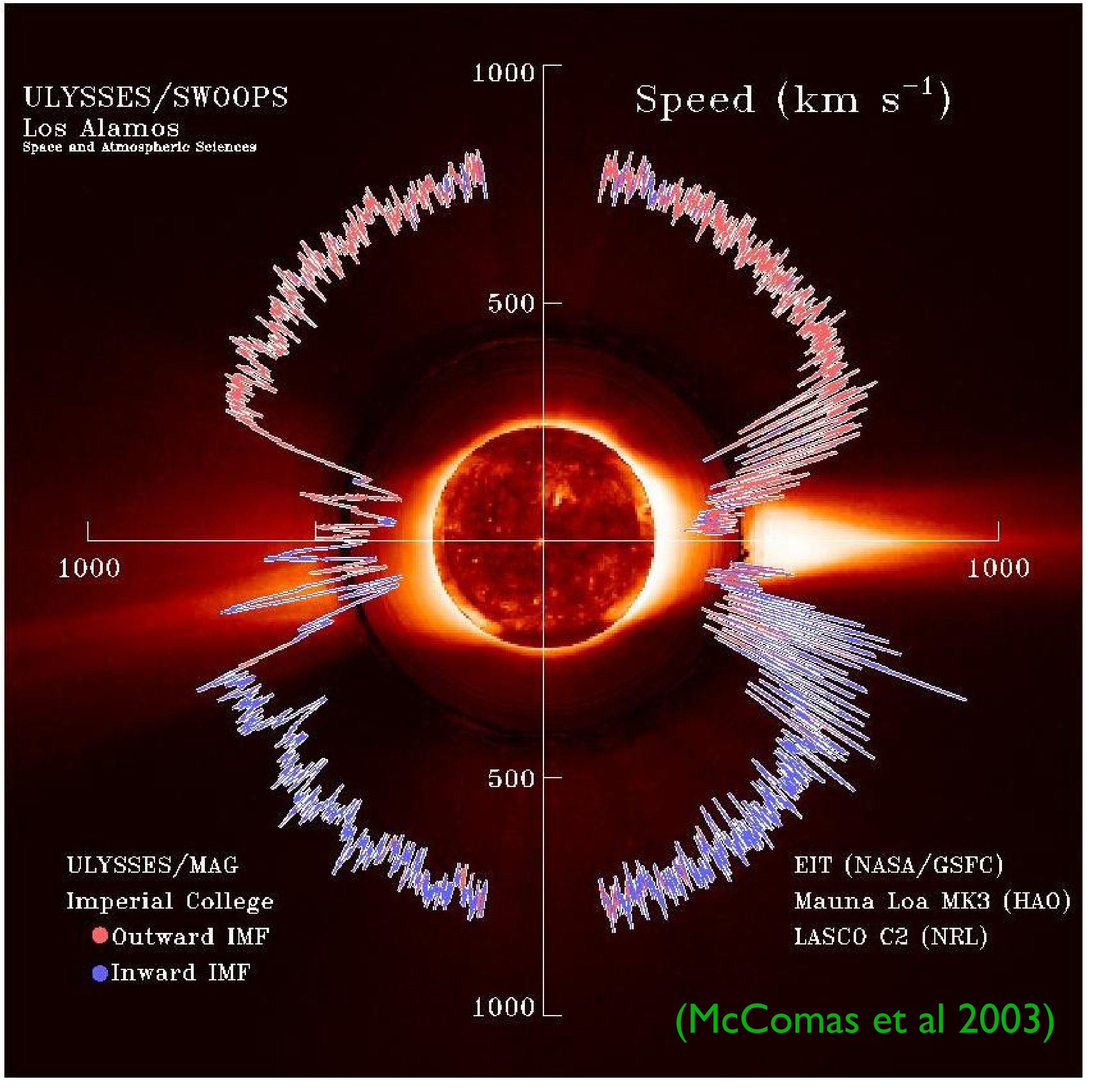
Solar-Wind Structure Depends on the Solar Cycle



- “Solar minimum” - very few sunspots.
- “Solar maximum” - many sunspots, solar flares, and coronal mass ejections.

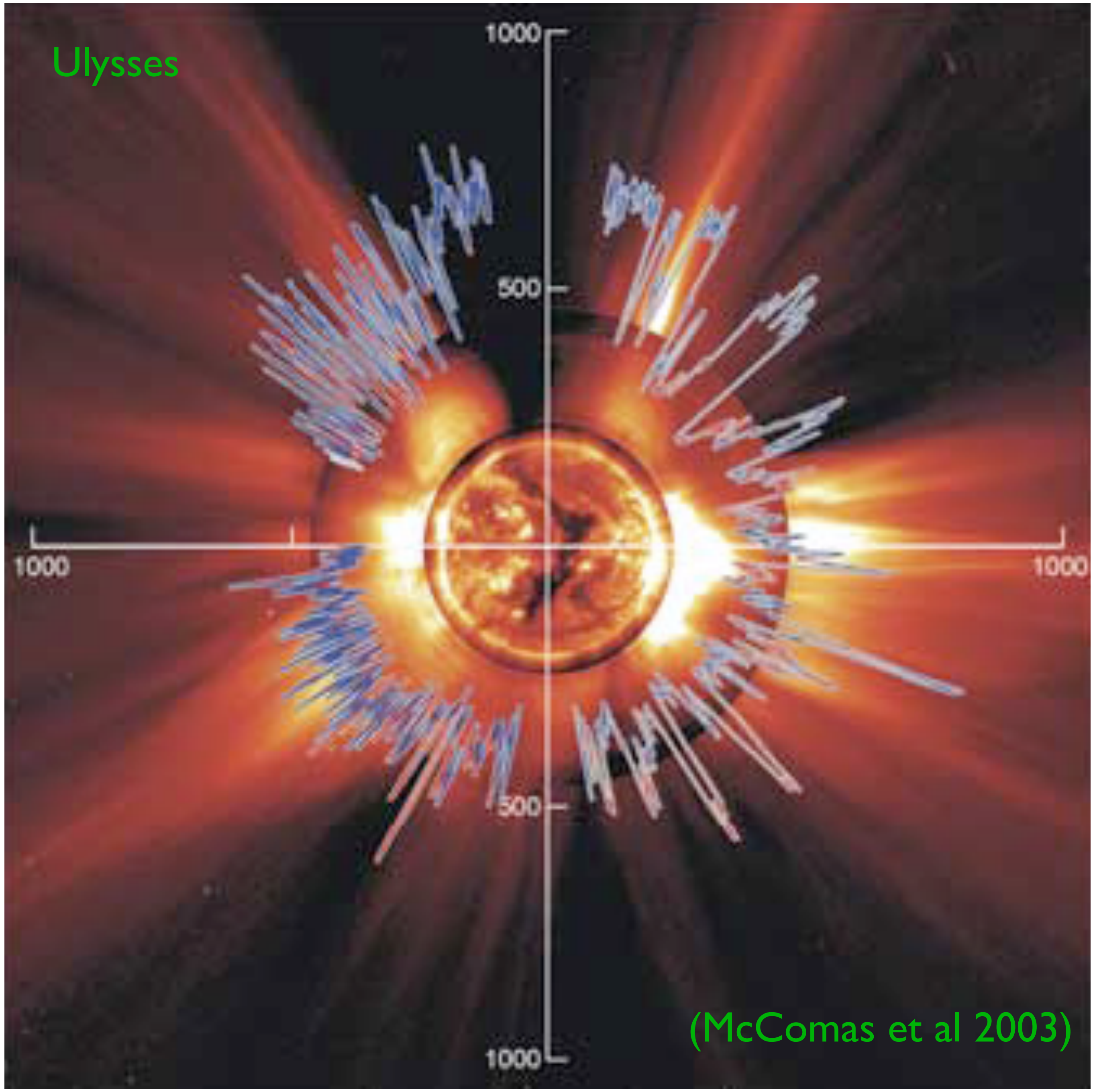
3D Structure Near Solar Minimum

- Fast wind (700-800 km/s) is the basic state of the flow near solar minimum
- Fast wind emanates primarily from open-field-line regions near the poles - "polar coronal holes"
- Slow wind (300-500 km/s) is confined to low latitudes (less so at solar maximum)

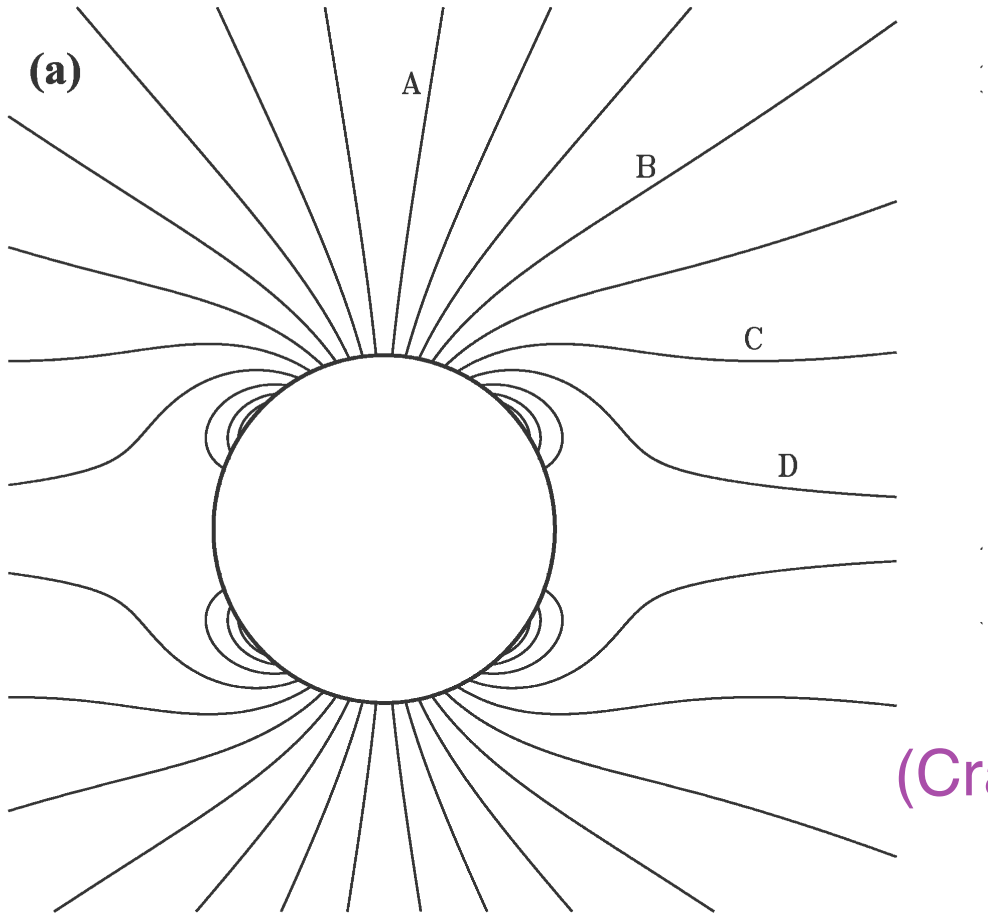


3D Structure Near Solar Maximum

- much more complex
- alternates between fast and slow wind at virtually all heliographic latitudes



What Causes the Different Speeds of the Fast Wind and Slow Wind?



(Cranmer 2005)

- Near the Sun, the solar wind flows along magnetic flux tubes.
- When the flux-tube cross section increases rapidly with increasing *r*, the outflow remains fairly slow in the low corona. This allows more heat to be conducted back to the Sun, removing energy from the wind, and reducing the asymptotic wind speed.

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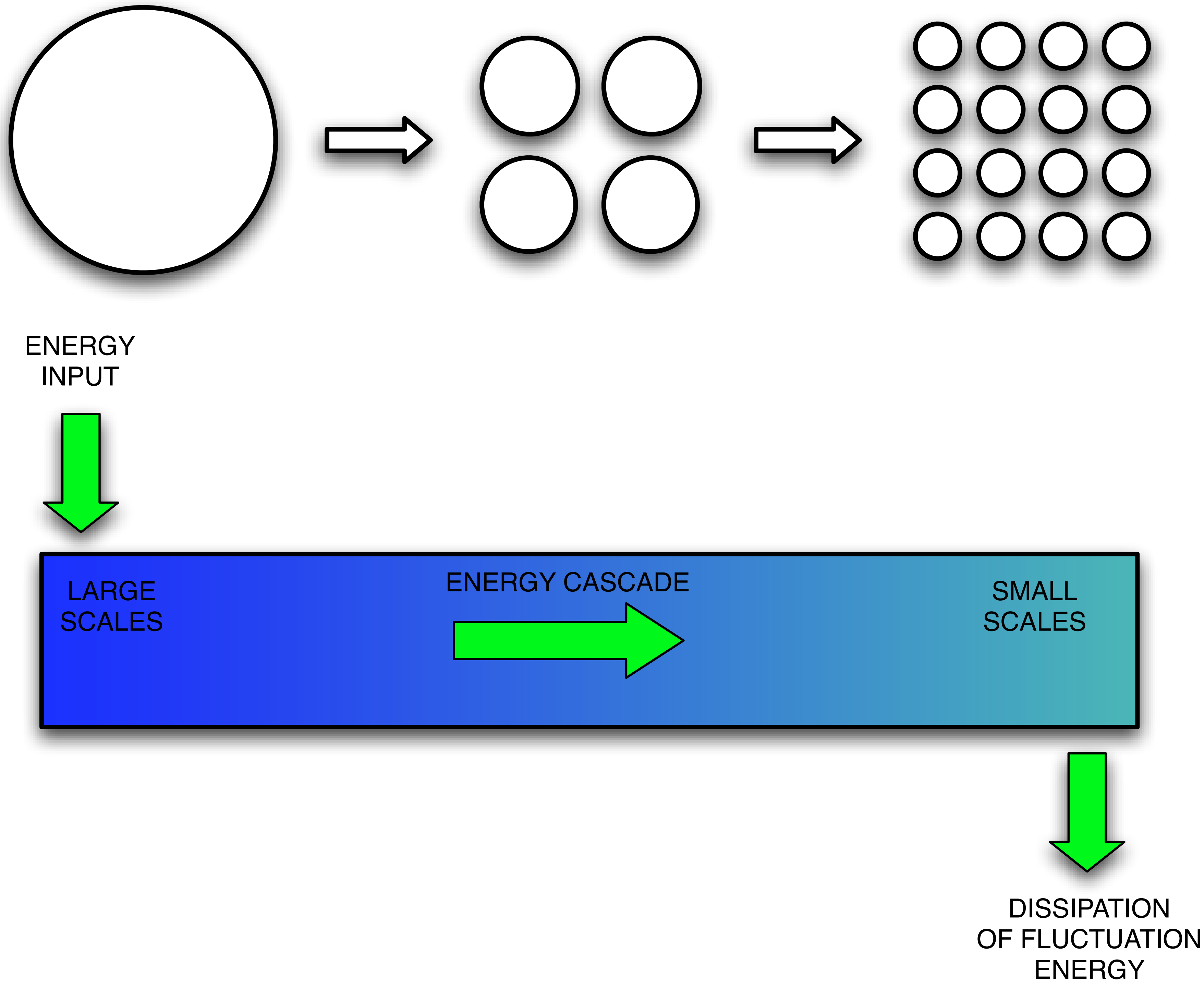
What Is Turbulence?

Operational definition for turbulence in plasmas and fluids: *turbulence consists of disordered motions spanning a large range of length scales and/or time scales.*

“Energy Cascade” in Hydrodynamic Turbulence

Canonical picture: larger eddies break up into smaller eddies



Plasma Turbulence Vs. Hydro Turbulence

- In plasmas such as the solar wind, turbulence involves electric and magnetic fields as well as velocity fluctuations.

- In some cases the basic building blocks of turbulence are not eddies but plasma waves or wave packets.

Where Does Turbulence Occur?

- Atmosphere (think about your last plane flight).
- Oceans.
- Sun, solar wind, interstellar medium, intracluster plasmas in clusters of galaxies...

What Causes Turbulence?

- Instabilities: some source of free energy causes the amplification of fluctuations which become turbulent. Example: convection in stars.
- Stirring. Example: stirring cream into coffee.
- Requirement: the medium can’t be too viscous. (Stirring a cup of coffee causes turbulence, but stirring a jar of honey does not.)

What Does Turbulence Do?

- Turbulent diffusion or mixing. Examples: cream in coffee, pollutants in the atmosphere.
- Turbulent heating. When small-scale eddies (or wave packets) dissipate, their energy is converted into heat. Example: the solar wind.

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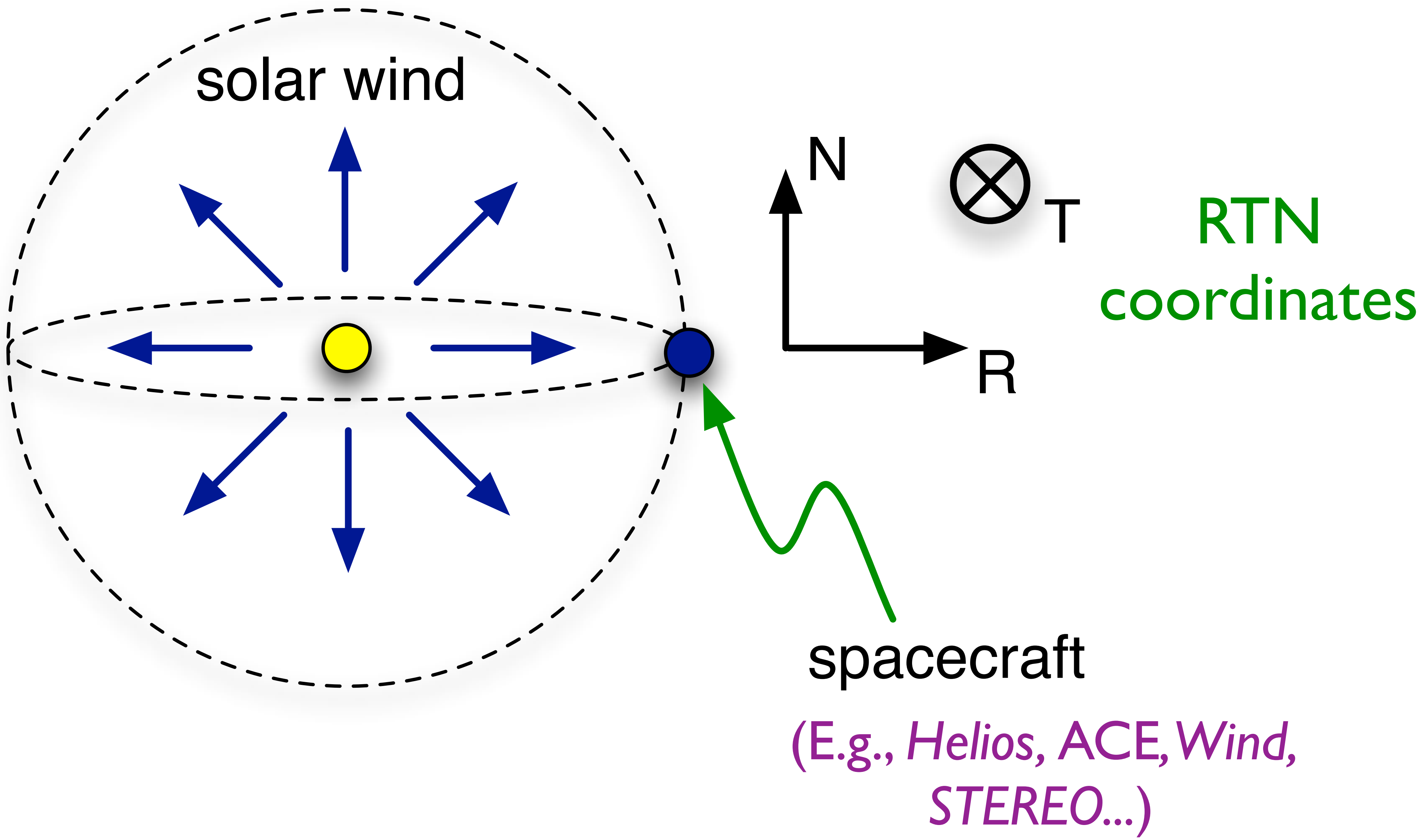
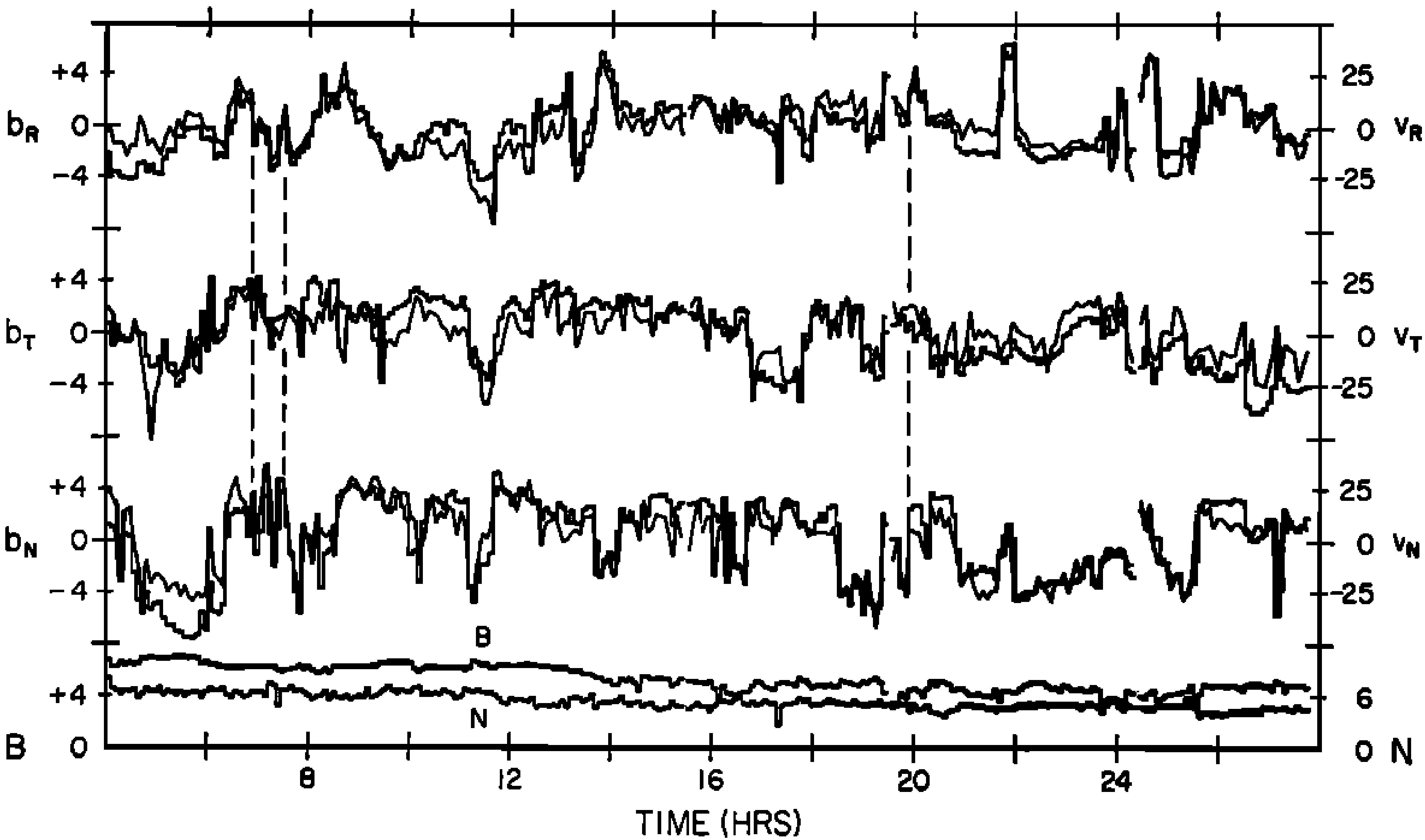
In Situ Measurements

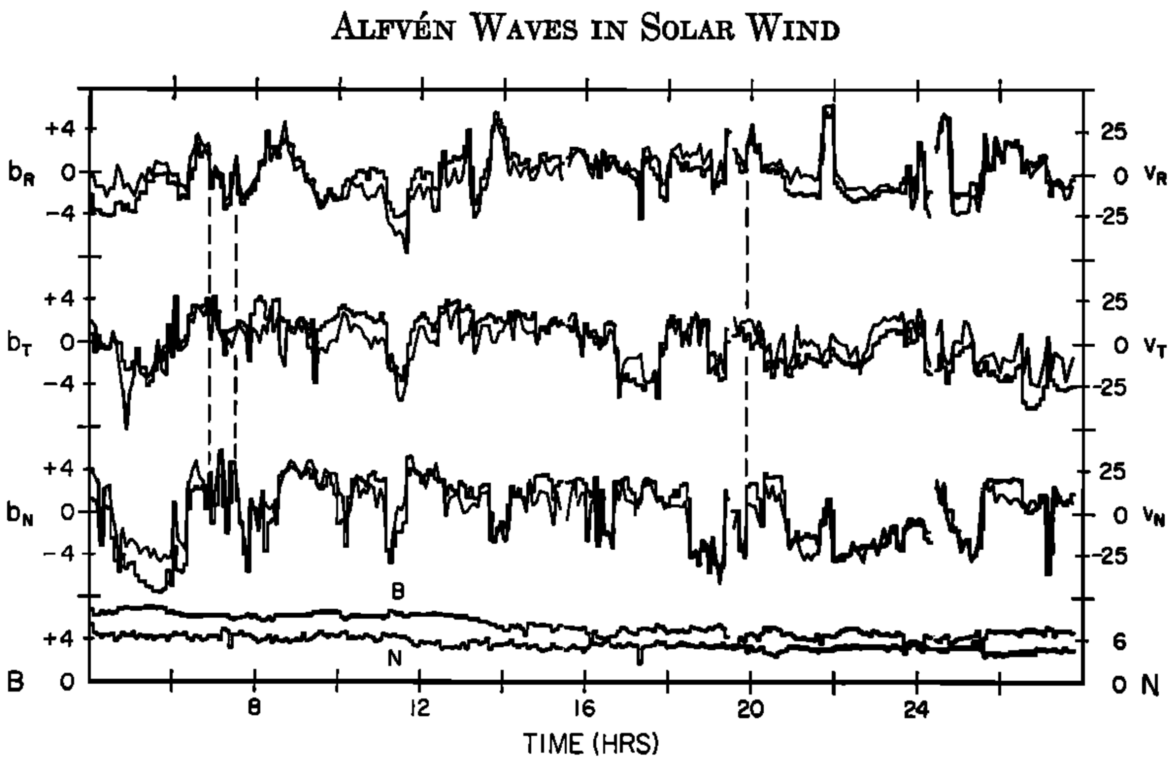


Quantities measured include ***v***, ***B***, ***E***, n, and T

Spacecraft Measurements of the Magnetic Field and Velocity



Data from the *Mariner 5* spacecraft (Belcher & Davis 1971)



Is This Turbulence?

- The velocity and magnetic field in these measurements appear to fluctuate in a random or disordered fashion.
- But how do we tell whether there are "velocity fluctuations spanning a large range of scales," as in our operational definition of turbulence?
- One way: by examining the power spectrum of the fluctuations.

The Magnetic Power Spectrum

Fourier transform of magnetic field

$$\tilde{\vec{B}}(f) = \int_{-T/2}^{T/2} \vec{B}(t)\, e^{2\pi i f t}\, dt$$

power spectrum

$$P(f) = \lim_{T\to\infty} \frac{1}{T} \langle \tilde{\vec{B}}(f) \cdot \tilde{\vec{B}}(-f) \rangle$$

- *B*(*t*) is the magnetic field vector measured at the spacecraft location.
- *T* is the duration of the measurements considered. (When power spectra are computed using real data, *T* can not be increased indefinitely; the resulting power spectra are then approximations of the above formulas.)
- <...> indicates an average over many such measurements.

The Magnetic Power Spectrum

Fourier transform of magnetic field

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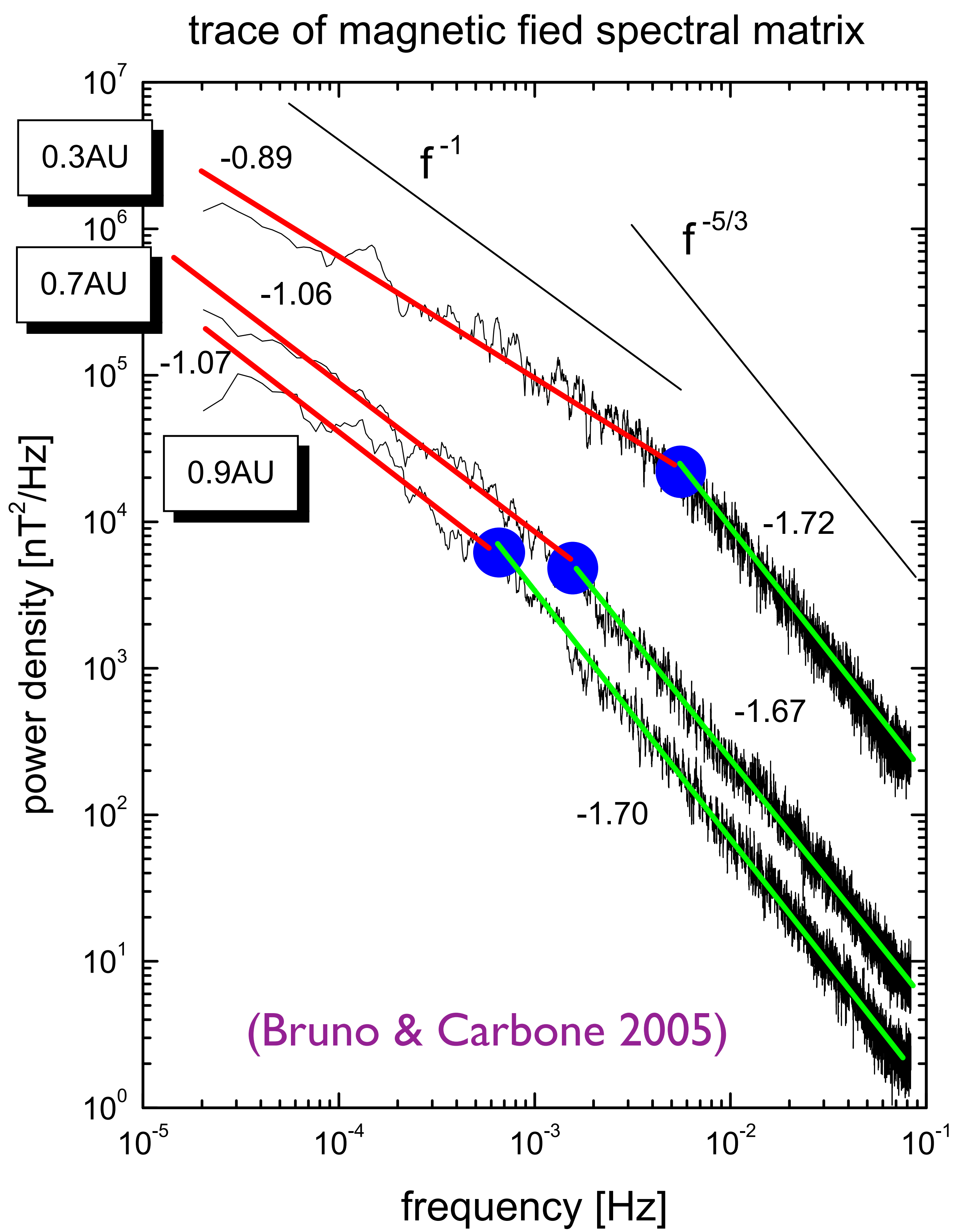
power spectrum

$$P(f) = \lim_{T\to\infty} \frac{1}{T} \langle \tilde{\vec{B}}(f) \cdot \tilde{\vec{B}}(-f) \rangle$$

- *B*(*f*) can be thought of as the part of *B* that oscillates with frequency *f*.
- When turbulence is present, *P*(*f*) is non-negligible over a broad range of frequencies. Typically, *P*(*f*) has a power-law scaling over frequencies varying by one or more powers of 10.

Magnetic Power Spectra in the Solar Wind

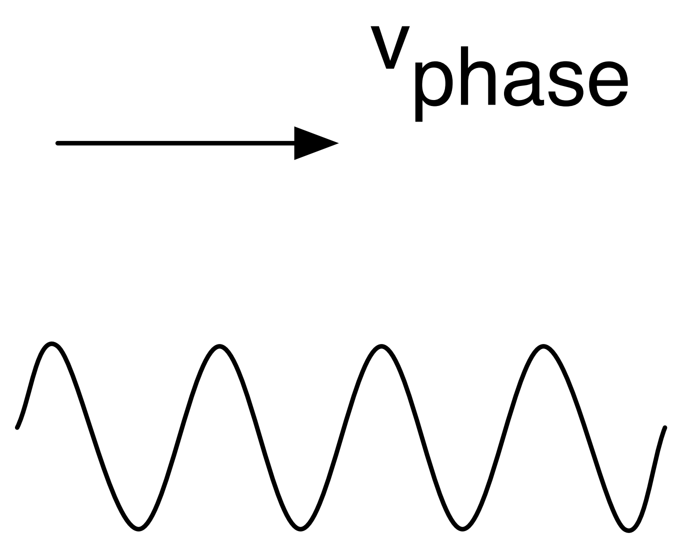


Is This Turbulence?

Yes - the magnetic fluctuations measured by the spacecraft span a broad range of timescales.

Similar spectra are observed at all locations explored by spacecraft in the solar wind.

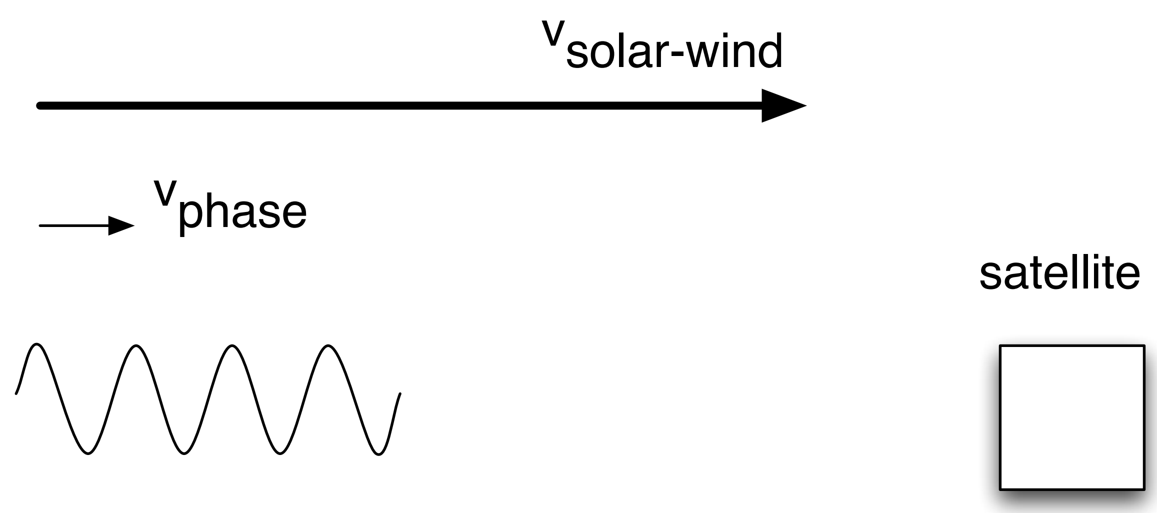
What Causes the Time Variation Seen in Spacecraft Measurements?



- Consider a traveling plasma wave (more on plasma waves soon). *Imagine you're viewing this wave as you move away from the Sun at the same velocity as the solar wind*. From your perspective, the time variation of the magnetic field is the result of the wave pattern moving past you at the wave phase speed *relative to the solar-wind plasma*, v_{phase}, which is typically ≈30 km/s in the solar wind near Earth.

- If the wavenumber of the wave is k, the angular frequency of the magnetic field in this case is kv_{phase}. This frequency characterizes the "intrinsic time variations" of the magnetic field in the solar wind frame.

But What if You Measure *B* Using a "Stationary" Spacecraft That Does Not Move with the Solar Wind?



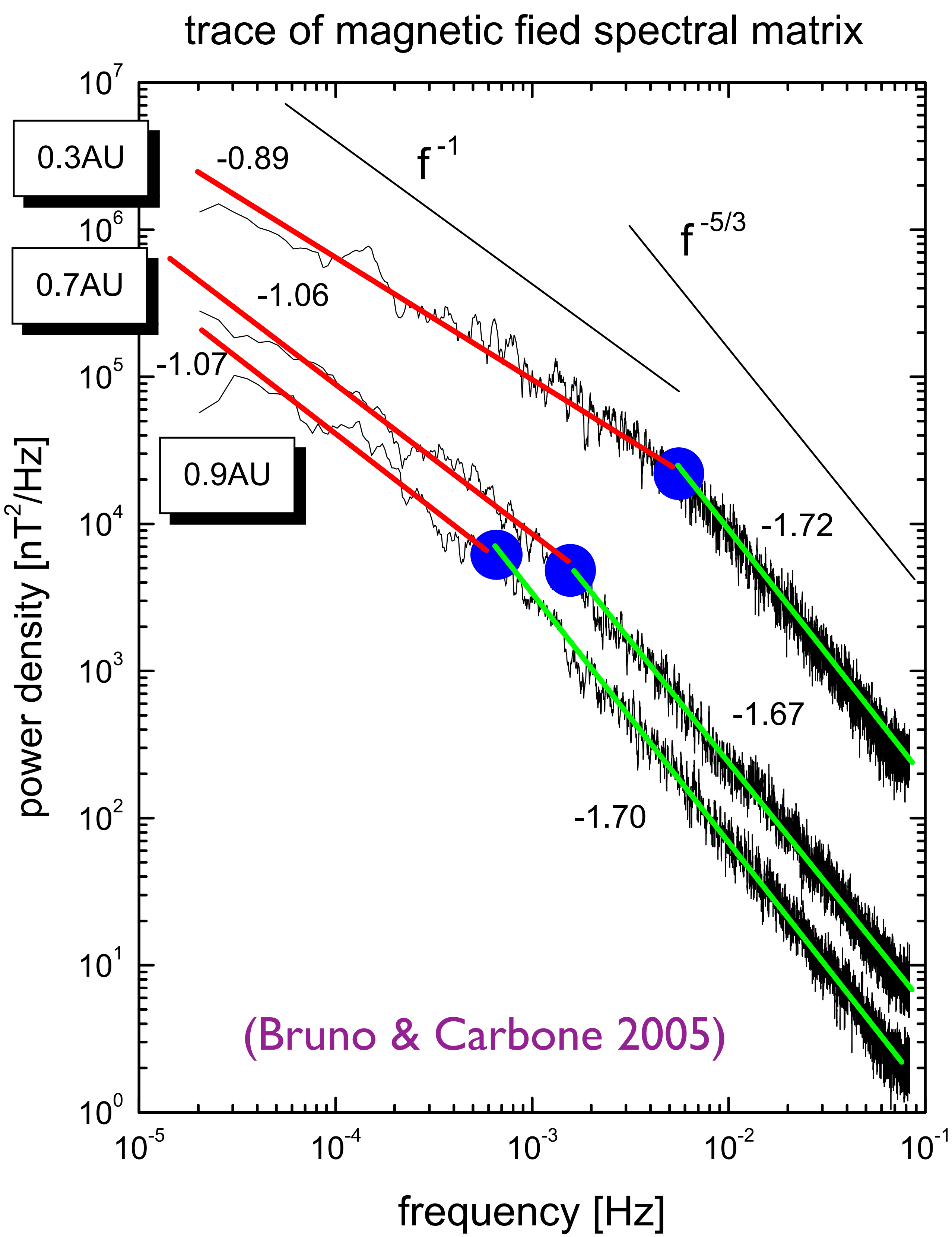
- Near Earth, the speed at which the solar wind flows past a satellite is highly supersonic, typically $>10v_{phase}$, where v_{phase} is the phase speed in the plasma rest frame.

- *"Taylor's Frozen-Flow Hypothesis":* Time variation measured by a spacecraft results primarily from the advection of spatially variations past the spacecraft at $v_{solar\text{-}wind}$, not from the "intrinsic time variation" in the plasma frame. E.g., the spacecraft would see almost the same thing if the fields were static in the solar-wind frame.

- If a wave with wavevector $\boldsymbol{k}$ is advected past the spacecraft (i.e., $\delta B \propto e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$), and the wave is static in the solar-wind frame, the spacecraft measures a magnetic oscillation with angular frequency $\omega = 2\pi f = \boldsymbol{k}\cdot\boldsymbol{v}_{solar\text{-}wind}$.

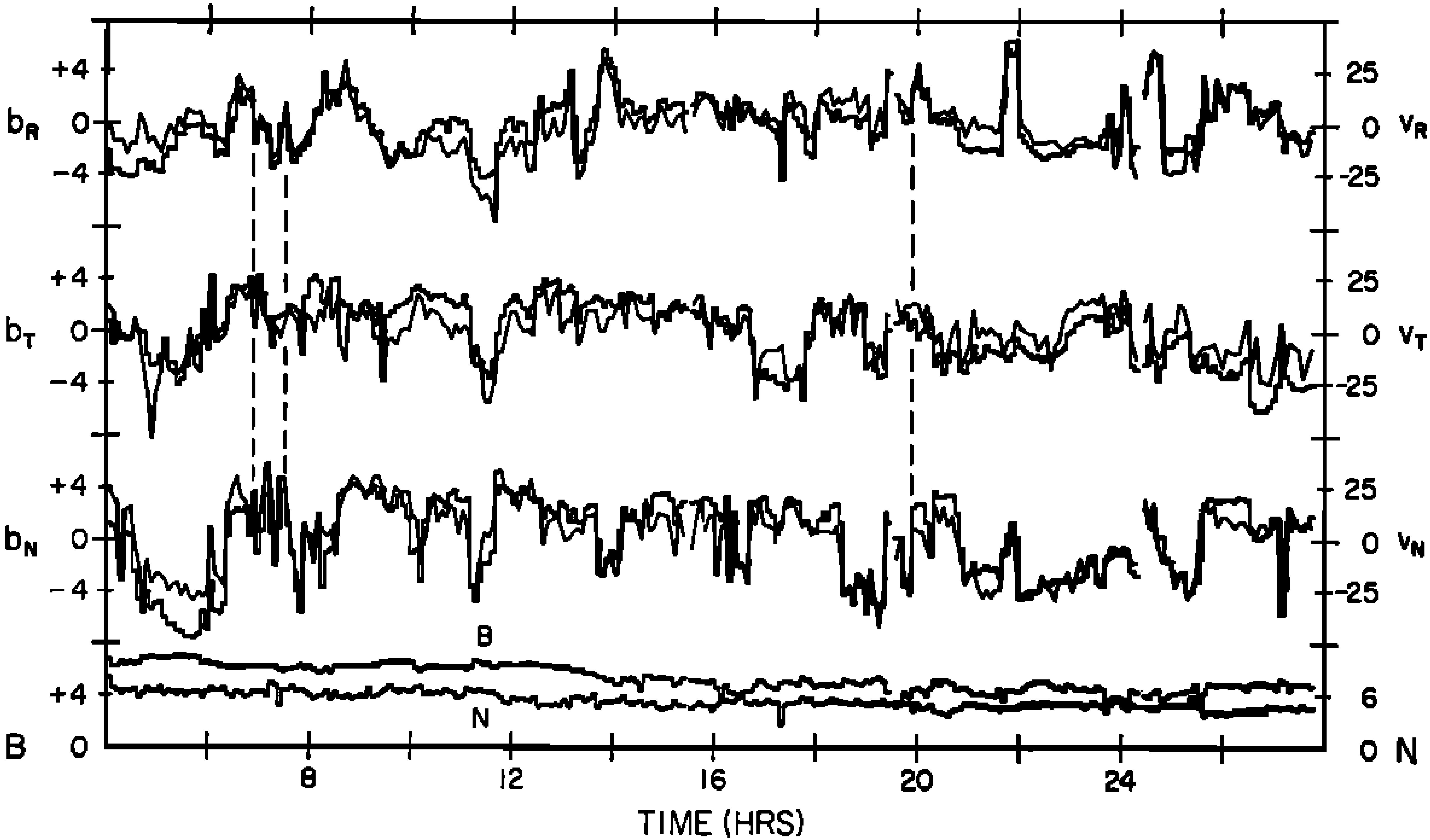
- Frequencies measured by a spacecraft thus tell us about $\boldsymbol{k}$ (spatial structure) rather than the intrinsic time variation that would be seen in the plasma rest frame.

Taylor's Frozen-Flow Hypothesis



The frequency spectra measured by satellites correspond to wavenumber spectra in the solar-wind frame.

Question: in the data below, the *v* and *B* fluctuations are highly correlated --- what does this mean? We'll come back to this...



Data from the *Mariner 5* spacecraft (Belcher & Davis 1971)

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Magnetohydrodynamics (MHD)

- In order to understand the power spectra seen in the solar wind, we need a theoretical framework for analyzing fluctuations on these lengthscales and timescales.

- In the solar wind near Earth, phenomena occurring at large length scales (exceeding ≈300 km) and long time scales (e.g., exceeding ≈10 s) can be usefully described within the framework of a fluid theory called magnetohydrodynamics (MHD).

- In MHD, the plasma is quasi-neutral, and the displacement current is neglected in Maxwell's equations (since the fluctuation frequencies are small).

Ideal, Adiabatic MHD

density

velocity

magnetic field

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{v})$$

$$\rho \left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = -\nabla \left(p + \frac{B^2}{8\pi} \right) + \frac{\vec{B} \cdot \nabla \vec{B}}{4\pi}$$

$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \nabla \right) \left(\frac{p}{\rho^\gamma} \right) = 0$$

pressure

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\vec{v} \times \vec{B} \right)$$

ratio of specific heats

(The phrase "ideal MHD" means that dissipative terms involving viscosity and resistivity have been neglected. I'll come back to these terms later.)

Ideal, Adiabatic MHD

"continuity equation"

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{v})$$

mass conservation

$$\rho \left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = -\nabla \left(p + \frac{B^2}{8\pi} \right) + \frac{\vec{B} \cdot \nabla \vec{B}}{4\pi}$$

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$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\vec{v} \times \vec{B} \right)$$

Ideal, Adiabatic MHD

Newton's 2nd law

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{v})$$

$$\rho \left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = -\nabla \left(p + \frac{B^2}{8\pi} \right) + \frac{\vec{B} \cdot \nabla \vec{B}}{4\pi}$$

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Ideal, Adiabatic MHD

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$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \nabla \right) \left(\frac{p}{\rho^\gamma} \right) = 0$$

adiabatic evolution. the specific entropy [$\propto \ln(p/\rho^\gamma)$] of a fluid element does not change in time.

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\vec{v} \times \vec{B} \right)$$

(An alternative, simple approximation is the isothermal approximation, in which $p = \rho c_s^2$, with the sound speed c_s = constant. More generally, this equation is replaced with an energy equation that includes thermal conduction and possibly other heating and cooling mechanisms.)

Ideal, Adiabatic MHD

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{v})$$

$$\rho \left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = -\nabla \left(p + \frac{B^2}{8\pi} \right) + \frac{\vec{B} \cdot \nabla \vec{B}}{4\pi}$$

$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \nabla \right) \left(\frac{p}{\rho^\gamma} \right) = 0$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\vec{v} \times \vec{B} \right)$$

Ohm's Law for a perfectly conducting plasma

Ideal, Adiabatic MHD

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{v})$$

$$\rho \left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = -\nabla \left(p + \frac{B^2}{8\pi} \right) + \frac{\vec{B} \cdot \nabla \vec{B}}{4\pi}$$

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$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\vec{v} \times \vec{B} \right)$$

- Magnetic forces: magnetic pressure and magnetic tension
- Frozen-in Law: magnetic field lines are like threads that are frozen to the plasma and advected by the plasma

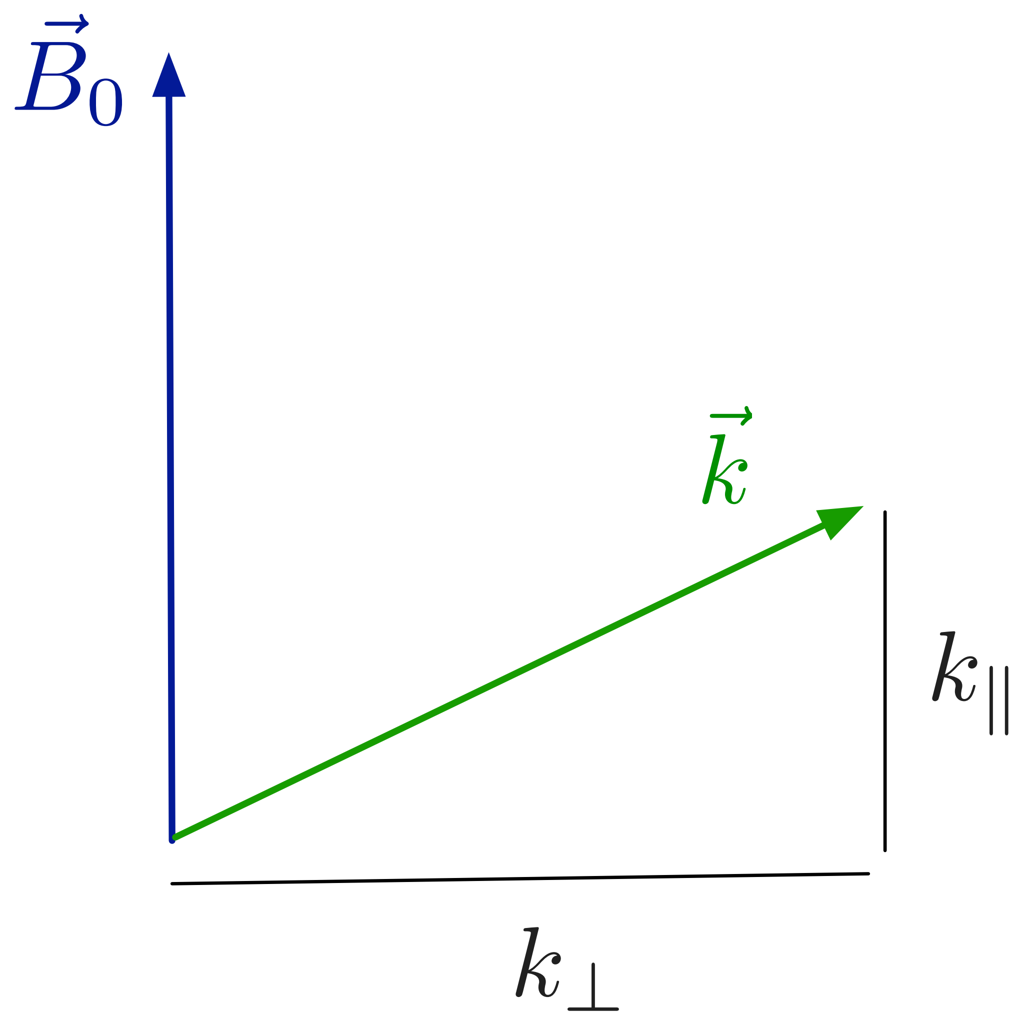
Waves: small-amplitude oscillations about some equilibrium

$$\delta\vec{v} \; = \; \delta\vec{v}_0 \, \cos(\vec{k}\cdot\vec{x} \, - \, \omega t \, + \, \phi_0)$$

wavelength λ = 2π/k

decreasing λ ⇔ increasing k

energy cascades to small λ, or equivalently to large k



As an example, let's look at MHD Waves in "low-beta" plasmas such as the solar corona.

$$\beta = \frac{8\pi p}{B^2} \ll 1, \quad \text{so magnetic pressure} \quad \frac{B^2}{8\pi} \quad \text{greatly exceeds } p$$

$$c_s = \text{sound speed} = \sqrt{\frac{\gamma p}{\rho}}$$

$$v_A = \text{“Alfvén speed”} = \frac{B}{\sqrt{4\pi\rho}}$$

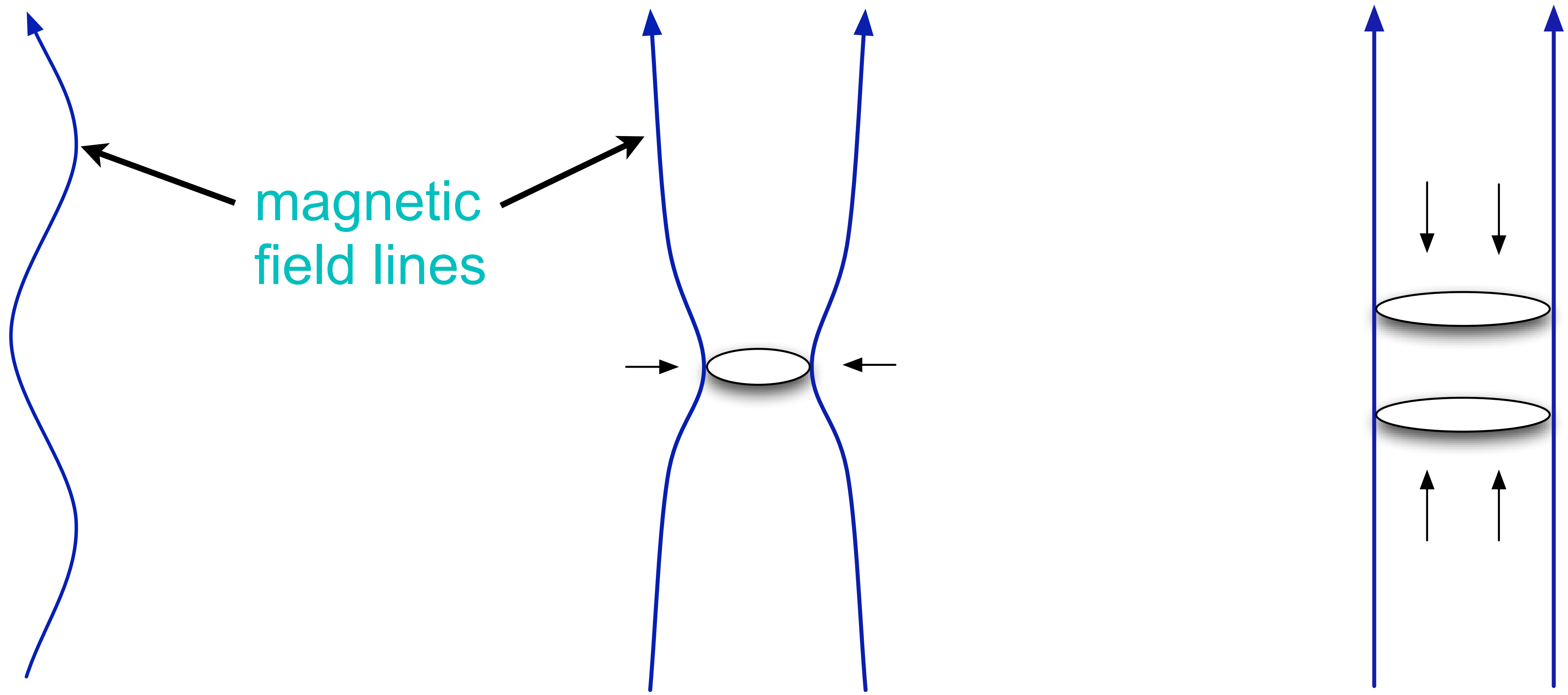
$$\longrightarrow \beta = \frac{2c_s^2}{\gamma v_A^2}$$

Plasma Waves at Low Beta

Alfvén wave

fast magnetosonic wave

slow magnetosonic wave



magnetic tension

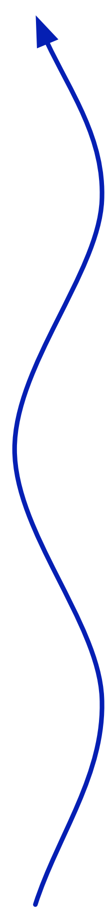
(like a wave propagating on a string)

magnetic pressure

thermal pressure

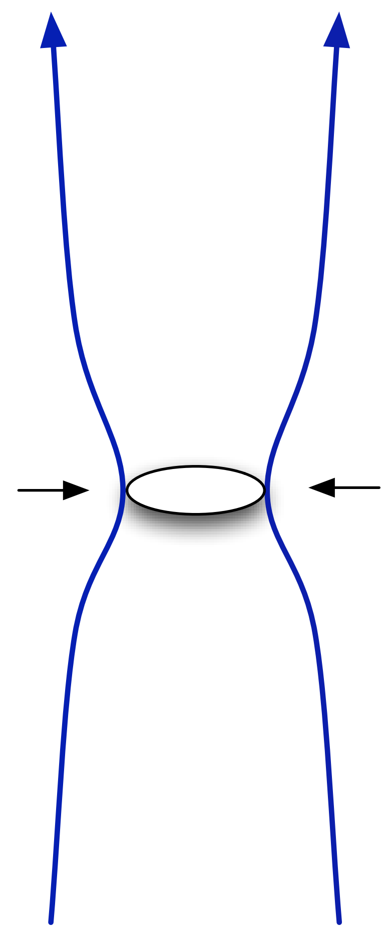
Plasma Waves at Low Beta

Alfvén wave



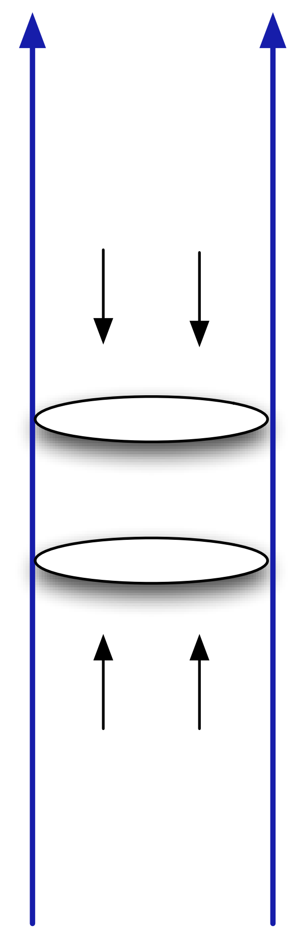
magnetic tension

fast magnetosonic wave



magnetic pressure

slow magnetosonic wave



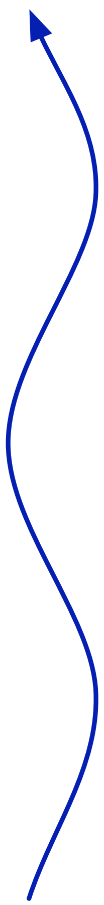
thermal pressure

$$\omega = k_{\|} v_{\mathrm{A}}$$

$$v_{\mathrm{A}} = B/\sqrt{4\pi\rho}$$ ← "Alfvén speed"

Plasma Waves at Low Beta

Alfvén wave

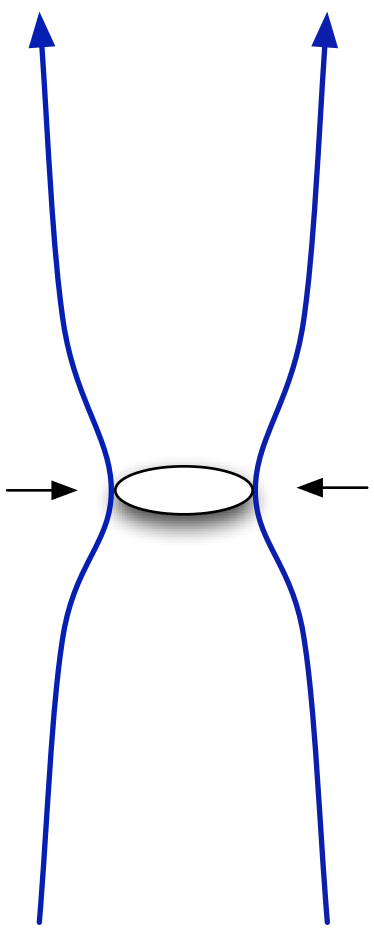


magnetic tension

$$\omega = k_{\parallel} v_{A}$$

$$v_{A} = B/\sqrt{4\pi\rho}$$

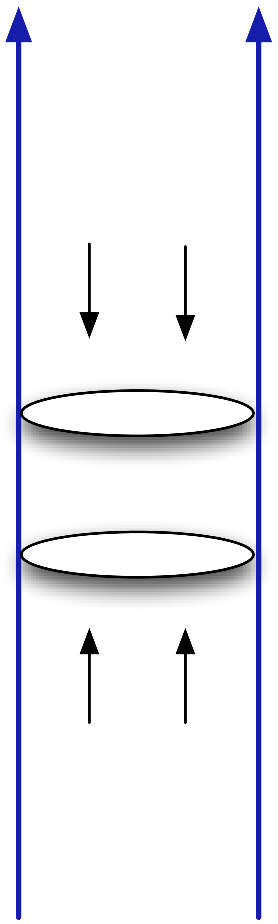
fast magnetosonic wave



magnetic pressure

$$\omega = k v_{A}$$

slow magnetosonic wave



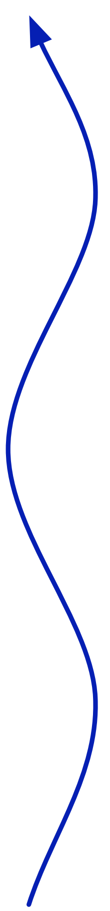
thermal pressure

$$\omega = k_{\parallel} c_{s}$$

sound speed $c_s = (\gamma p/\rho)^{1/2}$

Plasma Waves at Low Beta

Alfvén wave

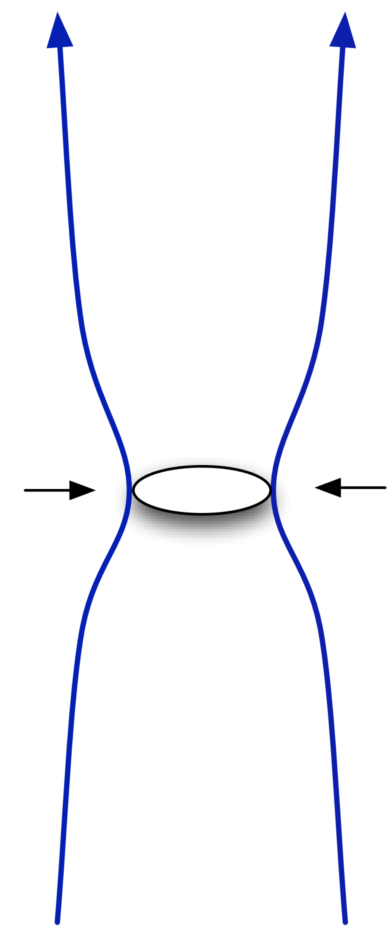


magnetic tension

$$\omega = k_{\parallel} v_A$$

virtually undamped
in collisionless plasmas
like the solar wind

fast magnetosonic wave

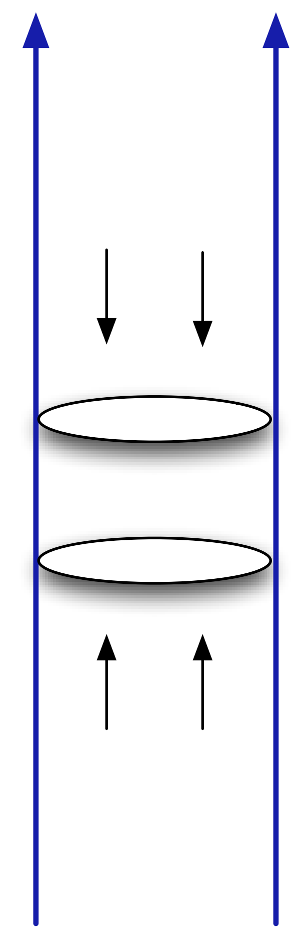


magnetic pressure

$$\omega = k v_A$$

damped in collisionless plasmas
(weakly at $\beta \ll 1$,
strongly at $\beta \approx 1$)

slow magnetosonic wave



thermal pressure

$$\omega = k_{\parallel} c_s$$

strongly damped
in collisionless plasmas

Properties of Alfvén Waves (AWs)

- Two propagation directions: parallel to $\vec{B}_0$ or anti-parallel to $\vec{B}_0$.

- $\delta\vec{v} = \pm\delta\vec{B}/\sqrt{4\pi\rho_0}$ for AWs propagating in the $\mp\vec{B}_0$ direction, and $\delta\rho = 0$

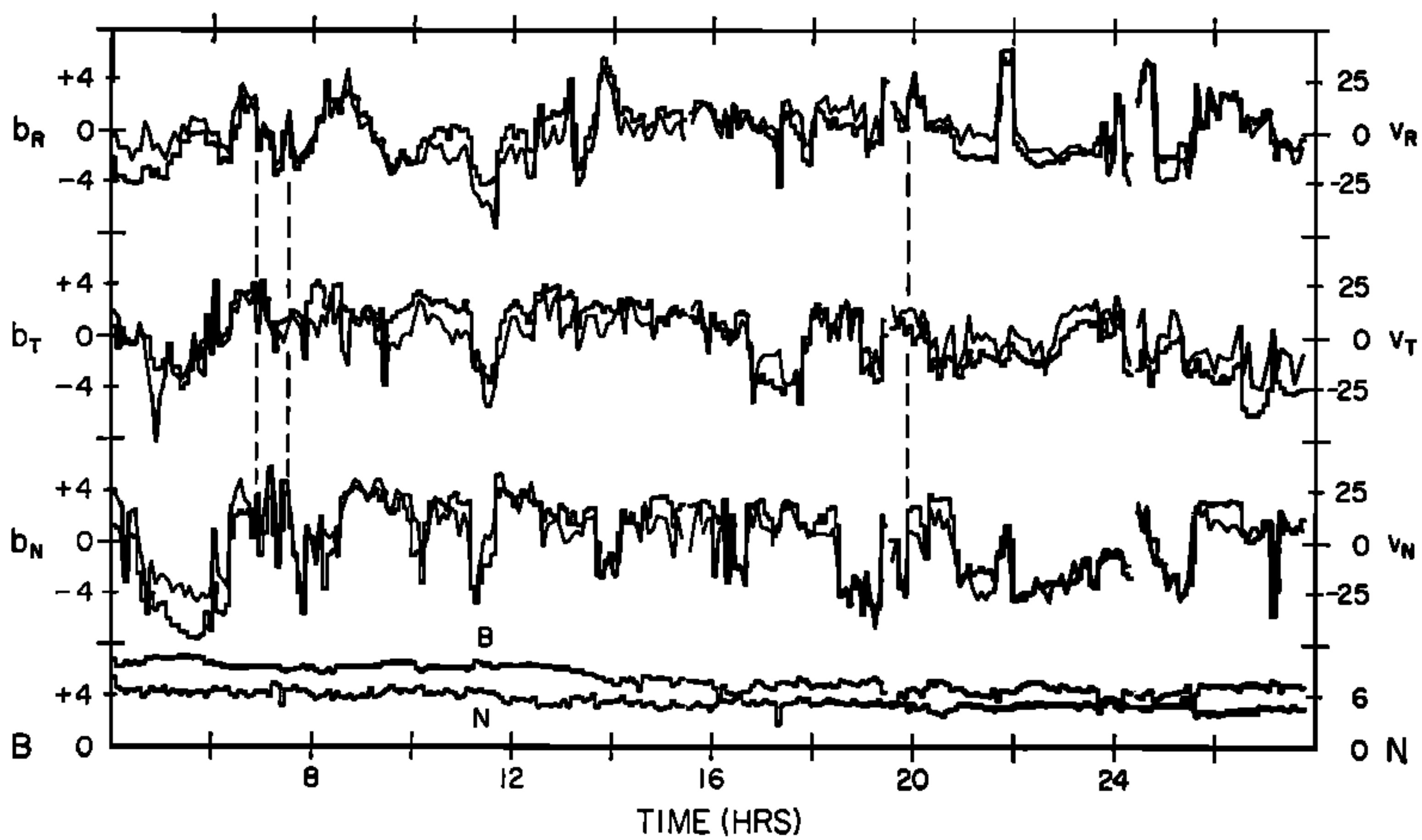
Properties of Alfvén Waves (AWs)

- Two propagation directions: parallel to $\vec{B}_0$ or anti-parallel to $\vec{B}_0$.

- $\delta \vec{v} = \pm \delta \vec{B} / \sqrt{4\pi\rho_0}$ for AWs propagating in the $\mp \vec{B}_0$ direction, and $\delta\rho = 0$

- In the solar wind $\delta\rho/\rho_0 \ll |\delta \vec{B}/B_0|$. Also, there are many intervals of time in which the relation $\delta \vec{v} = \pm \delta \vec{B} / \sqrt{4\pi\rho_0}$ is nearly satisfied, with the sign corresponding to propagation of AWs away from the Sun.

ALFVÉN WAVES IN SOLAR WIND



(Belcher & Davis 1971)

Properties of Alfvén Waves (AWs)

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- In the solar wind $\delta\rho/\rho_0 \ll |\delta\vec{B}/B_0|$. Also, there are many intervals of time in which the relation $\delta\vec{v} = \pm\delta\vec{B}/\sqrt{4\pi\rho_0}$ is nearly satisfied, with the sign corresponding to propagation of AWs away from the Sun.

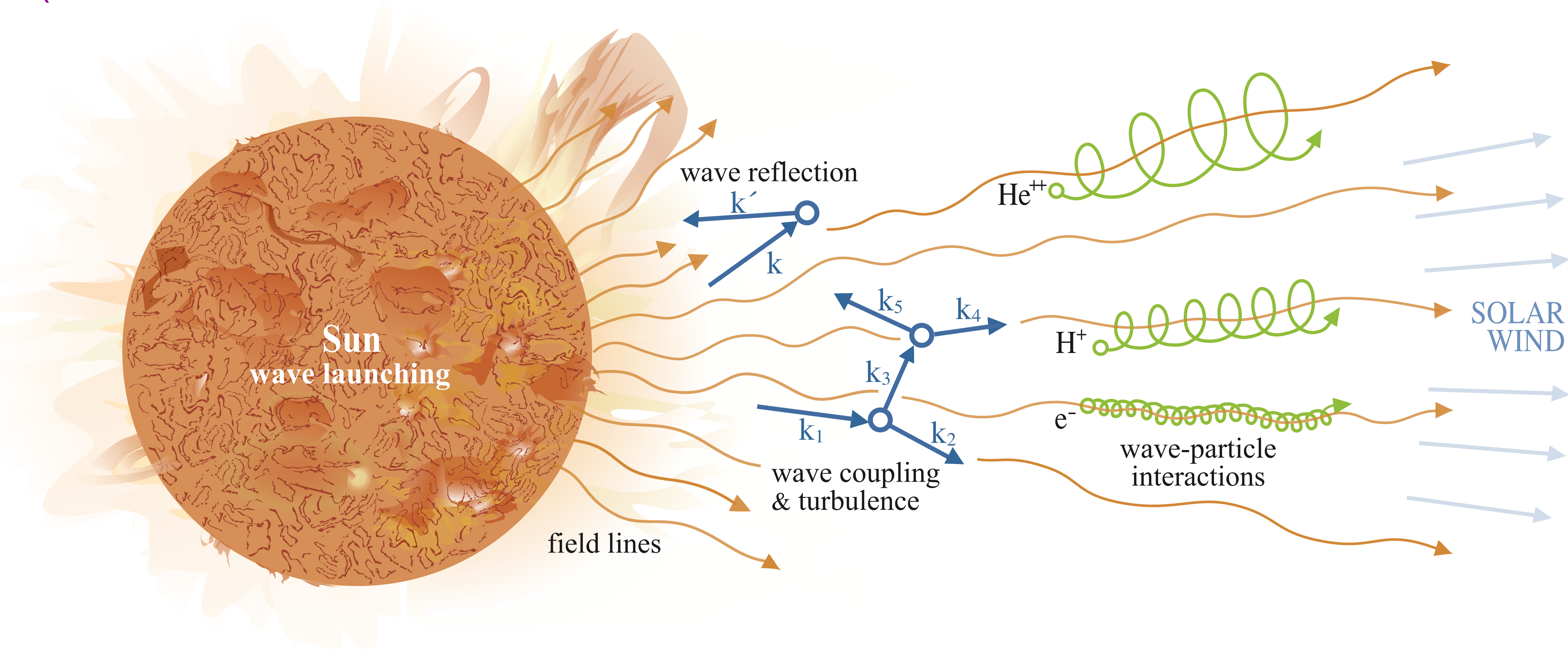
- For these reasons, and because AWs are the least damped of the large-scale plasma waves, AWs or nonlinear AW-like fluctuations likely comprise most of the energy in solar wind turbulence

In a moment, we'll consider a specialized form of MHD that contains Alfvén waves and nonlinear interactions between Alfvén waves, but neglects slow and fast magnetosonic waves.

But before jumping in to more details, to motivate us thru what's coming up, a few words about why Alfvén-wave turbulence is so interesting…

Coronal Heating and Solar-Wind Acceleration by Waves

(Parker 1965, Coleman 1968, Velli et al 1989, Zhou & Matthaeus 1989, Cranmer et al 2007)



- The Sun launches Alfven waves, which transport energy outwards
- The waves become turbulent, which causes wave energy to ‘cascade’ from long wavelengths to short wavelengths
- Short-wavelength waves dissipate, heating the plasma. This increases the thermal pressure, which, along with the wave pressure, accelerates the solar wind.

Incompressible MHD

$$\rho\left(\frac{\partial\vec{v}}{\partial t}+\vec{v}\cdot\nabla\vec{v}\right)=-\nabla\left(p+\frac{B^2}{8\pi}\right)+\frac{1}{4\pi}\vec{B}\cdot\nabla\vec{B}+\rho\nu\nabla^2\vec{v}$$

$$\frac{\partial\vec{B}}{\partial t}=\nabla\times(\vec{v}\times\vec{B})+\eta\nabla^2\vec{B}$$

$$\nabla\cdot\vec{v}=0$$

$$\rho=\ \text{constant}$$

kinematic viscosity

resistivity

Because Alfvén waves (AWs) satisfy $\nabla\cdot\boldsymbol{v} = 0$, incompressible MHD captures much of the physics of both small-amplitude AWs and AW turbulence.

Because the viscous and resistive terms contain ∇^2 they dominate for fluctuations with sufficiently small lengthscales.

Elsässer Variables, $\vec{a}^{\pm}$

$$\vec{B} = B_0\hat{z} + \delta\vec{B}$$

(B_0 = constant)

$$v_{\rm A} = B_0/\sqrt{4\pi\rho}$$

$$\vec{b} = \delta\vec{B}/\sqrt{4\pi\rho}$$

$$\Pi = \frac{1}{\rho}\left(p + \frac{B^2}{8\pi}\right)$$

represent AWs traveling parallel (a^-) or anti-parallel (a^+) to $\vec{B}_0$

$$\vec{a}^{\,\pm} = \vec{v} \pm \vec{b}$$

Substitute the above into the MHD eqns and obtain

$$\frac{\partial\vec{a}^{\,\pm}}{\partial t} \mp v_{\rm A}\frac{\partial\vec{a}^{\,\pm}}{\partial z} = -\nabla\Pi - \vec{a}^{\,\mp}\cdot\nabla\vec{a}^{\,\pm} + \{\text{ terms} \propto \text{to } \nu \text{ or } \eta\ \}$$

Conserved Quantities in Ideal, Incompressible MHD

$$\mathcal{E} = \int d^3x \left(\frac{\rho |\vec{v}|^2}{2} + \frac{|\delta \vec{B}|^2}{8\pi} \right) = \frac{\rho}{4} \int d^3x \left(|\vec{a}^{\,+}|^2 + |\vec{a}^{\,-}|^2 \right)$$ ⟵ energy

$$H_{\rm m} = \int d^3x \, \vec{A} \cdot \vec{B}$$ ⟵ magnetic helicity

cross helicity

$$H_{\rm c} = \int d^3x \, \vec{v} \cdot \vec{B} = \int d^3x \, \vec{v} \cdot \vec{B}_0 + \frac{\sqrt{\pi \rho}}{2} \int d^3x \left(|\vec{a}^{\,+}|^2 - |\vec{a}^{\,-}|^2 \right)$$

integral vanishes when there is no average flow along B_0

integral measures the difference in energy between AWs moving parallel and anti-parallel to B_0.

- These "quadratic invariants" are conserved in the "ideal" limit, in which the viscosity and resistivity are set to zero.

Conserved Quantities and Cascades

- MHD turbulence results from "nonlinear interactions" between fluctuations. These interactions are described mathematically by the nonlinear terms in the MHD equations (e.g., $a^{-} \cdot \nabla a^{+}$). When you neglect viscosity and resistivity, the equations conserve *E*, H_m, and H_c. The nonlinear terms in the equations thus can't create or destroy *E*, H_m, and H_c, but they can "transport" these quantities from large scales to small scales (a "*forward cascade*") or from small scales to large scales (an "*inverse cascade*").

- At sufficiently small scales, dissipation (via viscosity, resistivity, or collisionless wave-particle interactions) truncates a forward energy cascade, leading to turbulent heating of the ambient medium.

Forward and "Inverse" Cascades in 3D Incompressible MHD

(Frisch et al 1975)

- Energy cascades from large scales to small scales. (Large wave packets or eddies break up into smaller wave packets or eddies.)

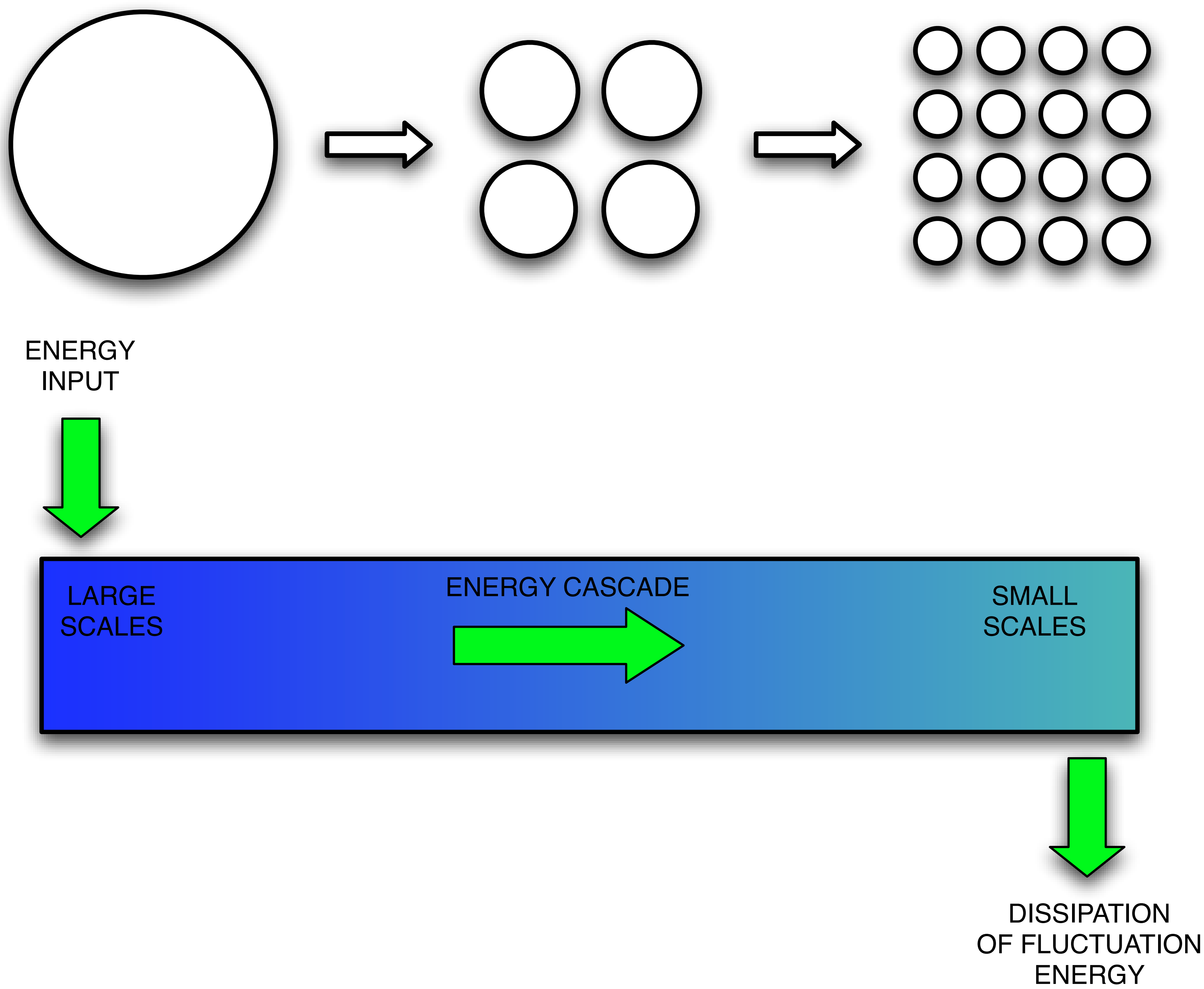
- Magnetic helicity cascades from small scales to large scales. (Helical motions associated with rotation cause the growth of large-scale magnetic fields, i.e., dynamos.)

The Inertial Range of Turbulence

- Suppose turbulence is stirred/excited at a large scale or "outer scale" *L.*
- Suppose that the turbulence dissipates at a much smaller scale *d*, the "dissipation scale."
- Lengthscales λ satisfying the inequality $d \ll \lambda \ll L$ are said to be in the "inertial range" of scales.
- Fluctuations with wavelengths in the inertial range are insensitive to the details of either the forcing at large scales or the dissipation at small scales.
- Systems with different types of large-scale forcing or small-scale dissipation may nevertheless possess similar dynamics and statistical properties in the inertial range ("universality").

“Energy Cascade” in Hydrodynamic Turbulence

Canonical picture: larger eddies break up into smaller eddies



Kolmogorov's Theory of Inertial-Range Scalings in Hydrodynamic Turbulence

$$\delta v_\lambda = \text{rms amplitude of velocity difference across a spatial separation } \lambda$$

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In the “inertial range,” ϵ is independent of λ.

$$\longrightarrow \delta v_\lambda \propto \lambda^{1/3}$$

Connection to Power Spectra

- Let $f = kv_{\rm solar-wind}/2\pi$, and $E(k)dk = P(f)df$, where $P(f)$ (or $E(k)$) is the frequency (or wavenumber) power spectrum of the *velocity* fluctuations. (This velocity power spectrum is defined just like the magnetic power spectrum introduced earlier in the talk, but with $\vec{B} \rightarrow \vec{v}$.)
- The total kinetic energy in velocity fluctuations per unit mass is $0.5\int_0^\infty E(k)dk$.
- The mean square velocity fluctuation at lengthscale $\lambda \equiv k_1^{-1}$ is given by

$$(\delta v_\lambda)^2 \simeq \int_{0.5k_1}^{2k_1} E(k)\,dk \sim k_1 E(k_1)$$

- If $\delta v_\lambda \propto \lambda^{1/3} \propto k_1^{-1/3}$, then $E(k) \propto k^{-5/3}$, and

$$P(f) \propto f^{-5/3}.$$

- We saw earlier in this talk that this type of scaling is seen in magnetic-field measurements. A similar scaling is also seen in velocity fluctuation measurements, although the exponent appears to be somewhat smaller than 5/3 (Podesta et al 2007).

Alfvén-Wave Turbulence

- Wave propagation adds an additional complication.
- Here, I'm going to walk you through some difficult physics, and try to convey some important ideas through diagrams rather than equations.
- These ideas are useful and have been influential in the field, but represent a highly idealized viewpoint that misses some physics and is not universally accepted.
- This is challenging material the first time you see it, but these notes will hopefully serve as a useful introduction, and one that you can build upon with further study if you wish to learn more.

Nonlinear terms - the basis of turbulence

No nonlinear terms $\rightarrow$ linear waves. Small nonlinear terms $\rightarrow$ fluctuations are still wavelike, but waves interact ("weak turbulence" or "wave turbulence"). Large nonlinear terms $\rightarrow$ strong turbulence, fluctuations are no longer wave-like.

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$$\frac{\partial \vec{a}^{\pm}}{\partial t} \mp v_A \frac{\partial \vec{a}^{\pm}}{\partial z} = -\nabla \Pi - \vec{a}^{\mp} \cdot \nabla \vec{a}^{\pm}$$

Note that the nonlinear terms vanish unless a^+ and a^- are both nonzero. Nonlinear interactions result from "collisions between oppositely directed wave packets" (Iroshnikov 1963, Kraichnan 1965).

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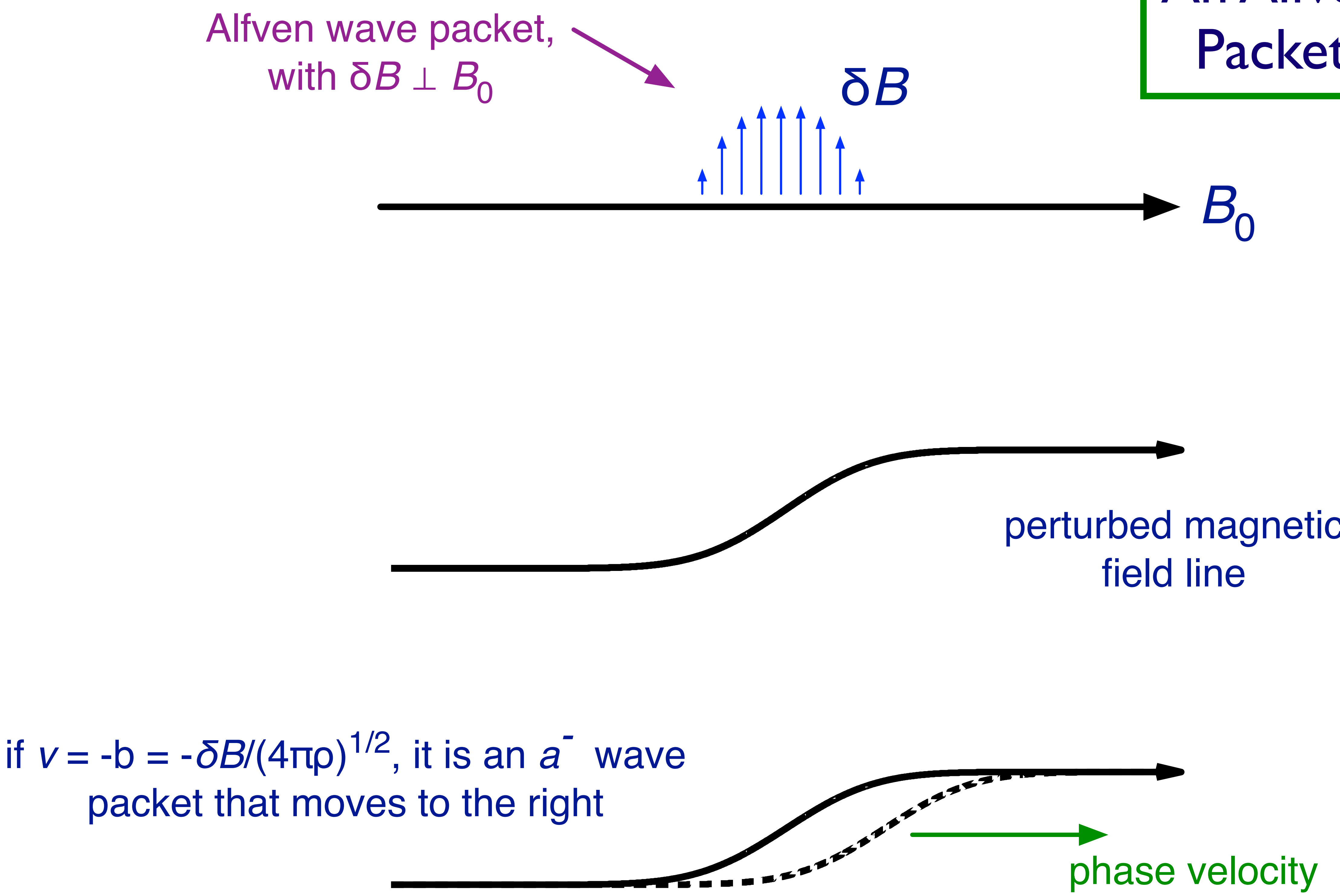
$$\frac{\partial \vec{a}^{-}}{\partial t} + (v_A \hat{z} + \vec{a}^{+}) \cdot \nabla \vec{a}^{-} = -\nabla \Pi$$

If $\vec{a}^+ = 0$, the $\vec{a}^-$ waves follow the background field $B_0 \hat{z}$. When $\vec{a}^+ \neq 0$ and $a^+ \ll v_A$, the $\vec{a}^-$ waves approximately follow the field lines corresponding to $B_0 \hat{z}$ and the part of $\delta \vec{B}$ associated with the $\vec{a}^+$ waves (Maron & Goldreich 2001).

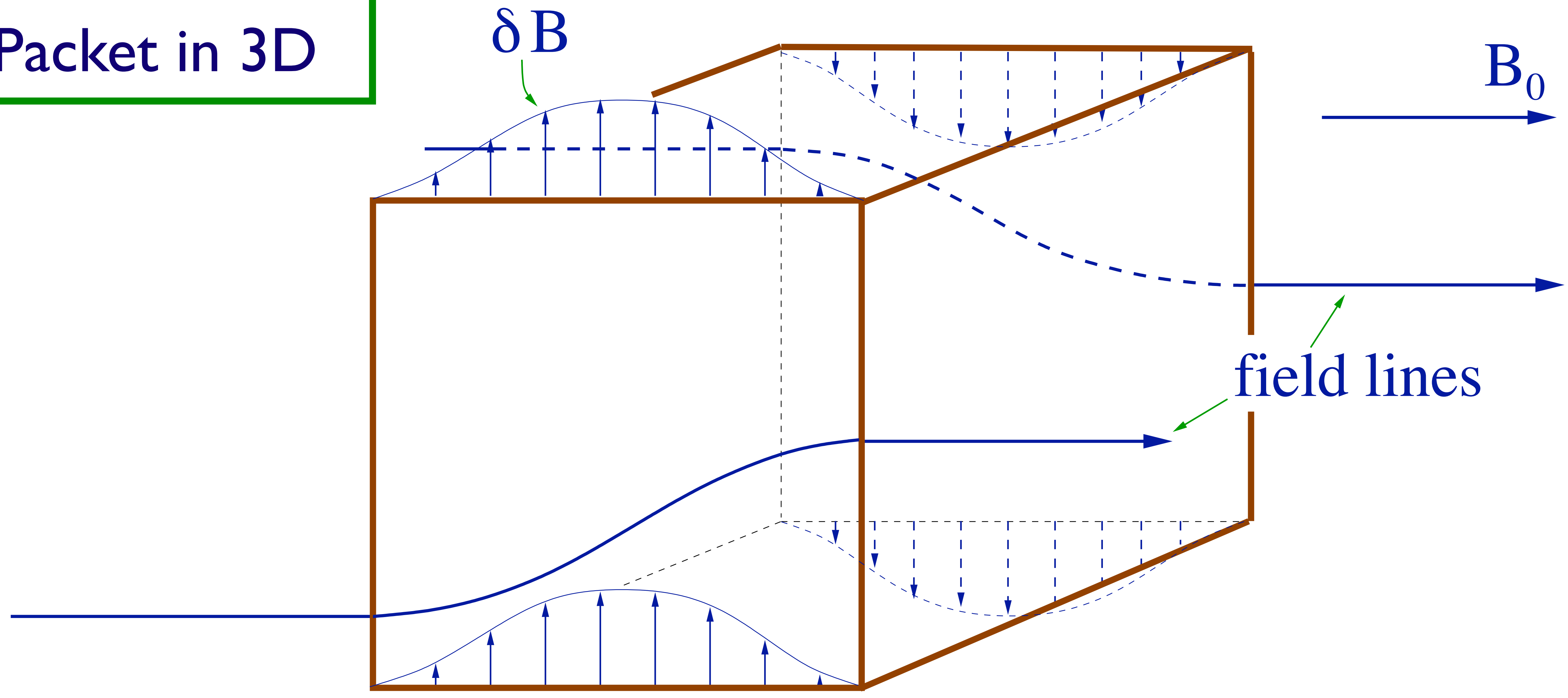
the way that wave packets displace field lines is the key
to understanding nonlinear wave-wave interactions

An Alfven Wave Packet in 1D



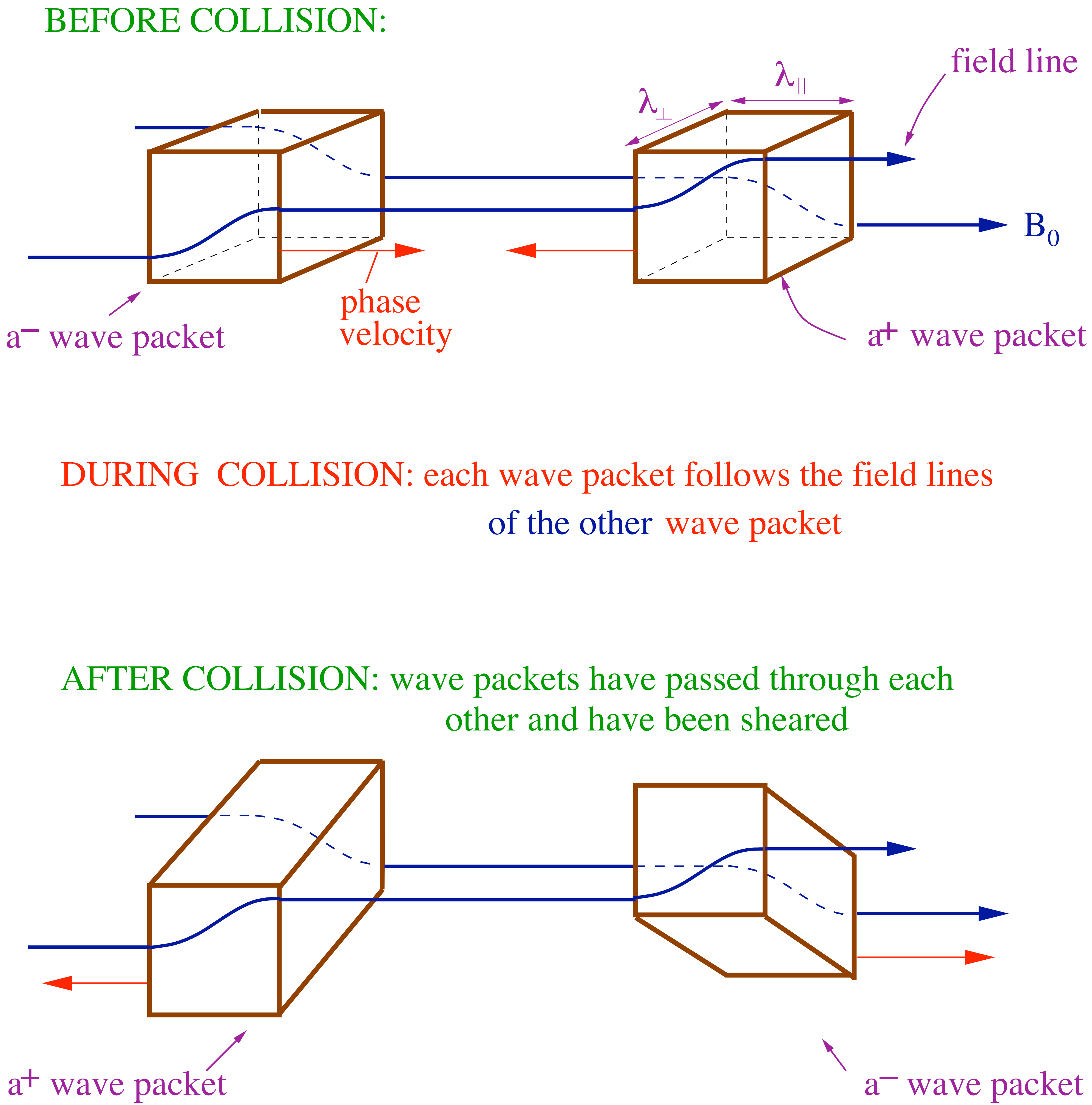
An "incoming" a^+ wave packet from the right would follow the perturbed field line, moving to the left and down.

An Alfven Wave Packet in 3D

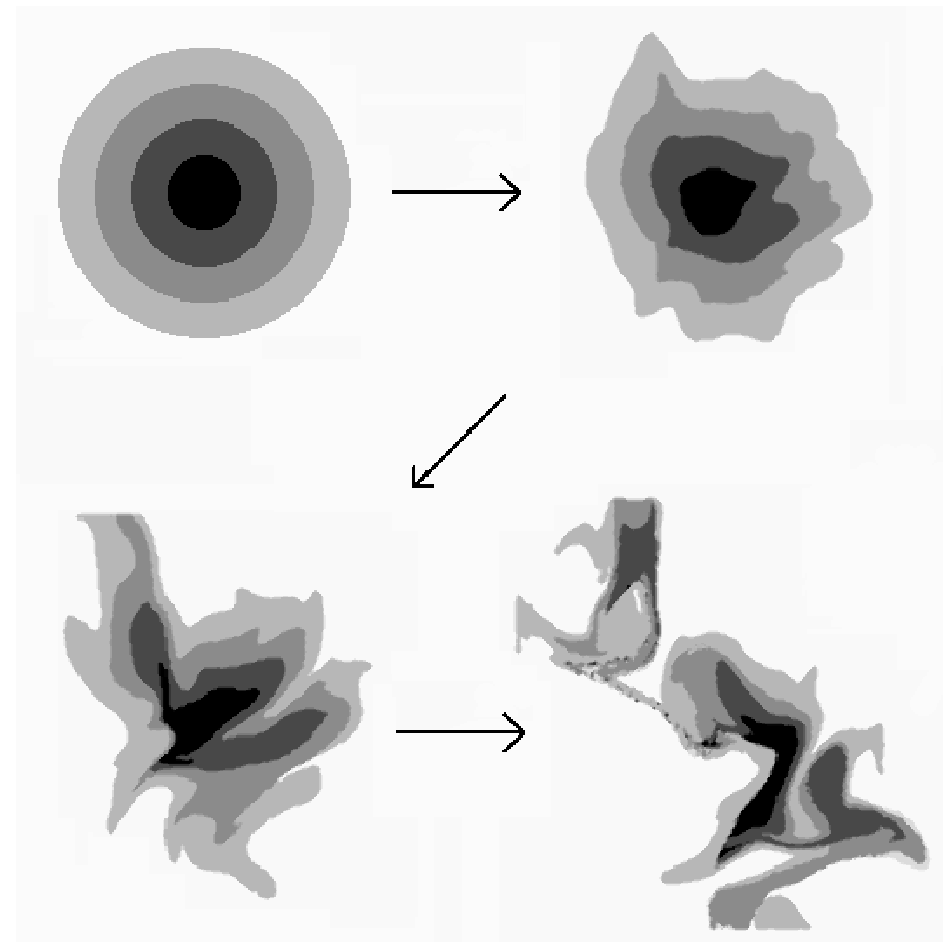


If $\vec{v} = -\vec{b}$, then $a^+ = 0$ and this is an a^- wave packet that propagates to the right without distortion.

An "incoming" a^+ wave packet approaching from the right would follow the perturbed field lines, moving left and down in the plane of the cube nearest to you and moving to the left and up in the plane of the cube farthest from you.

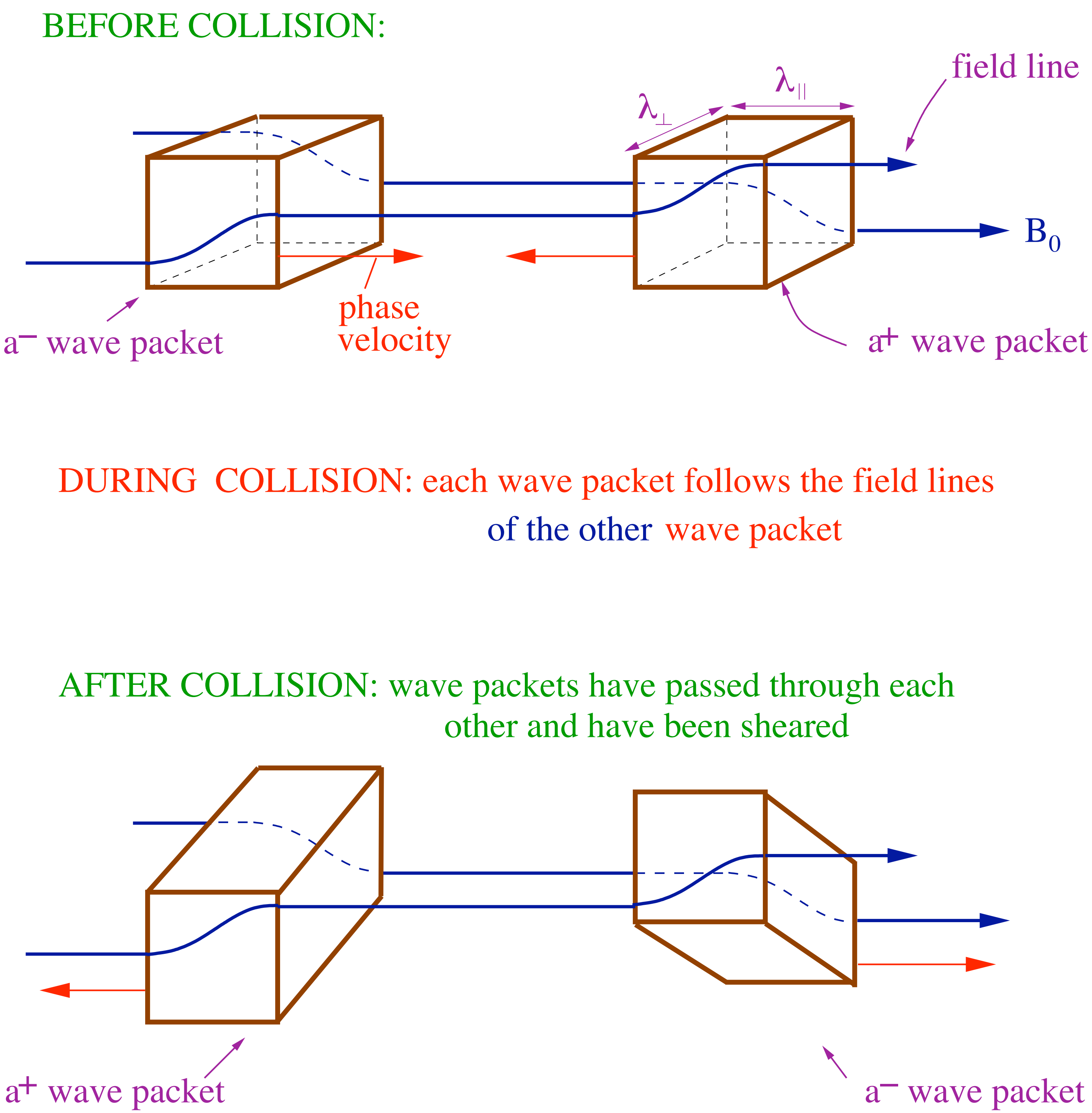

BEFORE COLLISION:
field line
$\lambda_{\parallel}$
$\lambda_{\perp}$
B_0
phase
velocity
a^- wave packet
a^+ wave packet
DURING COLLISION: each wave packet follows the field lines
of the other wave packet
AFTER COLLISION: wave packets have passed through each
other and have been sheared
a^+ wave packet
a^- wave packet

Shearing of a wave packet by field-line wandering



Maron & Goldreich (2001)

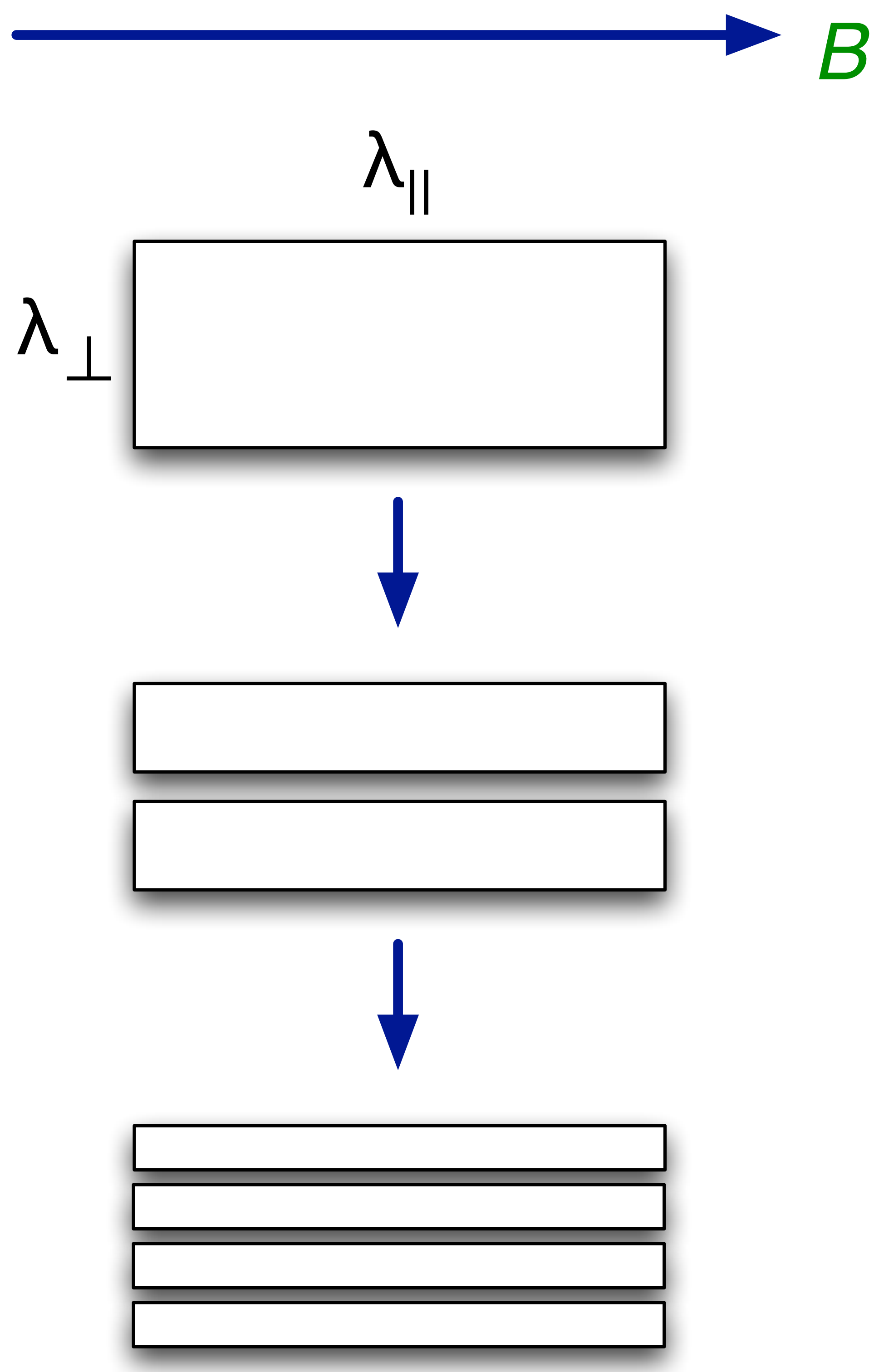
Anisotropic energy cascade



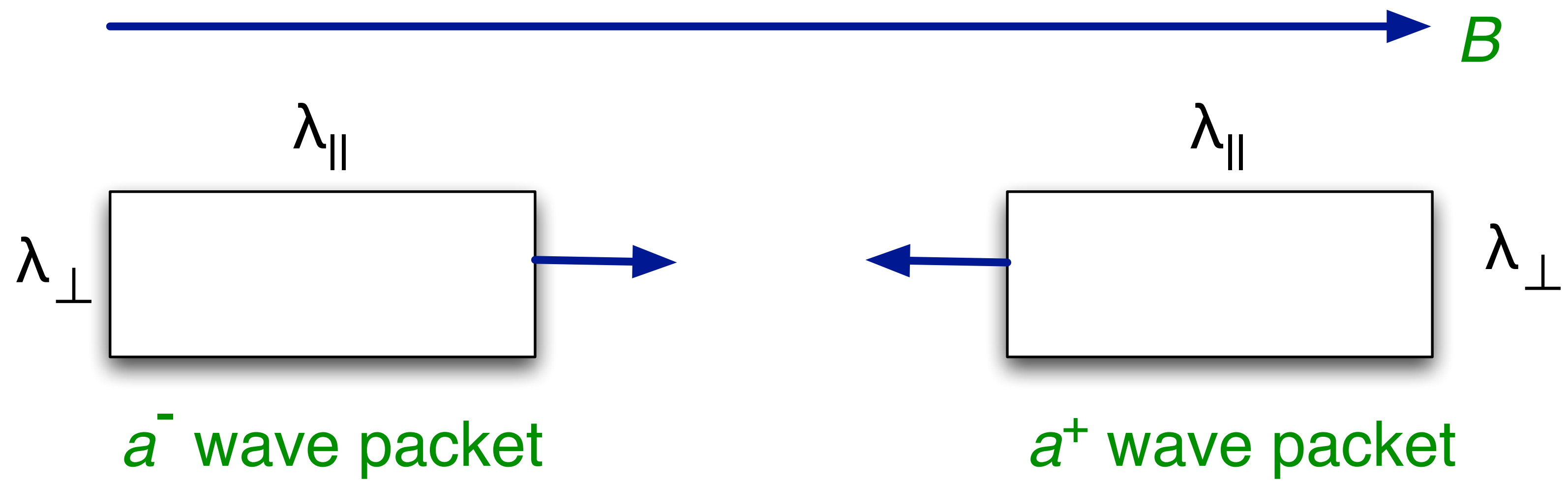
In *weak* turbulence, neither wave packet is changed appreciably during a single "collision," so, e.g., the right and left sides the "incoming" a+ wave packet are affected in almost exactly the same way by the collision. This means that the structure of the wave packet along the field line is altered only very weakly (at 2nd order). You thus get small-scale structure transverse to the magnetic field, but not along the magnetic field. (Large perpendicular wave numbers, not large parallel wave numbers.) (Shebalin, Matthaeus, & Montgomery 1983, Ng & Bhattacharjee 1997, Goldreich & Sridhar 1997)

Anisotropic Cascade

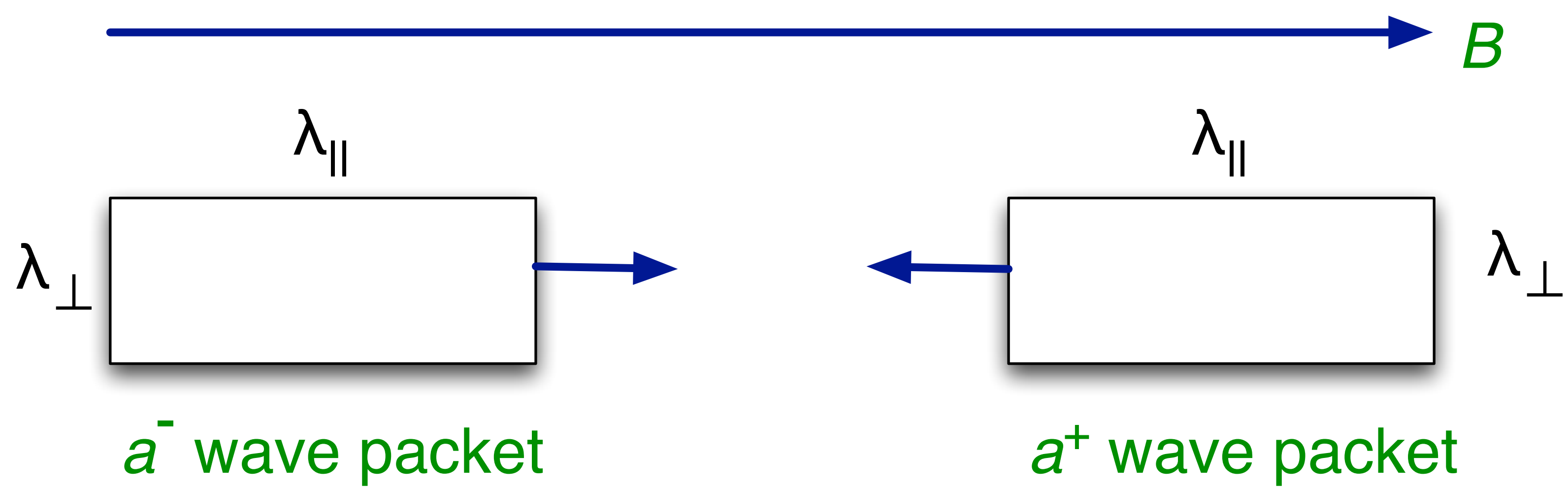
(Shebalin, Montgomery, & Matthaeus 1983)



- As energy cascades to smaller scales, you can think of wave packets breaking up into smaller wave packets.
- During this process, the length $\lambda_{||}$ of a wave packet measured parallel to *B* remains constant, but the length $\lambda_{\perp}$ measured perpendicular to *B* gets smaller.
- Fluctuations with small $\lambda_{\perp}$ end up being very anisotropic, with $\lambda_{||} >> \lambda_{\perp}$



- $\delta v_{\lambda_\perp}$ = rms velocity difference across a distance $\lambda_\perp$ in the plane perpendicular to $\vec{B}$ = velocity fluctuation of wave packets of $\perp$ size $\lambda_\perp$.



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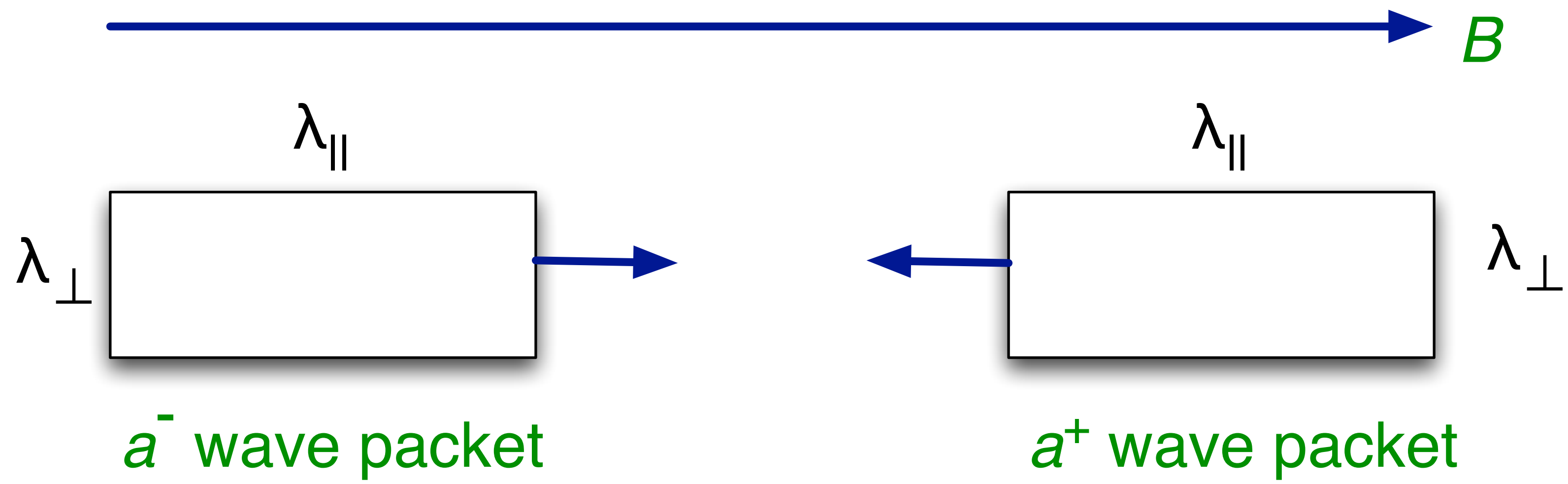
- The contribution of one of these wave packets to the local value of $\vec{v} \cdot \nabla \vec{v}$ is $\sim (\delta v_{\lambda_\perp})^2/\lambda_\perp$. (For AWs, $\vec{v} \perp \vec{B}_0$.)



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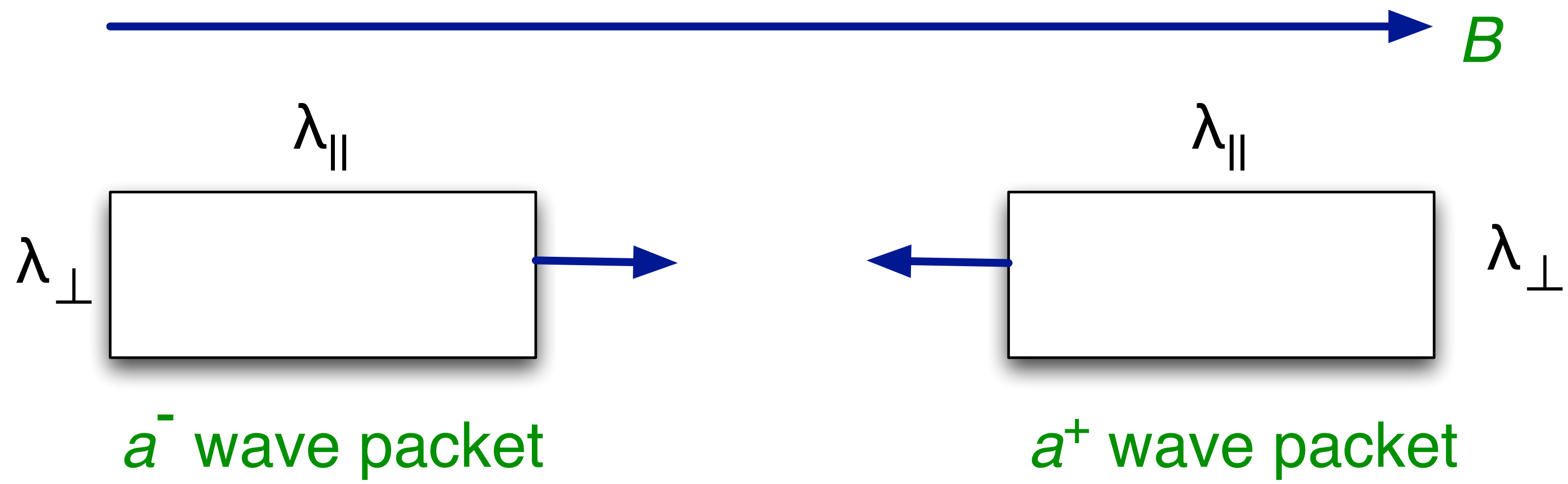

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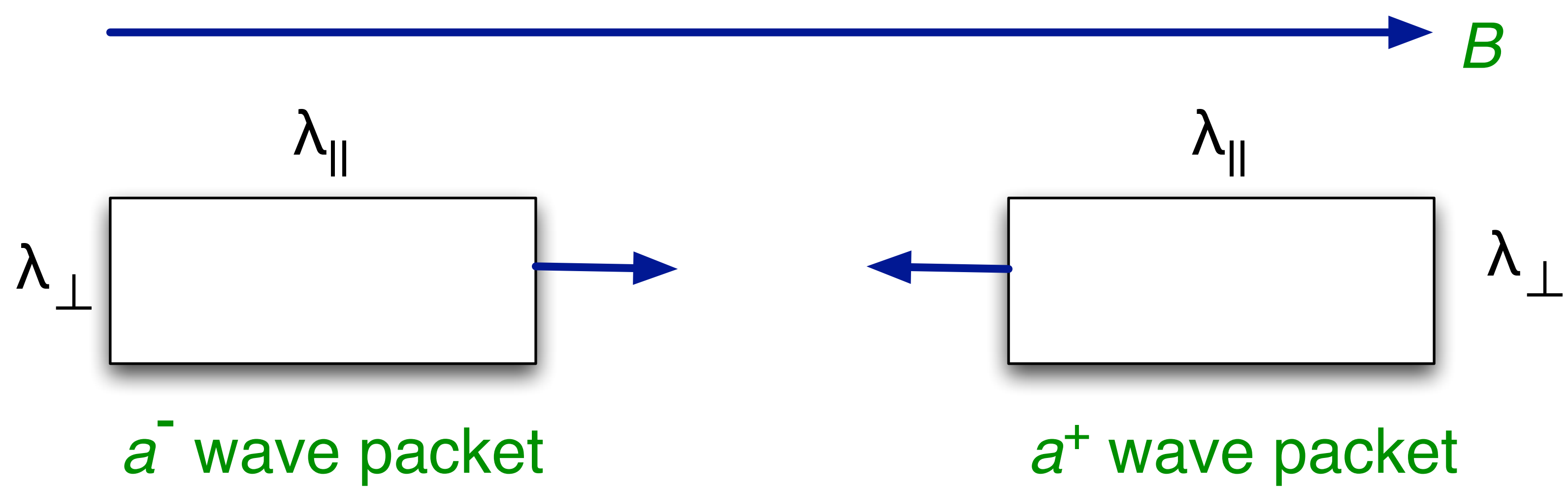

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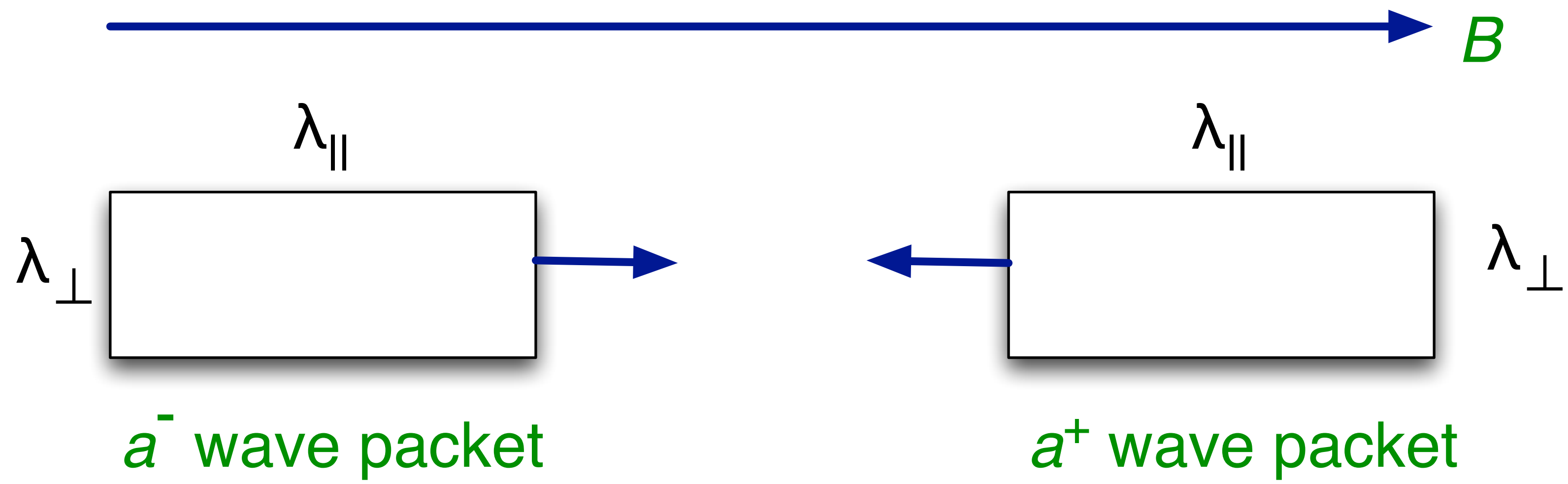

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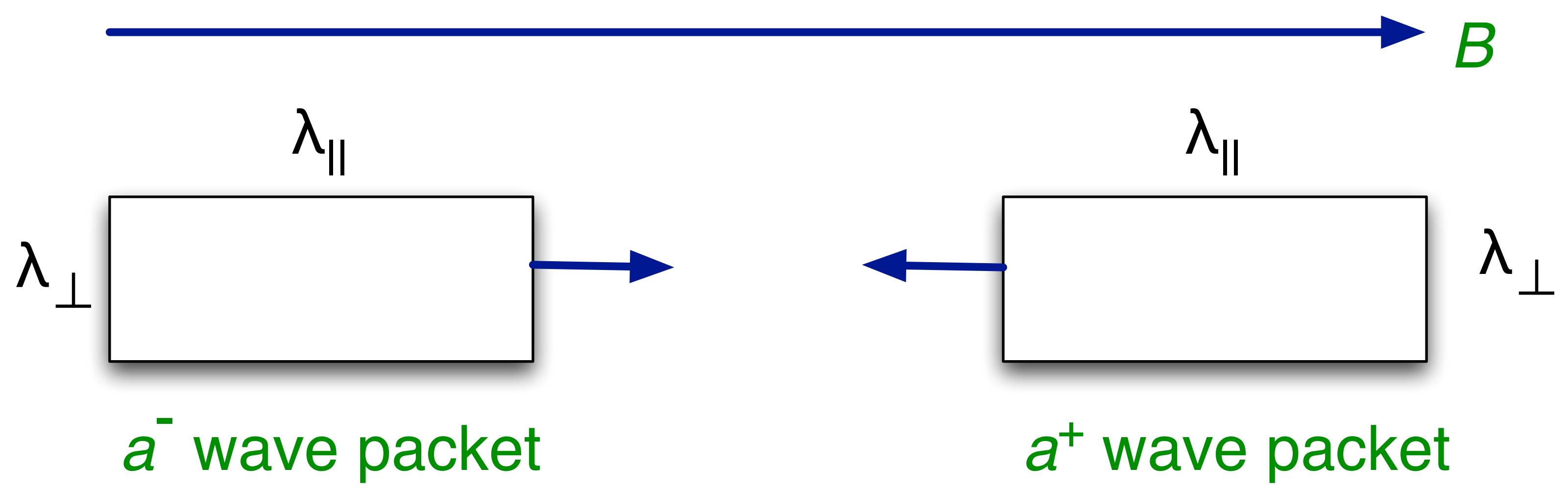
(Ng & Bhattacharjee 1997, Goldreich & Sridhar 1997)



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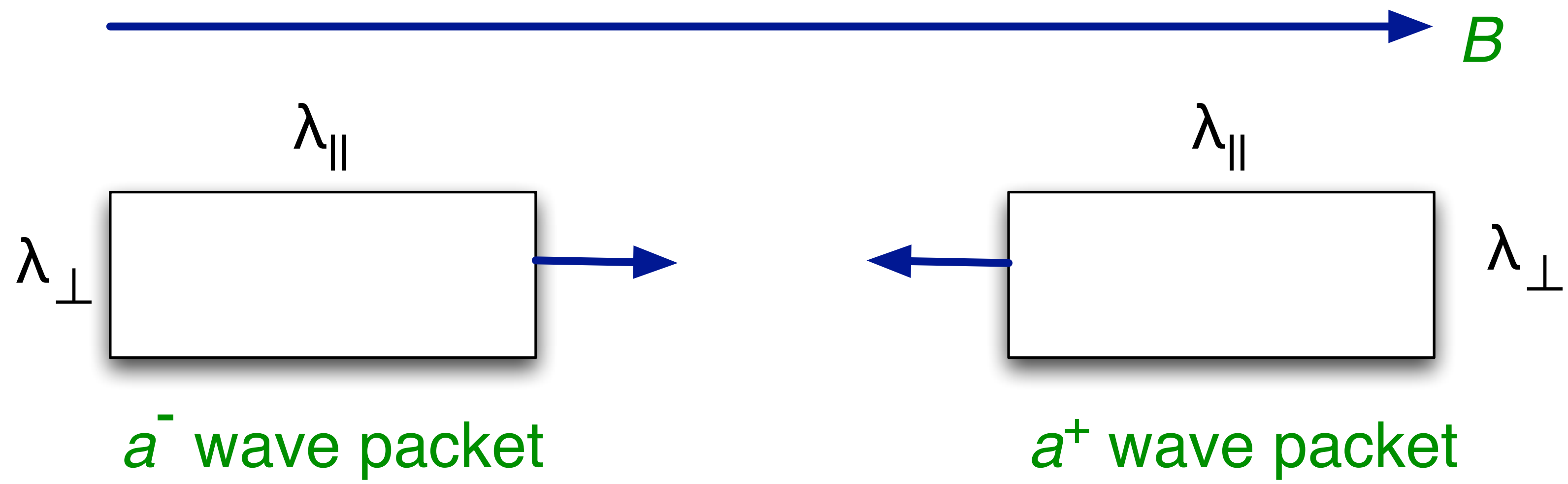

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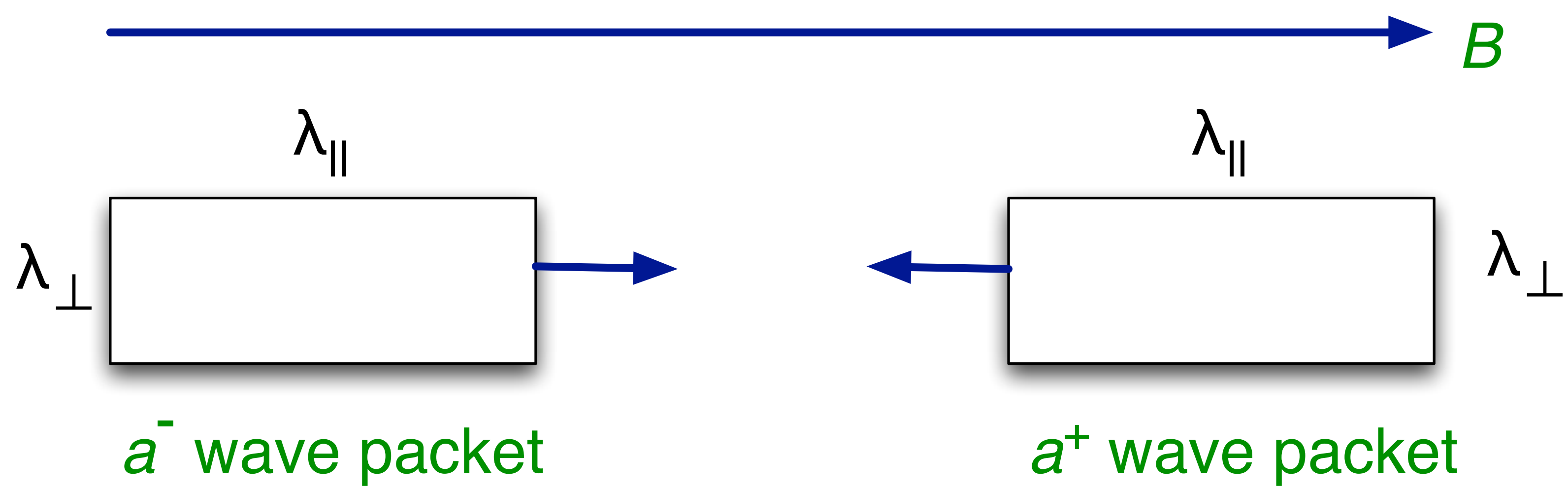

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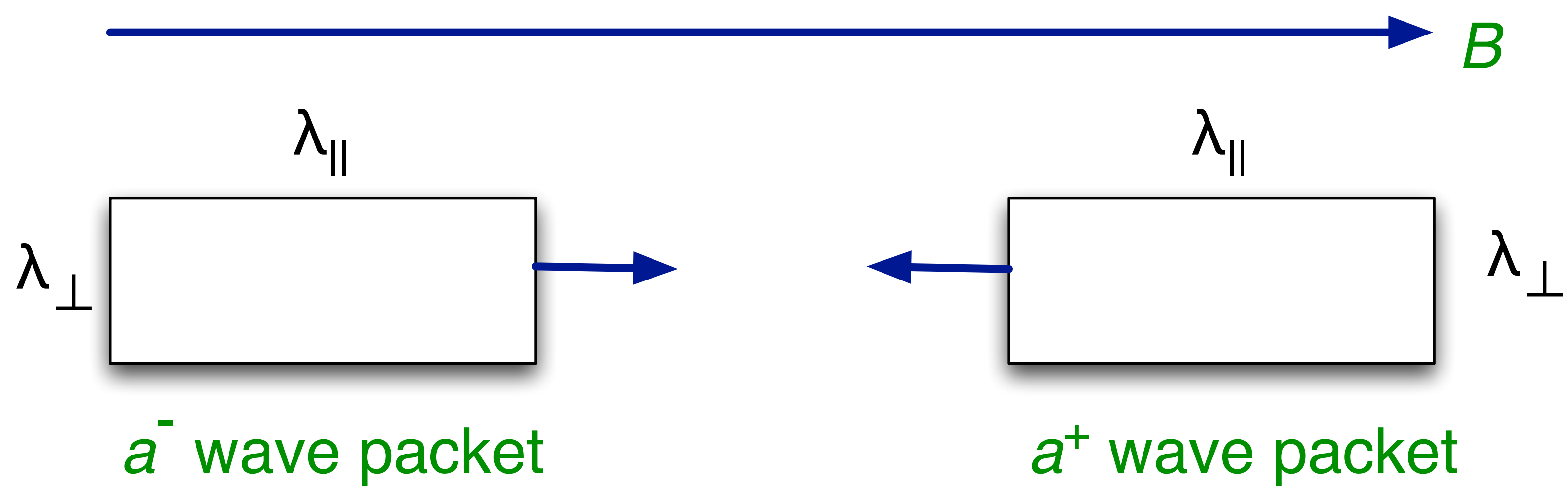

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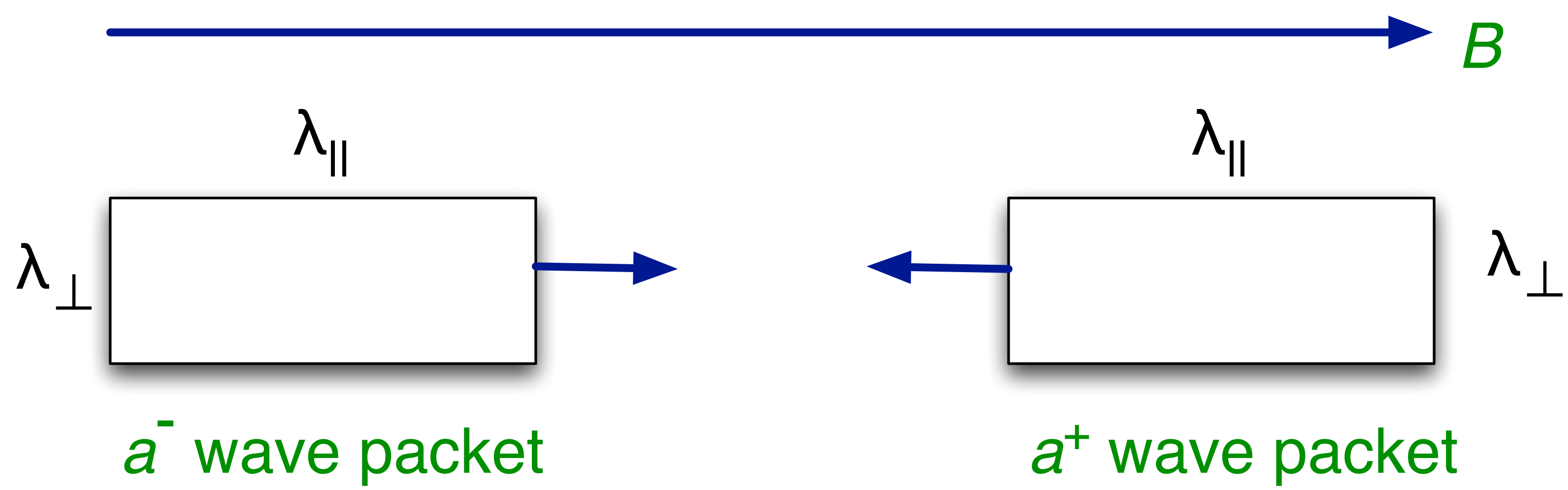

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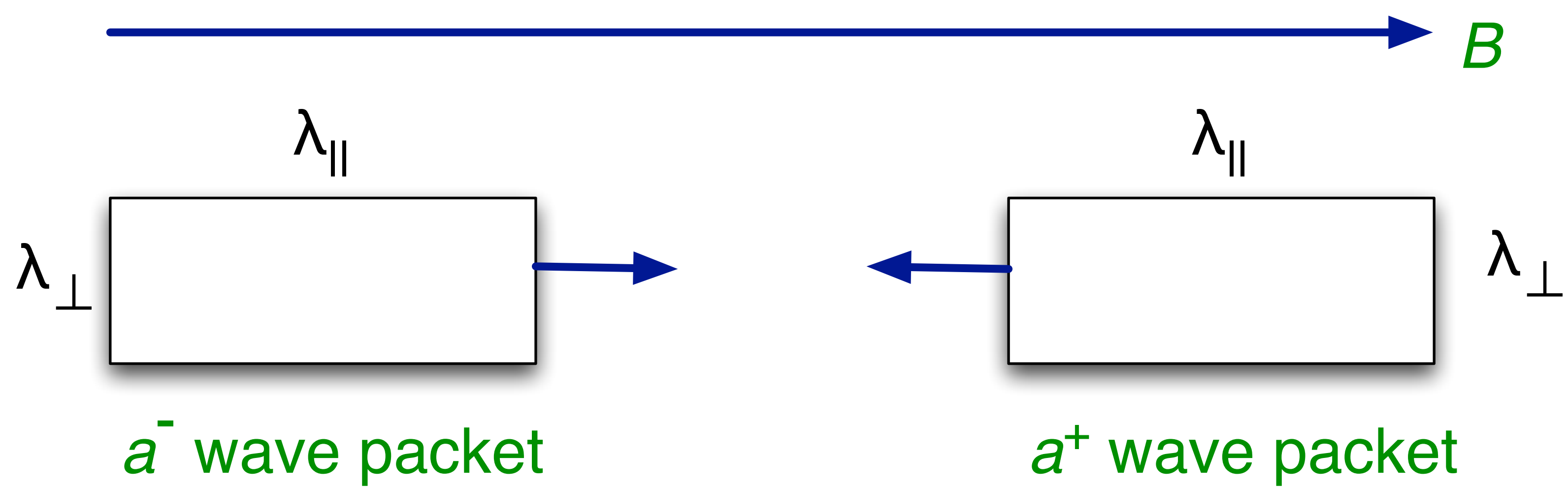
(Ng & Bhattacharjee 1997, Goldreich & Sridhar 1997)



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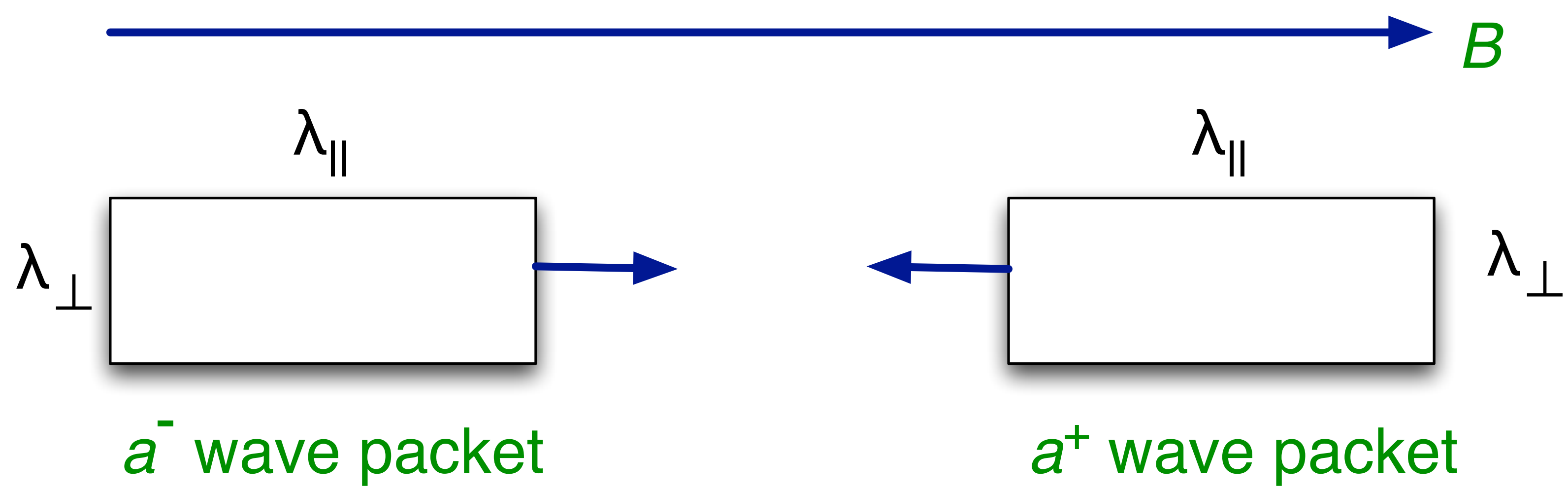
$$\tau_c \sim \frac{v_A \lambda_\perp^2}{\lambda_\| \delta v_{\lambda_\perp}^2}$$



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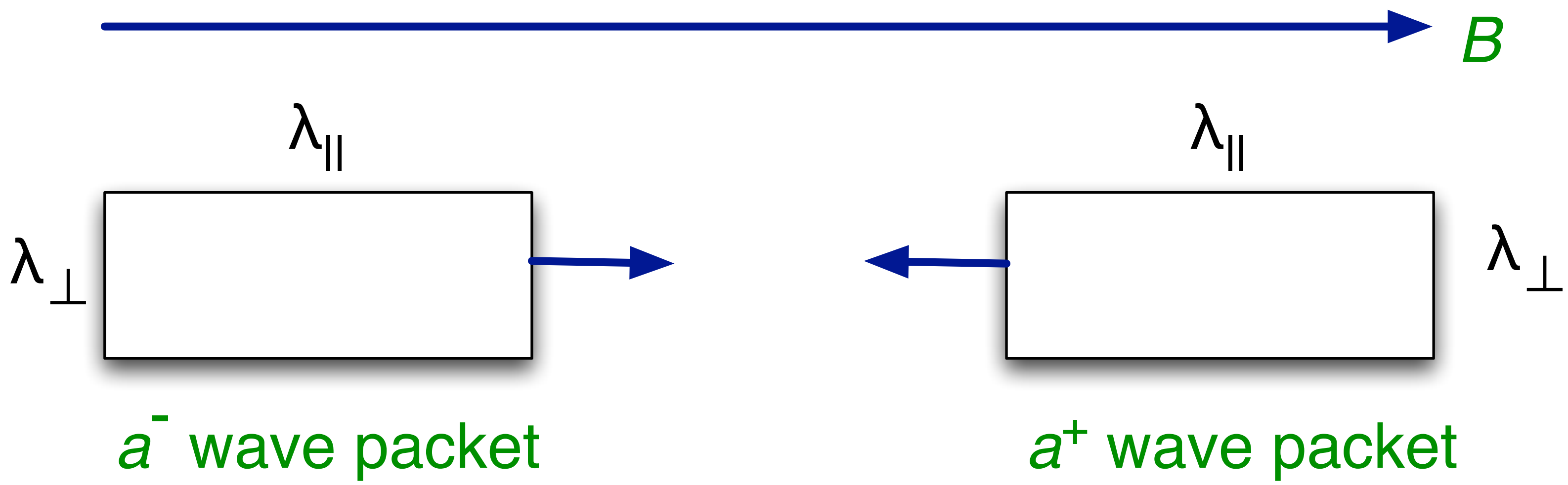

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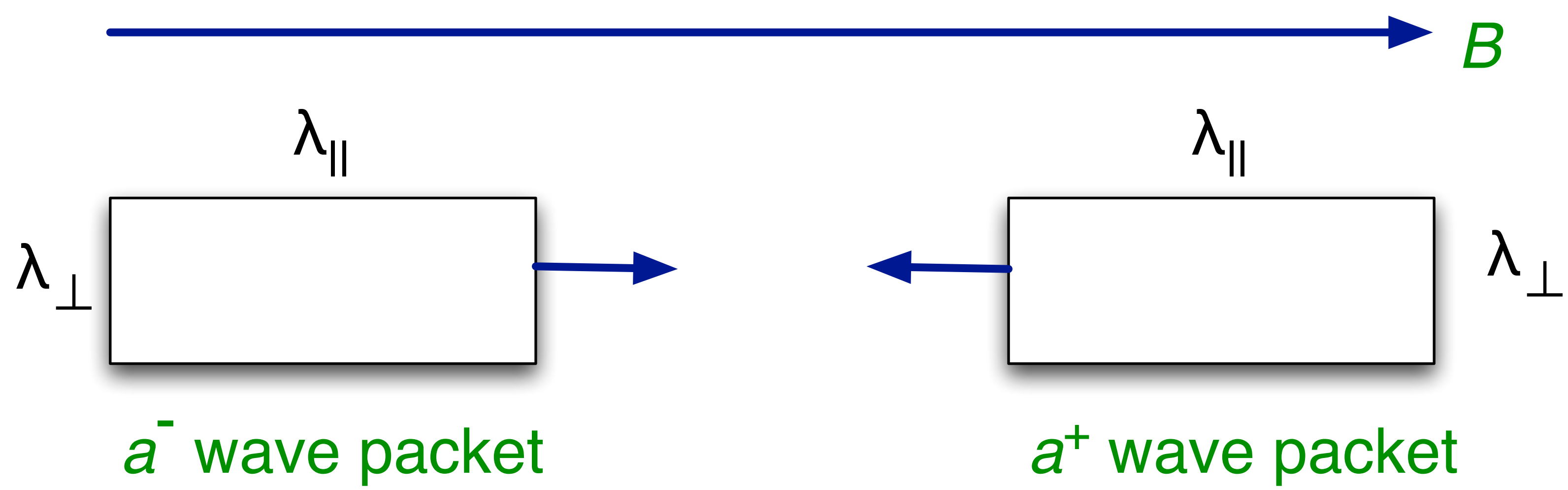
- Noting that ϵ is independent of $\lambda_\perp$ within the inertial range, and that $\lambda_\|$ is constant, we obtain $v_{\lambda_\perp} \propto \lambda_\perp^{1/2}$. (Ng & Bhattacharjee 1997)



- The cascade time is therefore $\tau_{\rm c} \sim N\lambda_{\|}/v_{\rm A} \sim \chi^{-2}\lambda_{\|}/v_{\rm A}$. Recalling that $\chi = \lambda_{\|}\delta v_{\lambda_\perp}/(\lambda_\perp v_{\rm A})$, we obtain

$$\tau_{\rm c} \sim \frac{v_{\rm A}\lambda_\perp^2}{\lambda_{\|}\delta v_{\lambda_\perp}^2}$$

- The cascade power ϵ is $\delta v_{\lambda_\perp}^2/\tau_{\rm c}$, or

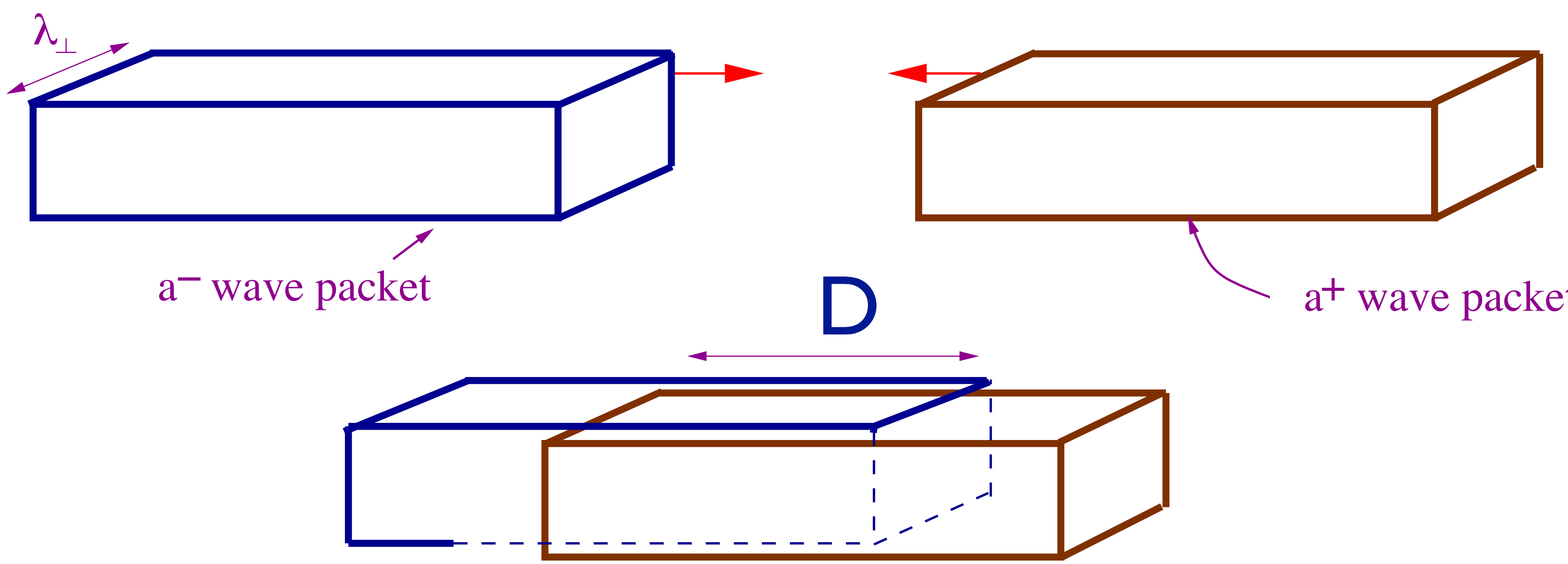
$$\epsilon \sim \frac{\delta v_{\lambda_\perp}^4 \lambda_{\|}}{v_{\rm A}\lambda_\perp^2}$$

corresponds to $E(k_\perp) \propto k_\perp^{-2}$

- Noting that ϵ is independent of $\lambda_\perp$ within the inertial range, and that $\lambda_{\|}$ is constant, we obtain $v_{\lambda_\perp} \propto \lambda_\perp^{1/2}$.

(Ng & Bhattacharjee 1997)

- Substituting this scaling into the expression for χ, we find that $\chi \propto \lambda_\perp^{-1/2}$. At sufficiently small scales, χ will increase to ~ 1, and the turbulence will become strong!



- after colliding wave packets have inter-penetrated by a distance D satisfying the relation

$$\frac{D}{v_{\rm A}} \times \frac{\delta v_{\lambda_\perp}}{\lambda_\perp} \sim 1$$

  the leading edge of each wave packet will have been substantially sheared/altered relative to the trailing edge. The parallel length of the wave packet therefore satisfies $\lambda_\parallel \lesssim D$, or equivalently $\chi \lesssim 1$.

- In weak turbulence, $\chi \ll 1$ but χ grows to ~ 1 as $\lambda_\perp$ decreases. Once χ reaches a value ~ 1 (strong turbulence), χ remains ~ 1, the state of "critical balance." (Higdon 1983; Goldreich & Sridhar 1995)

Critically Balanced, Strong AW Turbulence

(Higdon 1983; Goldreich & Sridhar 1995)

- In critical balance,
$$\chi = \frac{\lambda_\|}{v_\mathrm{A}} \times \frac{\delta v_{\lambda_\perp}}{\lambda_\perp} \sim 1$$
and the linear time scale $\lambda_\|/v_\mathrm{A}$ is comparable to the nonlinear time scale $\lambda_\perp/\delta v_{\lambda_\perp}$ at each perpendicular scale $\lambda_\perp$, and the turbulence is said to be "strong."

- the energy cascade obeys the same arguments as hydrodynamic turbulence: $\tau_\mathrm{c} \sim \lambda_\perp/\delta v_{\lambda_\perp}$ and $\epsilon \sim \delta v_{\lambda_\perp}^2/\tau_\mathrm{c} \sim \delta v_{\lambda_\perp}^3/\lambda_\perp$.

- Since the cascade power ϵ is independent of $\lambda_\perp$ in the inertial range, $\delta v_{\lambda_\perp} \propto \lambda_\perp^{1/3}$.

- the condition $\chi \sim (\lambda_\|/v_\mathrm{A}) \times (\delta v_{\lambda_\perp}/\lambda_\perp) \sim 1$ then implies that $\lambda_\| \propto \lambda_\perp^{2/3}$.

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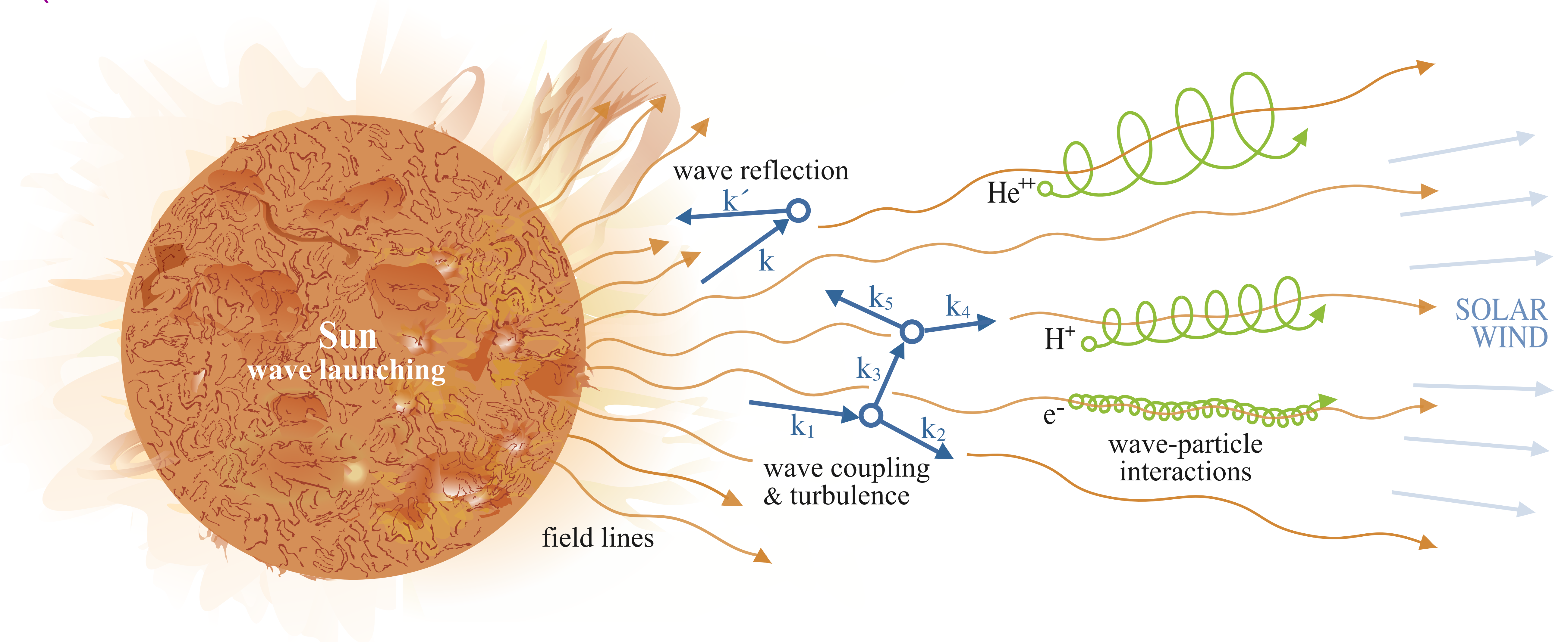
corresponds to
$E(k_\perp) \propto k_\perp^{-5/3}$

Outline

- I. 3D Structure of the Solar Wind
- II. Introduction to Turbulence
- III. Measurements of Solar-Wind Turbulence
- IV. Magnetohydrodynamic (MHD) Turbulence
- V. Reflection-Driven MHD Turbulence and the Origin of the Solar Wind.

Coronal Heating and Solar-Wind Acceleration by Waves

(Parker 1965, Coleman 1968, Velli et al 1989, Zhou & Matthaeus 1989, Cranmer et al 2007)



- The Sun launches Alfven waves, which transport energy outwards
- The waves become turbulent, which causes wave energy to ‘cascade’ from long wavelengths to short wavelengths
- Short-wavelength waves dissipate, heating the plasma. This increases the thermal pressure, which, along with the wave pressure, accelerates the solar wind.

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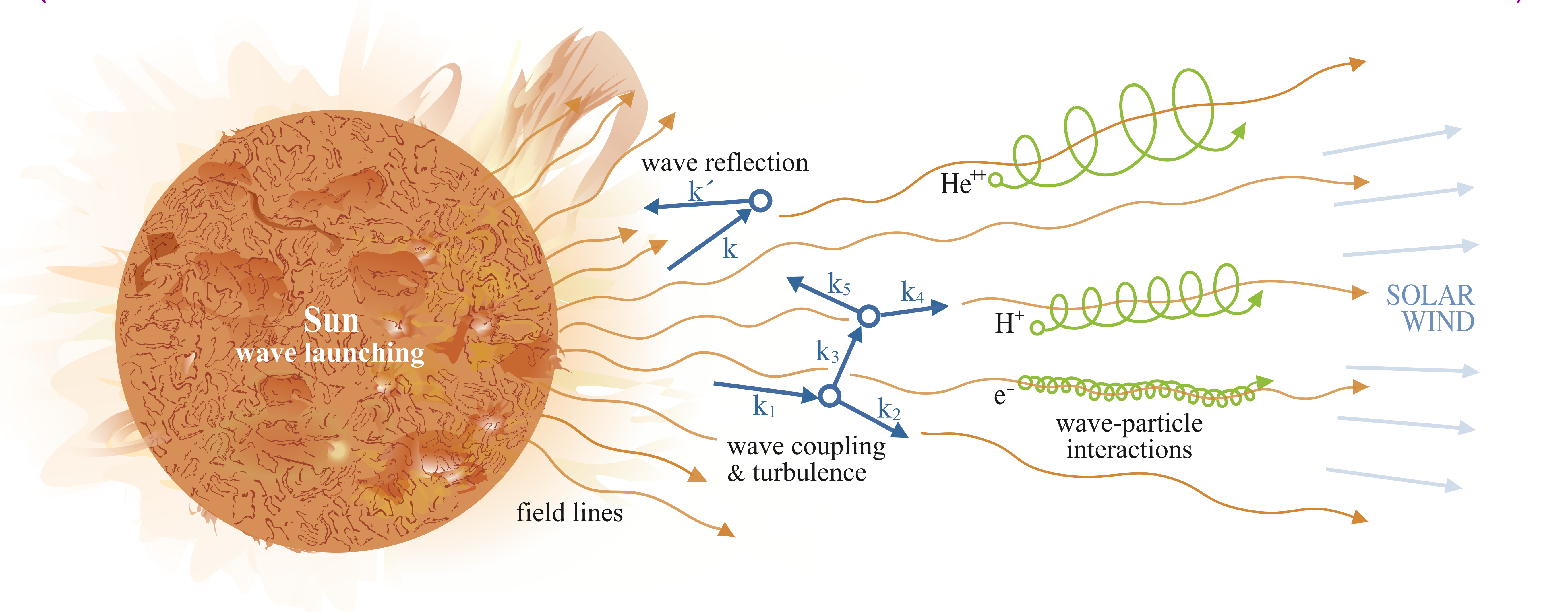

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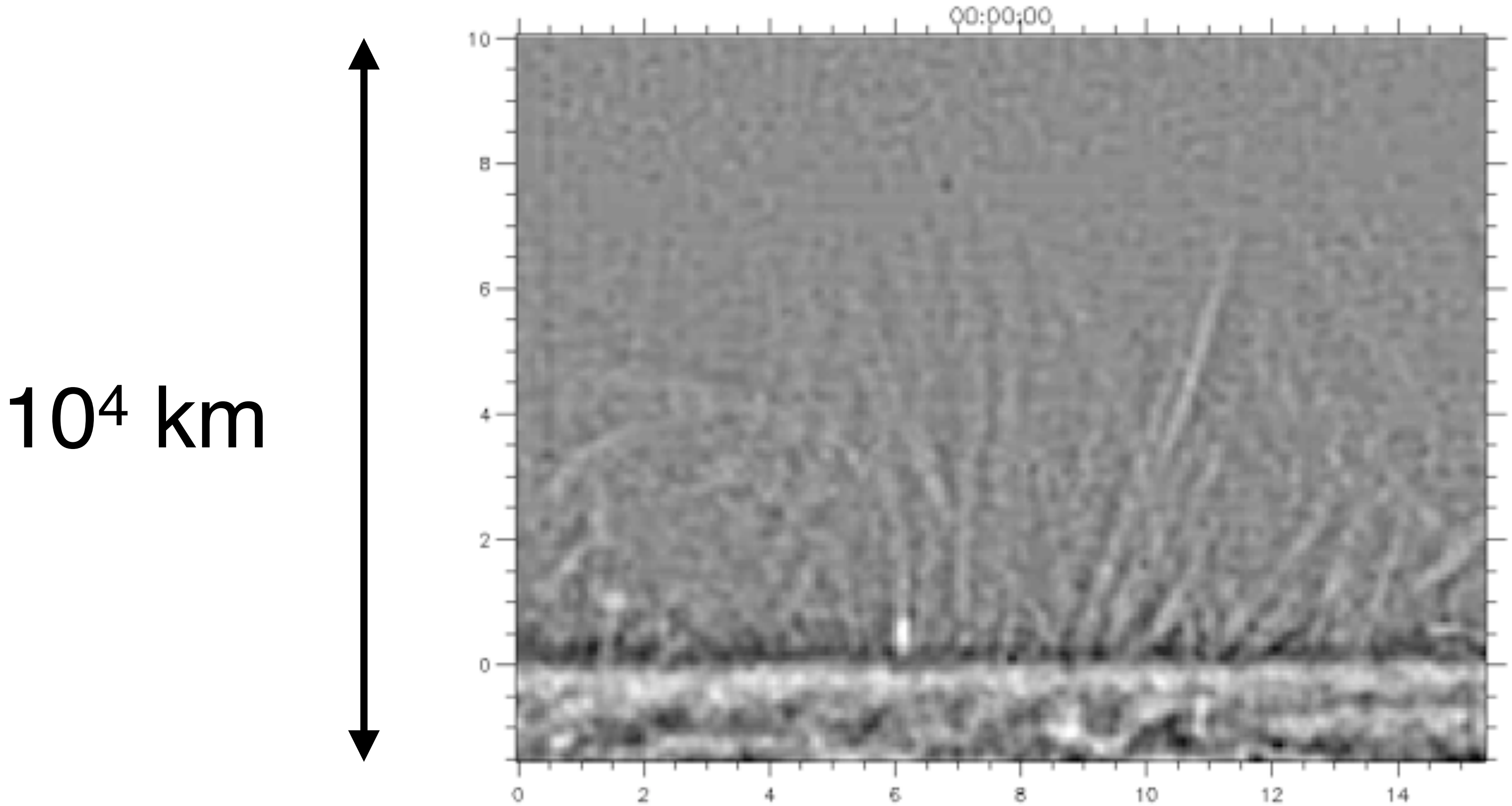
Observations of Alfven Waves in the Low Corona

(DePontieu et al 2007)



- Thomson-scattered light seen by *Hinode*'s Solar Optical Telescope.
- $F \sim (\rho \langle dv^2 \rangle) v_A \sim 10^5$ erg cm^{-2} s^{-1} — **sufficient to power solar wind.**
- These Alfvén waves may be launched by photospheric motions or magnetic reconnection.

Coronal Heating and Solar-Wind Acceleration by Waves

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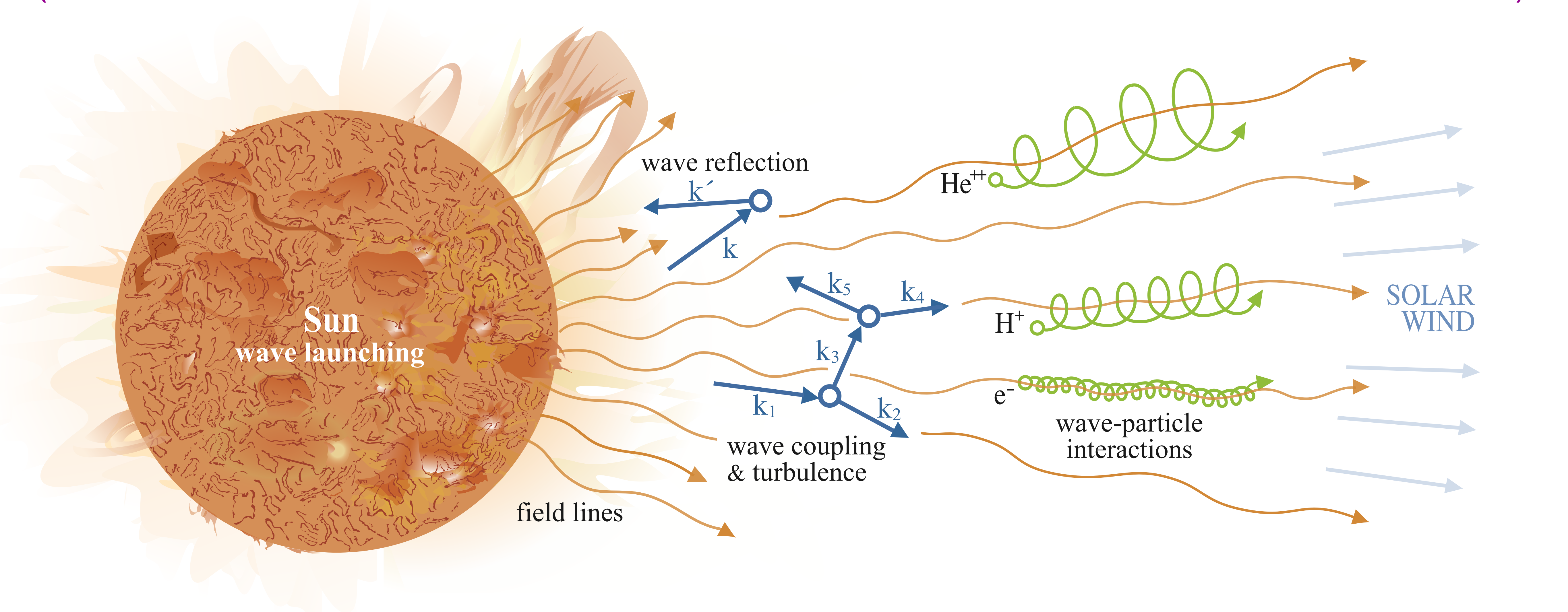

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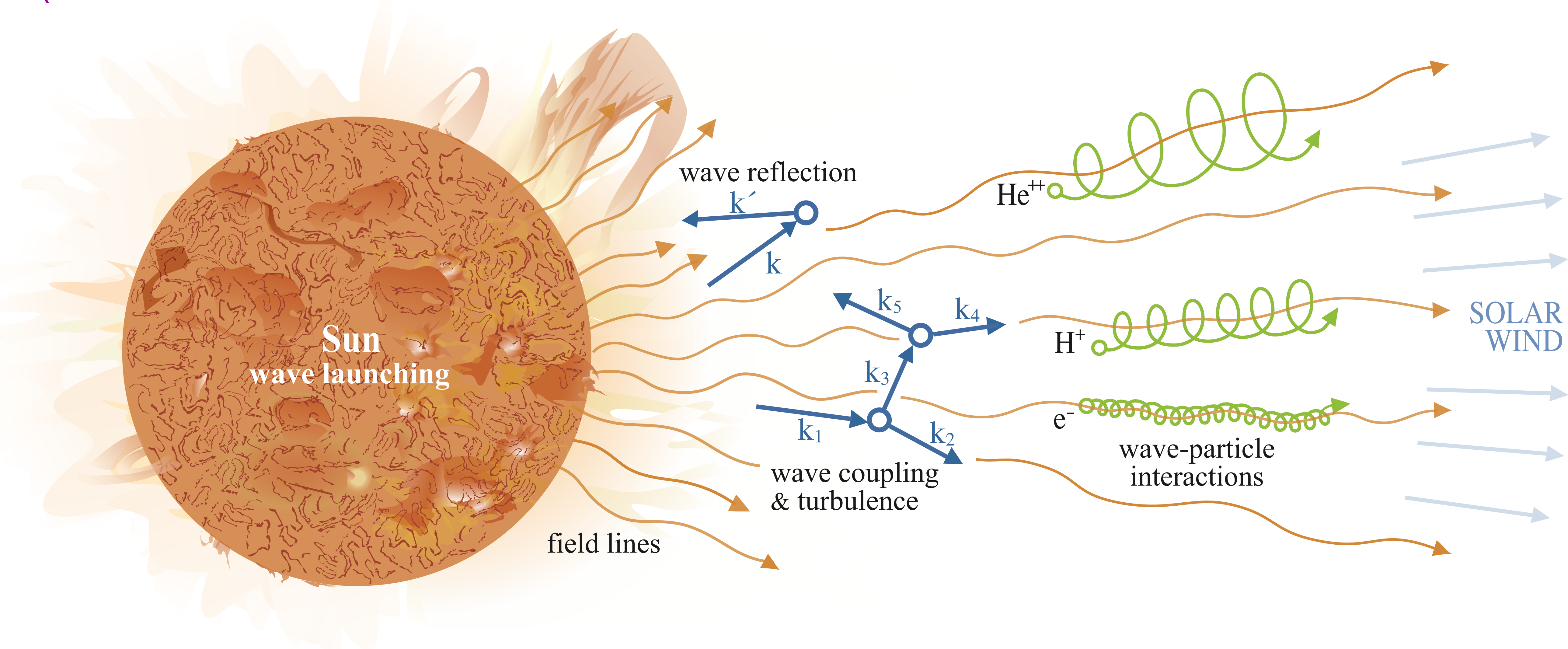

- Alfven wave damping is so weak that turbulence is needed in order to speed up the dissipation of the wave energy, so that the waves can dissipate and transfer their energy to the plasma.

Ideal, Adiabatic MHD

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla \left(p + \frac{B^2}{8\pi} \right) + \frac{1}{4\pi} \mathbf{B} \cdot \nabla \mathbf{B}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \frac{p}{\rho^\gamma} = 0$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

- Magnetic forces: magnetic pressure and magnetic tension
- Frozen-in Law: magnetic field lines are like threads that are frozen to the plasma and advected by the plasma.
- Alfven waves are like waves on a string, where the magnetic field plays the role of the string.

Transverse, Non-Compressive Fluctuations

(Whang 1980, Zhou & Matthaeus 1989; Velli et al 1989)

- $\mathbf{v} = \mathbf{U} + \delta\mathbf{v} \qquad \mathbf{B} = \mathbf{B}_0 + \delta\mathbf{B}$
- $\delta\mathbf{v} \perp \mathbf{B}_0 \qquad \delta\mathbf{B} \perp \mathbf{B}_0 \qquad \nabla \cdot \delta\mathbf{v} = 0$ (transverse, non-compressive approximation)
- $\mathbf{z}^{\pm} = \delta\mathbf{v} \mp \delta\mathbf{B}/\sqrt{4\pi\rho}$ (Elsasser variables)
- $\mathbf{v}_{\mathrm{A}} = \mathbf{B}_0/\sqrt{4\pi\rho} \qquad p_{\mathrm{tot}} = p + B^2/8\pi.$ (Alfven velocity and total pressure)

$$\frac{\partial \mathbf{z}^{\pm}}{\partial t} + (\quad \pm \mathbf{v}_{\mathrm{A}}) \cdot \nabla \mathbf{z}^{\pm} \cdot$$

$$= -\left(\mathbf{z}^{\mp} \cdot \nabla \mathbf{z}^{\pm} + \frac{\nabla p_{\mathrm{tot}}}{\rho}\right)$$

For a stationary (U=0) and homogeneous background.

- $\mathbf{z}^{+}$ represents Alfvén waves (AWs) propagating parallel to $\mathbf{B}_0$
- $\mathbf{z}^{-}$ represents AWs propagating anti-parallel to $\mathbf{B}_0$.

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For a stationary (U=0) and homogeneous background.

The nonlinear term leads to energy cascade, but only in the presence of both z^+ and z^-. If all the waves propagate in same direction, they don't interact. **Only counter-propagating waves interact nonlinearly to produce turbulence.**

Transverse, Non-Compressive Fluctuations

(Whang 1980, Zhou & Matthaeus 1989; Velli et al 1989)

- $\mathbf{v} = \mathbf{U} + \delta\mathbf{v} \qquad \mathbf{B} = \mathbf{B}_0 + \delta\mathbf{B}$
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- $\mathbf{v}_{\mathrm{A}} = \mathbf{B}_0/\sqrt{4\pi\rho} \qquad p_{\mathrm{tot}} = p + B^2/8\pi.$ (Alfven velocity and total pressure)

$$\frac{\partial \mathbf{z}^{\pm}}{\partial t} + (\mathbf{U} \pm \mathbf{v}_{\mathrm{A}}) \cdot \nabla \mathbf{z}^{\pm} + \mathbf{z}^{\mp} \cdot \nabla (\mathbf{U} \mp \mathbf{v}_A)$$
$$+ \frac{1}{2}(\mathbf{z}^{-} - \mathbf{z}^{+}) \left(\nabla \cdot \mathbf{v}_{\mathrm{A}} \mp \frac{1}{2} \nabla \cdot \mathbf{U} \right) = -\left(\mathbf{z}^{\mp} \cdot \nabla \mathbf{z}^{\pm} + \frac{\nabla p_{\mathrm{tot}}}{\rho} \right)$$

Here we have allowed for background flow and inhomogeneity, and we have taken **U** to be parallel to $\mathbf{B}_0$.

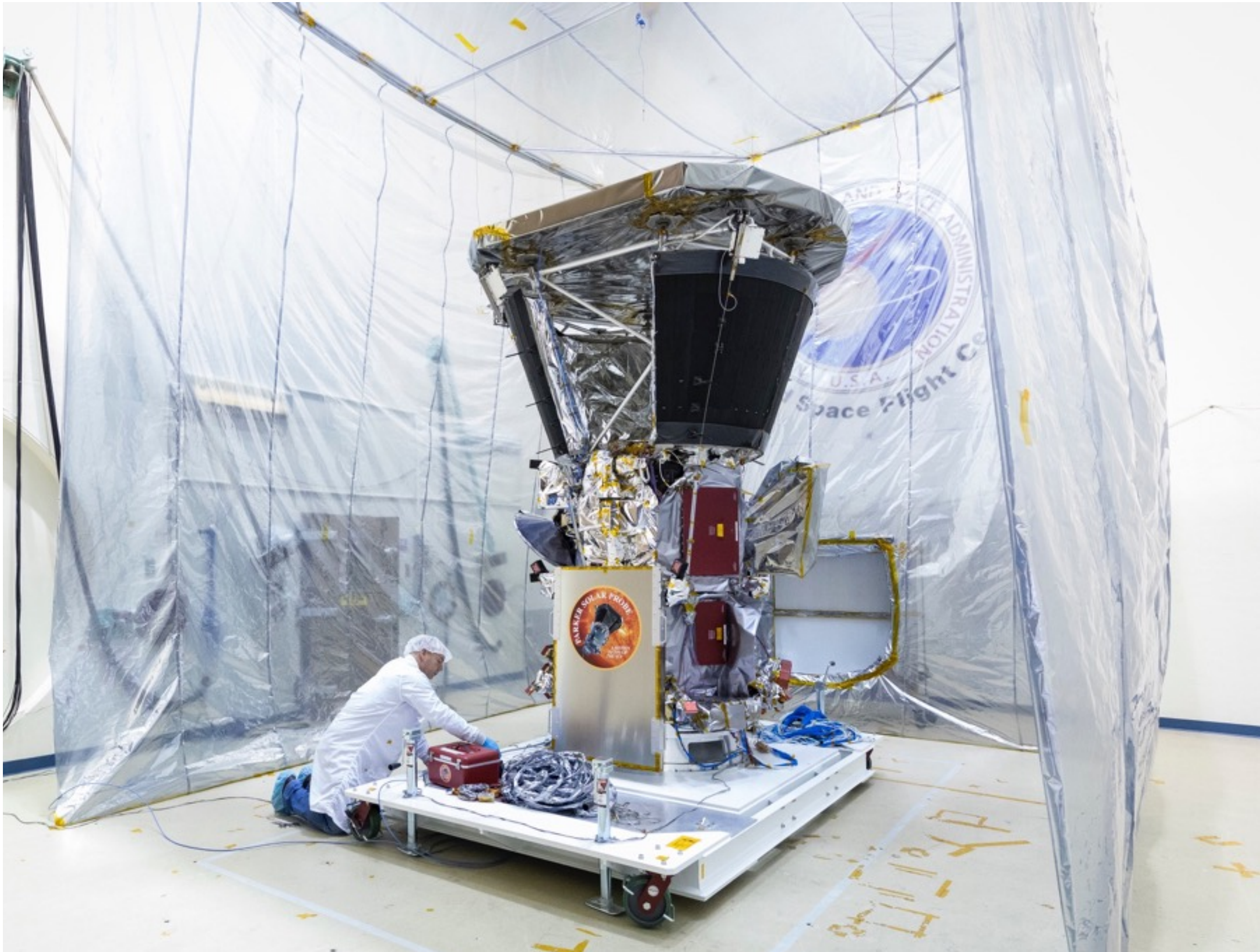
Inward-propagating waves (z^-) and outward-propagating waves (z^+) are coupled via linear terms, which lead to (non-WKB) wave reflection.
REFLECTION PROVIDES THE INWARD-PROPAGATING WAVES NEEDED FOR ALFVÉN-WAVE TURBULENCE!

There are now a number of models and numerical simulations showing that reflection-driven Alfvén-wave turbulence is a promising mechanism for powering the solar wind, particularly the fast wind.

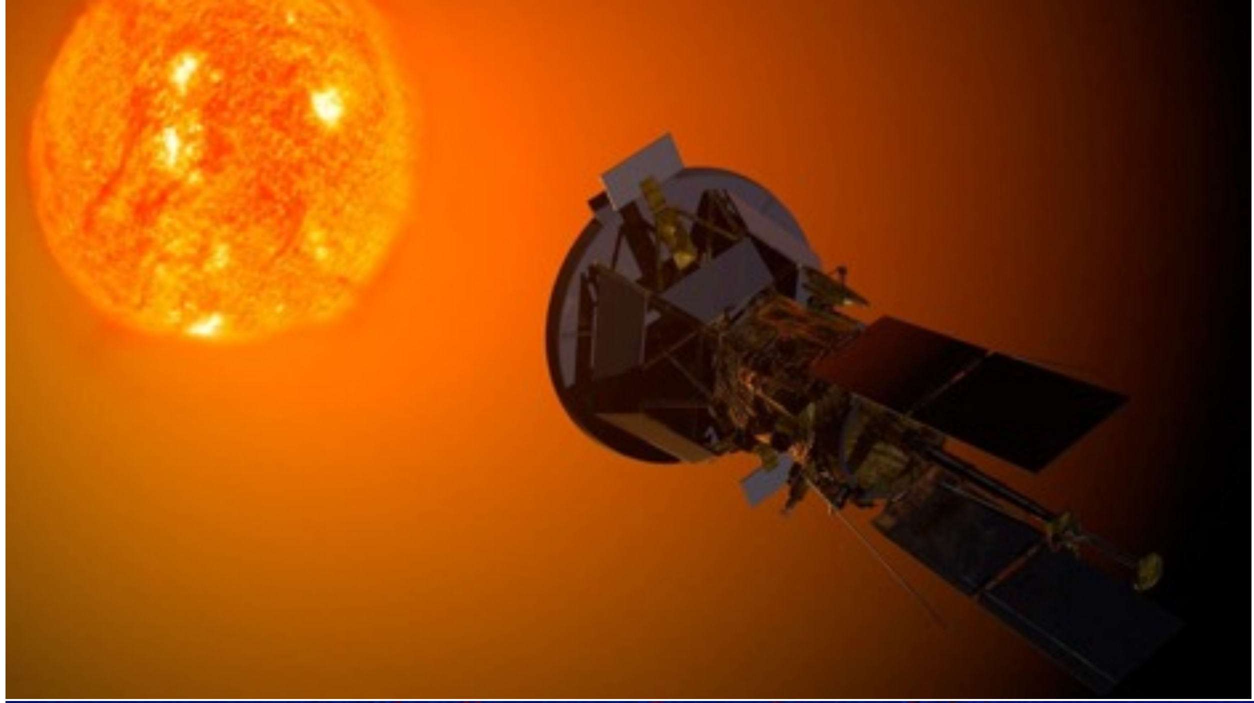
But how can we tell if these models are accurate? The solar-wind acceleration region is so far away…

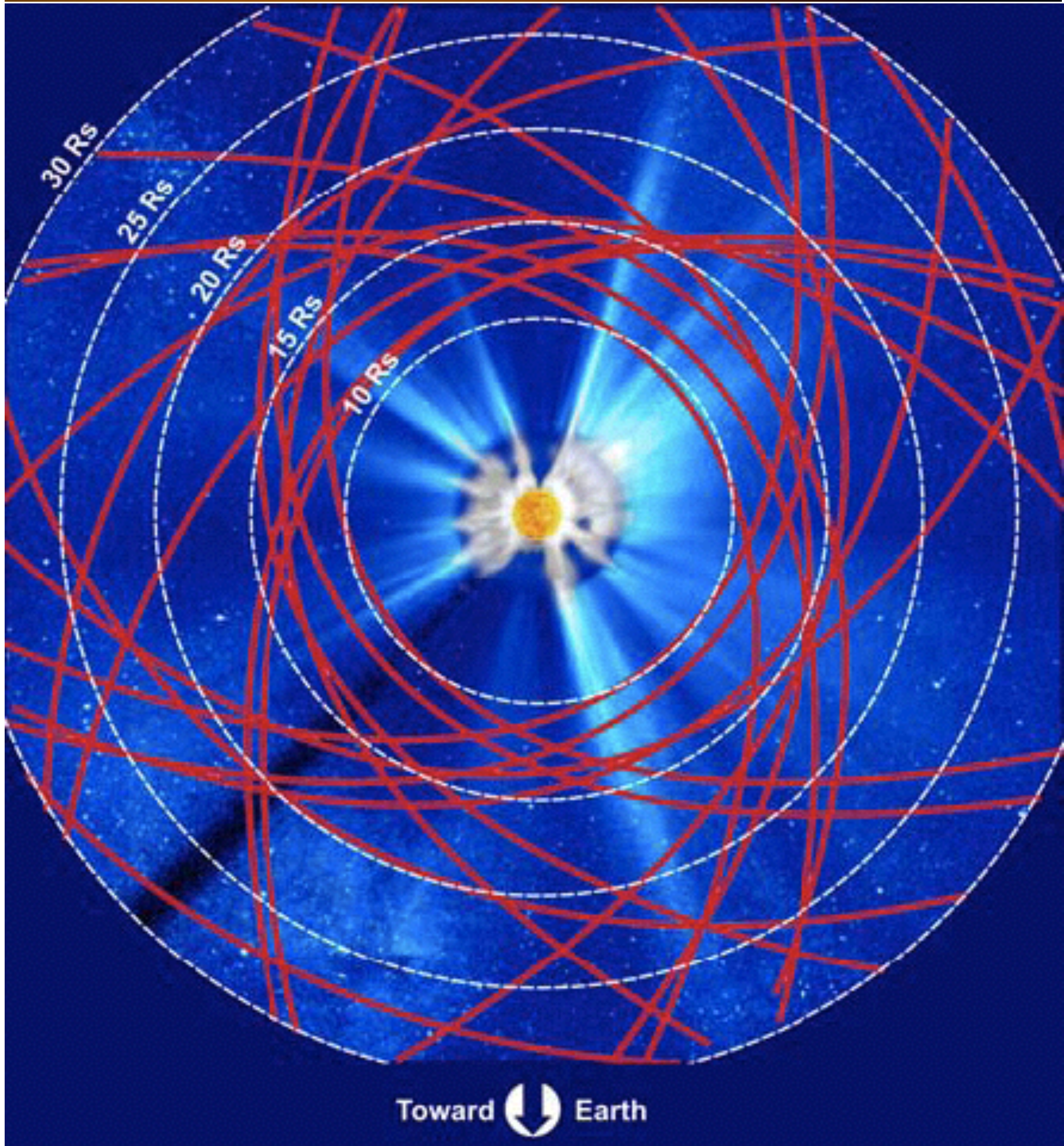

August 12, 2018

NASA's Parker Solar Probe



Parker Solar Probe





- Several passes to within 10 solar radii of Sun.
- First in situ measurements ever of the solar-wind acceleration region.
- Will measure *E, B*, ***u***, *T, f*(*v*), energetic particles.
- Could also provide important insights into other astrophysical systems.

Summary

- Turbulence is measured at all locations that spacecraft have explored in the solar wind.
- Alfvén-wave turbulence (i.e., non-compressive MHD turbulence) likely accounts for most of the energy in solar-wind turbulence.
- Phenomenological models (e.g., critical balance) offer insights into the dynamics and scalings of MHD turbulence.
- Alfvén waves may provide the energy required to power the solar wind, and Alfvén-wave turbulence may be the mechanism that allows the wave energy to dissipate and heat the solar-wind plasma.
- Parker Solar Probe will provide the in situ measurements needed to determine the mechanisms that heat and accelerate the solar wind within the solar-wind acceleration region.

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