Solar-Wind Acceleration and Coronal Heating

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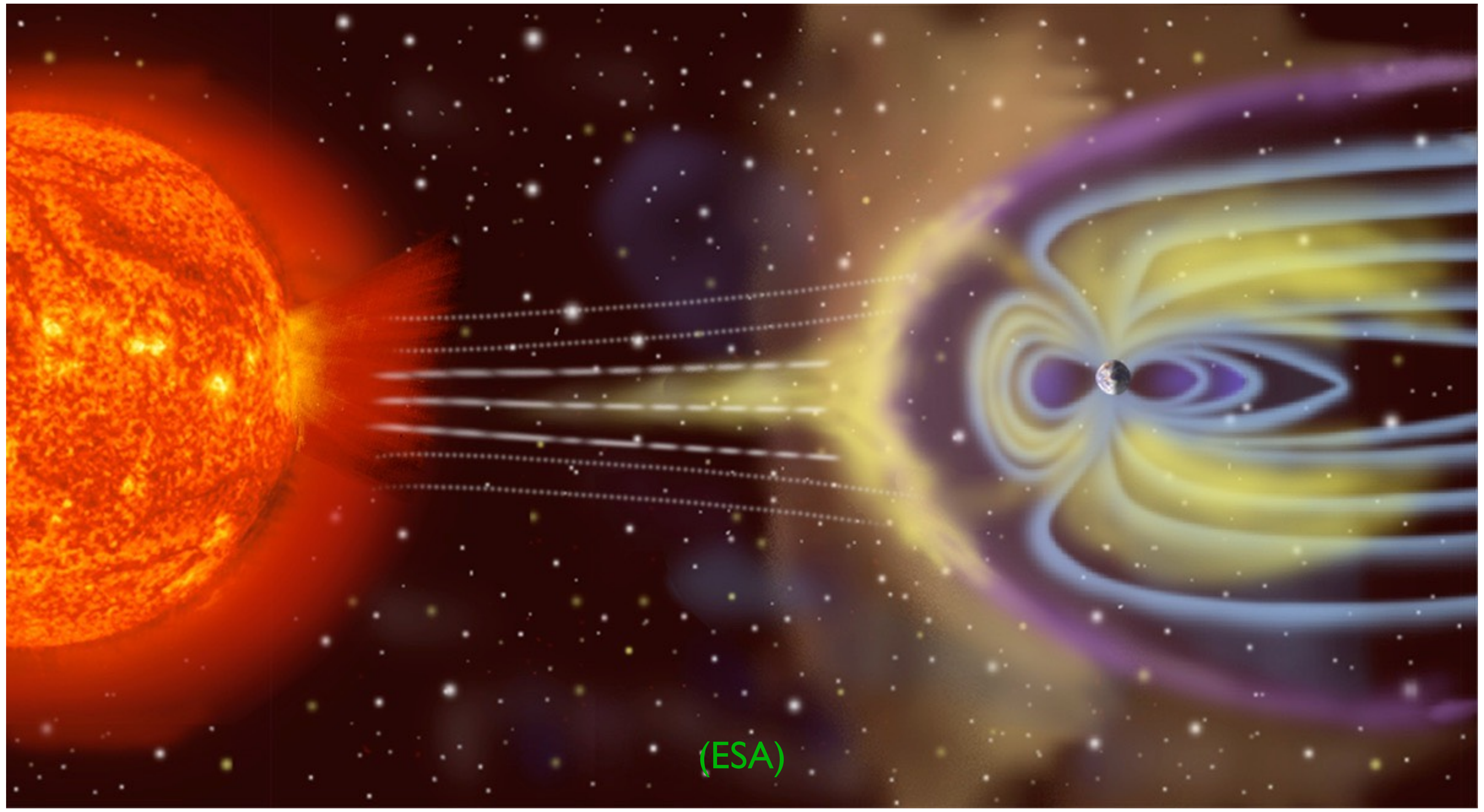
Heliophysics Summer School, June 22, 2021

Outline

- Background: the solar wind and the challenge of explaining its origin.
- Reflection-driven Alfvén-wave turbulence
- Approximate analytic solution to the coupled problems of coronal heating and solar-wind acceleration

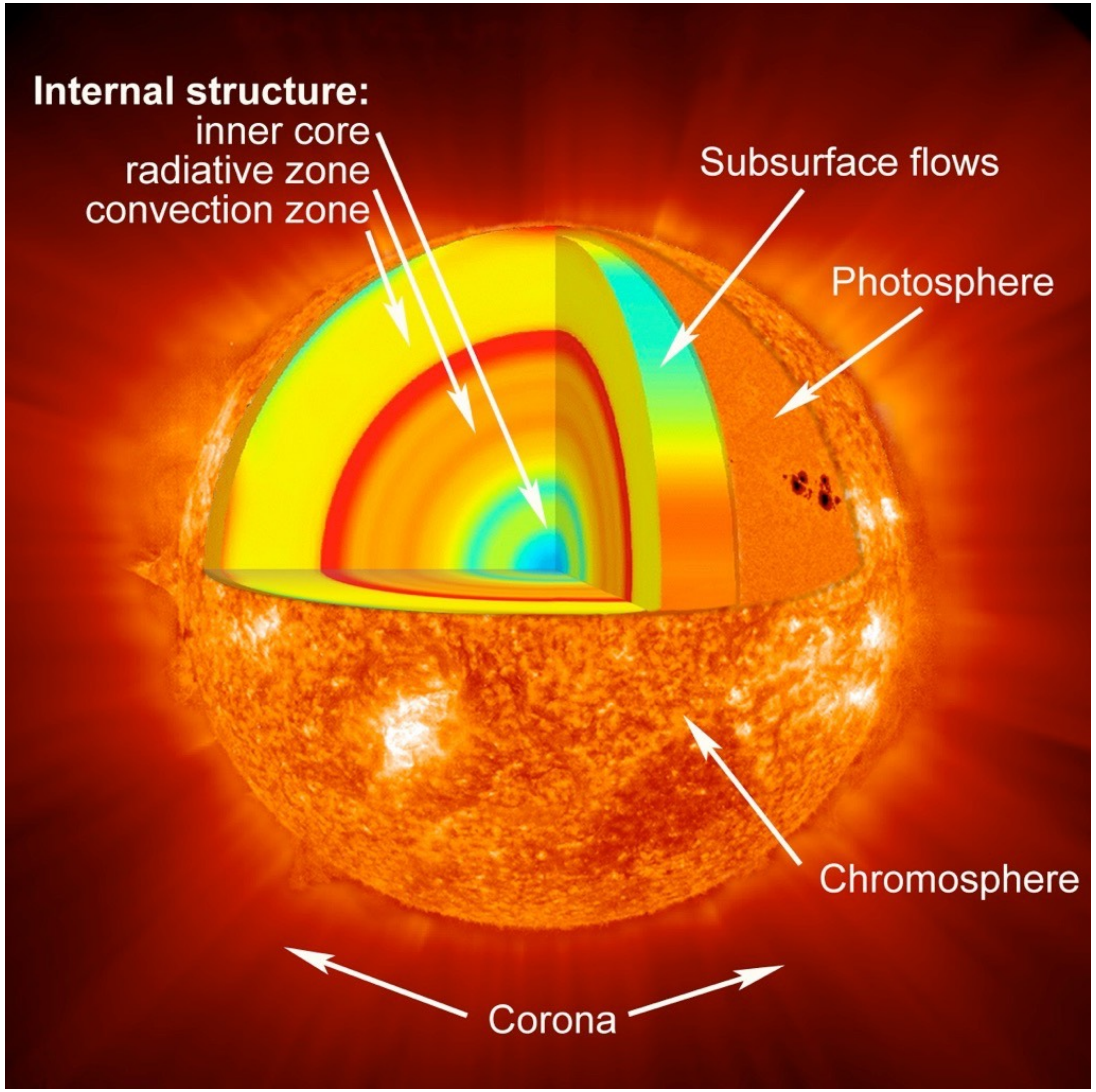
My focus: deriving analytic formulas and presenting intuitive explanations for the solar mass loss rate $\dot{M}$, the asymptotic wind speed U_∞, and the temperature of the corona $T_{\rm corona}$

The Solar Wind: a Quasi-Steady, Radial Outflow of Plasma from the Sun



- Near Earth, $U \sim 300 - 800$ km s^{-1}, $T \sim 10^5$ K, $n \sim 5$ cm^{-3}.
- $\dot{M} \sim 10^{-14} M_\odot$ yr^{-1}, $\qquad L_{\rm mech} \sim 10^{-6} L_\odot$.
- Theatre for all of space physics and space weather, a model for other astrophysical outflows, and a laboratory for plasma physics.

The Solar Interior and Atmosphere

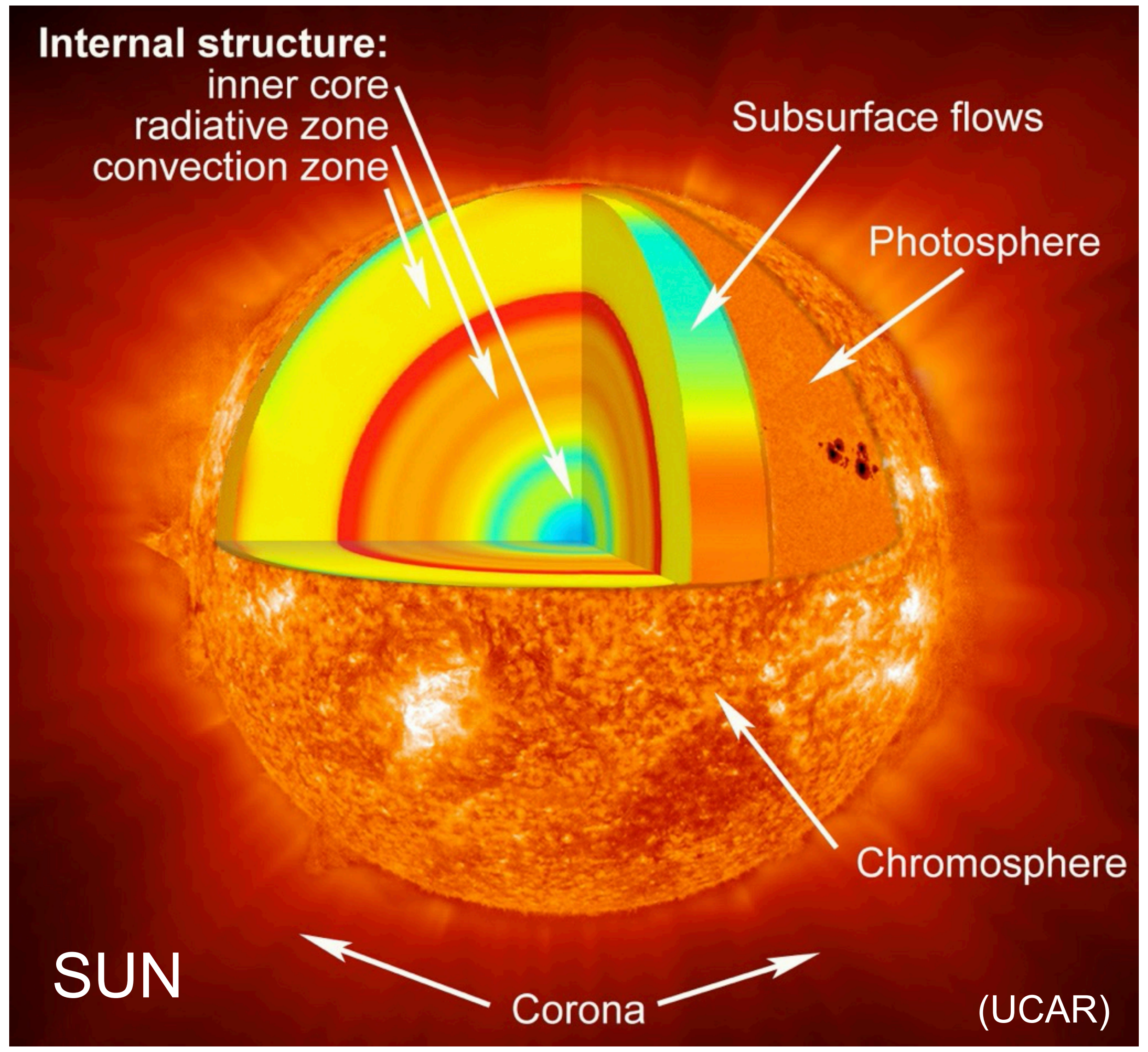


Distance Scale

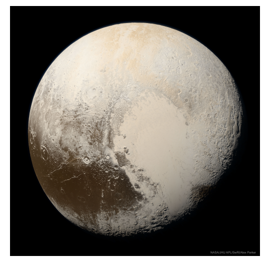
'sub-Alfvénic region' $(U < v_A)$

0	$R_\odot$	r_c	r_A	1 au	40 au	80 au	3×10^5 au
	$(7 \times 10^5$ km$)$	$(\sim 4R_\odot)$	$(\sim 10R_\odot)$	$(215R_\odot)$			
		$U = c_s$	$U = v_A$				
		critical point	Alfvén critical point	Earth	Pluto	heliospheric termination shock	Proxima Centauri (nearest star)

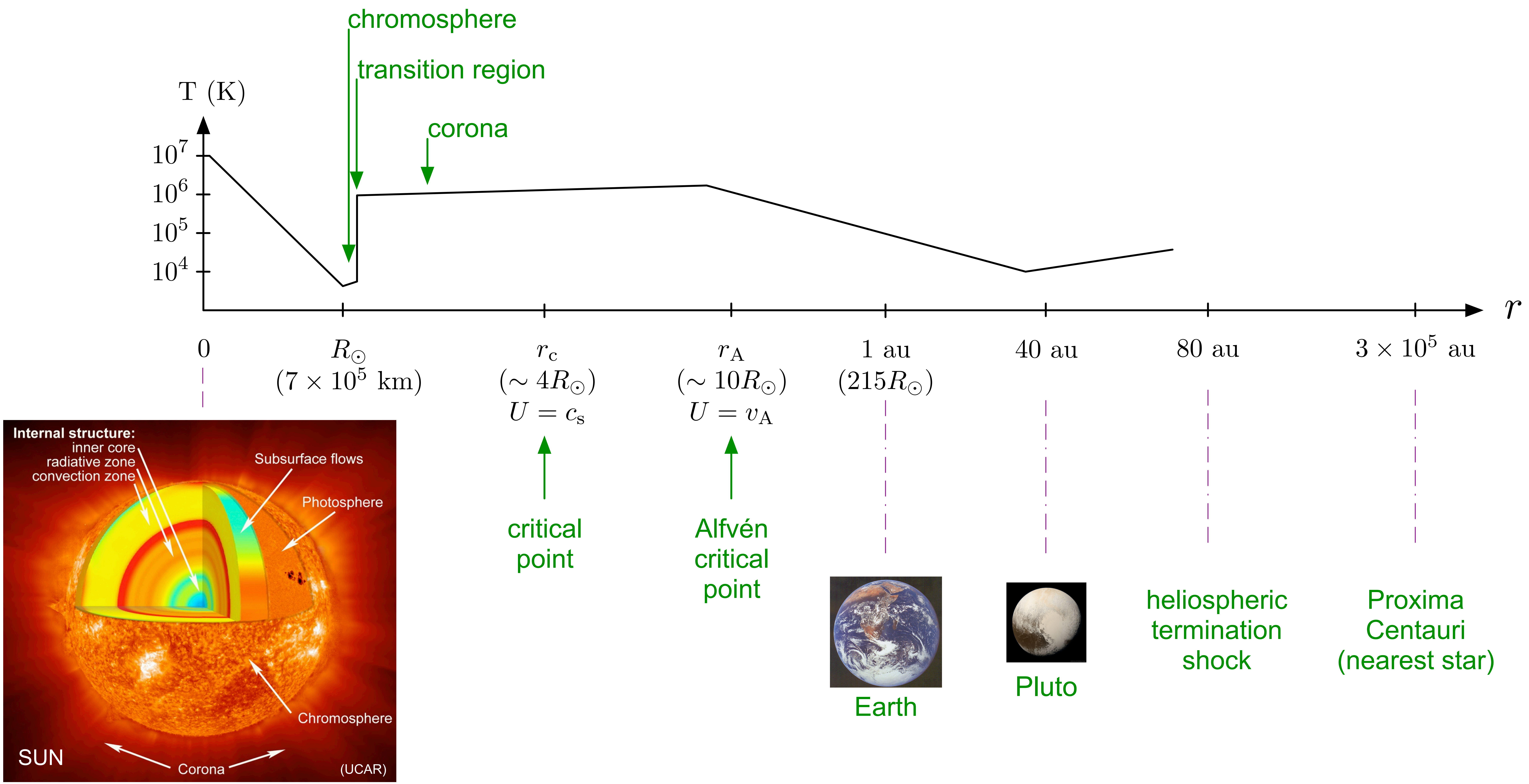
r




Earth


Pluto

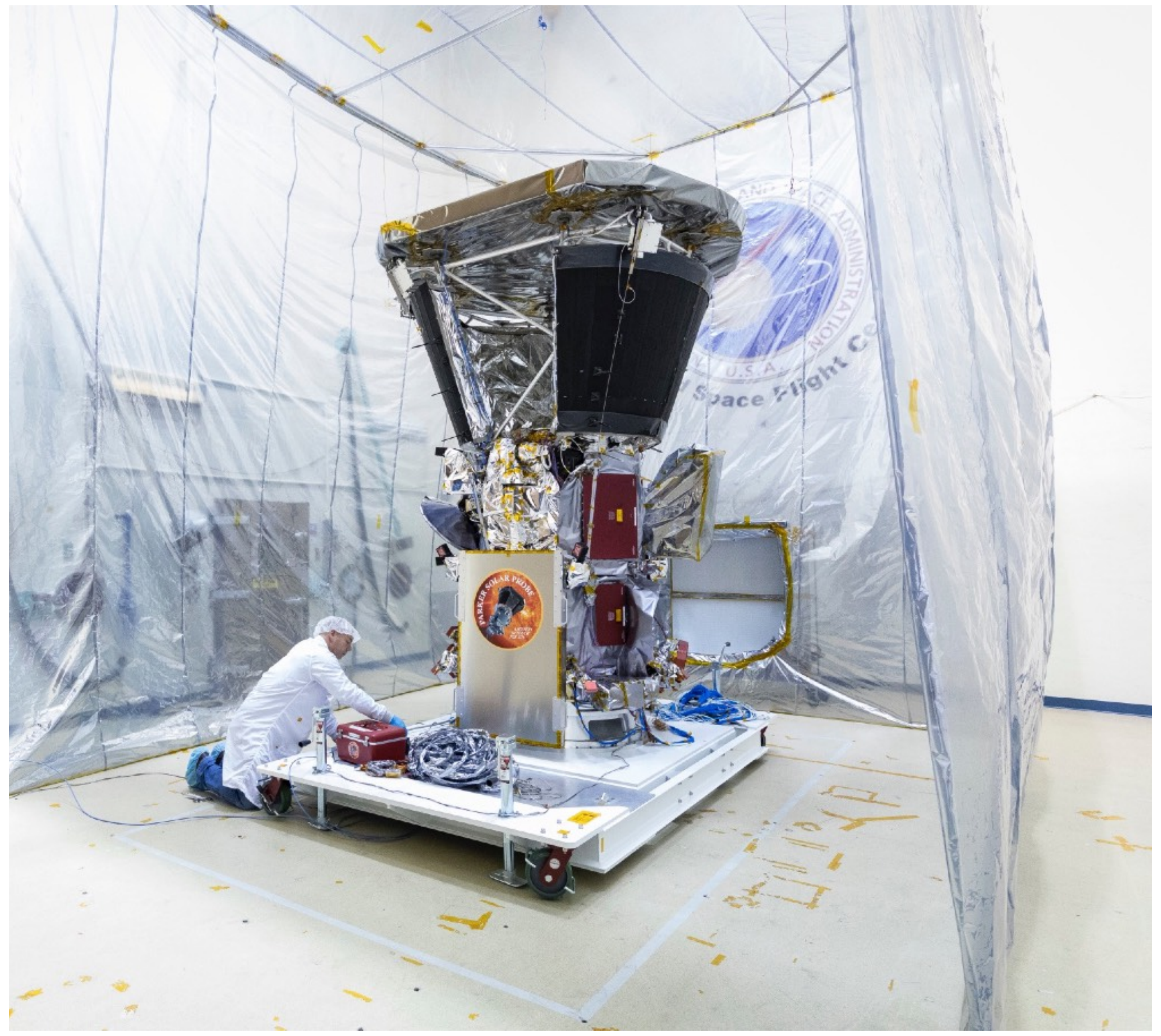
Temperature Profile



Parker's Pioneering Model of the Solar Wind

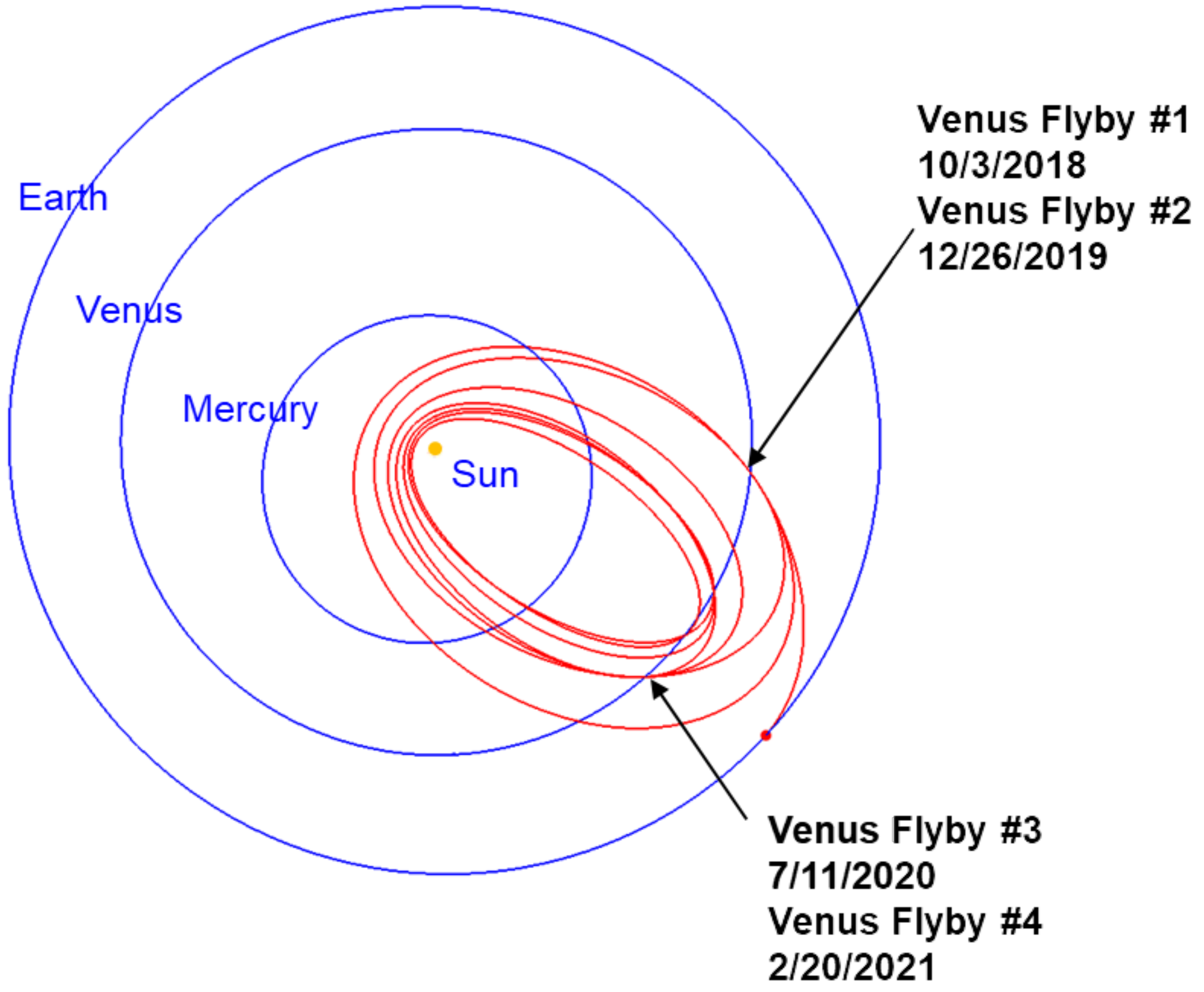
1. Spherical symmetry, steady state, no rotation.
2. Imposes a hot coronal temperature (~10^6 K) as an inner boundary condition.
3. The wind is heated by conduction and accelerated by the pressure gradient.
4. Parker predicted the super-sonic solar wind before it was discovered, but this model cannot explain the large solar-wind speeds for realistic coronal temperatures and offers no explanation for why the corona is so hot.
5. Conclusion: near the Sun, the dominant energy flux in the solar wind is not a heat flux, but something else that both heats the corona and powers the outflow.
6. Open problem: what is this dominant energy flux, and how is it converted into the kinetic energy of the outflowing solar wind?

Parker Solar Probe (PSP)



Finishing touches on the spacecraft

Launch, August 12, 2018



Venus flybys will gradually reduce
PSP's perihelion to less than 9.9 solar radii

- One of the mission's primary science objectives: to trace the flow of energy that heats and accelerates the solar corona and solar wind

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(Ideal) Magnetohydrodynamics (MHD)

Continuity eqn: $$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0$$

Momentum eqn: $$\rho \left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v} \right) = -\nabla \left(p + \frac{B^2}{8\pi} \right) + \frac{1}{4\pi} \boldsymbol{B} \cdot \nabla \boldsymbol{B}$$

Induction eqn: $$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B})$$

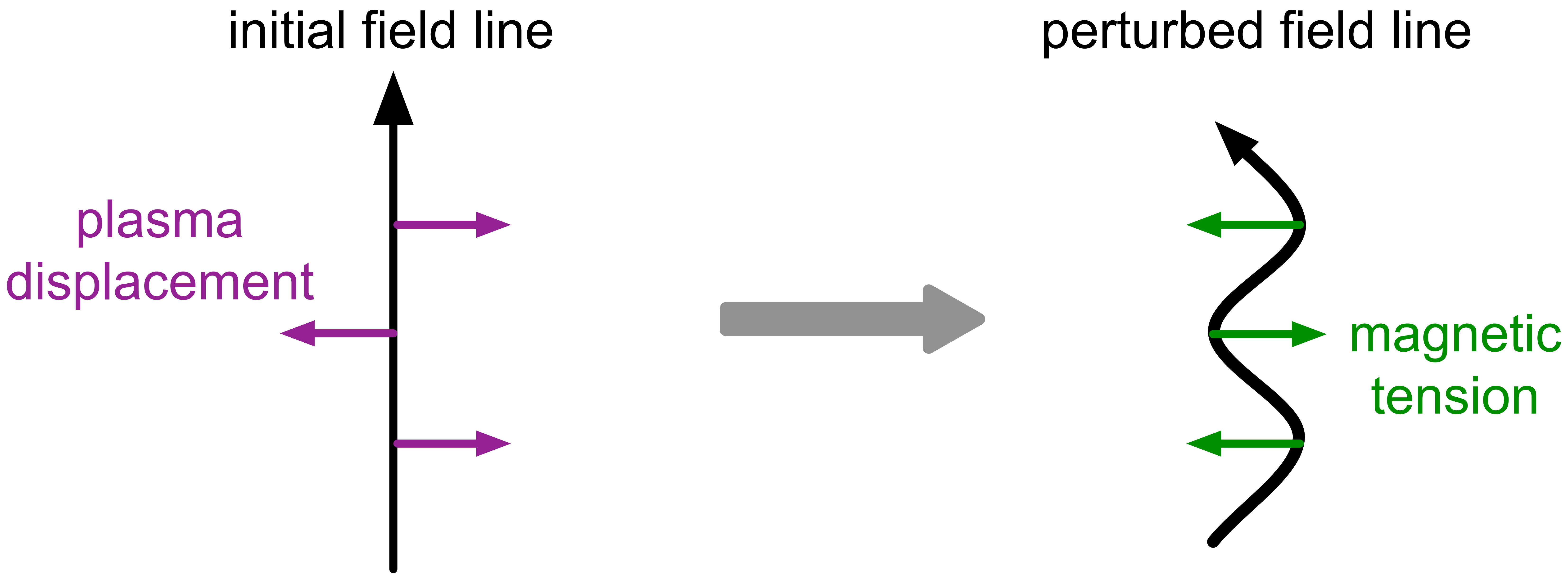
ρ = density
$\boldsymbol{v}$ = velocity
$\boldsymbol{B}$ = magnetic field
p = pressure

Closed by an energy equation.

Frozen-in law: magnetic-field lines are like threads that are frozen to the plasma and advected at the plasma velocity $\boldsymbol{v}$

Magnetic forces: magnetic pressure and magnetic tension

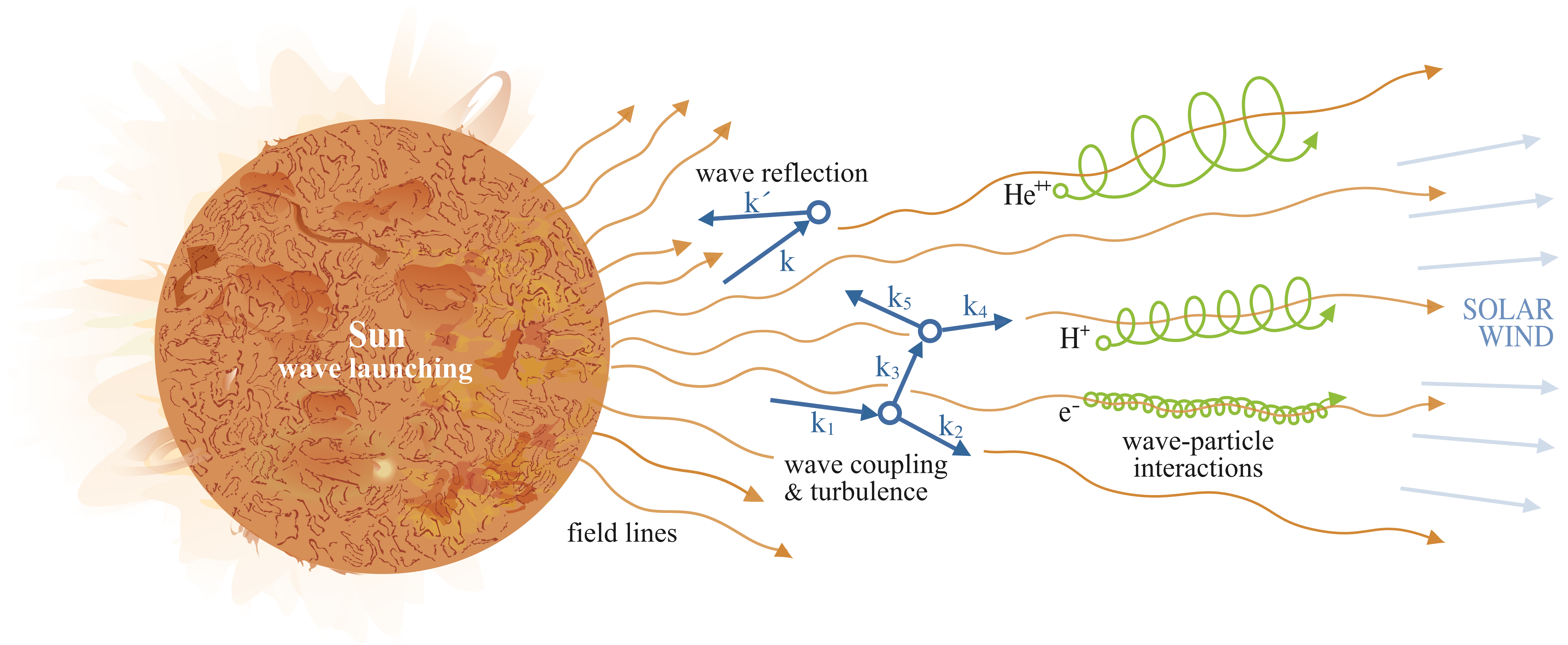
Alfvén Waves



- Propagate along magnetic field lines at the Alfvén speed $v_A = B_0/\sqrt{4\pi\rho}$

Solar-Wind Acceleration by Reflection-Driven Alfvén-Wave Turbulence

(Parker 1965, Coleman 1968, Hollweg 1973, Velli et al 1989, Matthaeus et al 1999, Cranmer et al 2007)



- The Sun launches Alfvén waves, which transport energy outwards
- The waves undergo partial reflection because of radial variations in the Alfvén speed
- Counter-propagating waves interact and become turbulent, which causes wave energy to ‘cascade’ from long wavelengths to short wavelengths
- Short-wavelength waves dissipate, heating the plasma. This increases the thermal pressure, which, along with the wave pressure, accelerates the solar wind.

WKB Evolution - No Reflection, No Dissipation

(Parker 1965, Bretherton & Garrett 1968, Heinemann & Olbert 1980)

wave action conservation

$$\frac{(1+y)^2(z^+)^2}{y} = \text{constant}$$

- z^+ = twice the rms velocity of outward-propagating Alfvén waves
- $y = \left[\frac{\rho(r)}{\rho(r_\mathrm{A})}\right]^{1/2}$

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- z^+ = twice the rms velocity of outward-propagating Alfvén waves
- $y = \left[\frac{\rho(r)}{\rho(r_A)}\right]^{1/2}$
- Near the Sun, $y \gg 1$, $y \cdot (z^+)^2 =$ constant, $z^+ \propto \rho^{-1/4}$. Amplitude increases as waves propagate into lower density. Like a whip cracking.
- At $r \gg r_A$, $y \ll 1$, $(z^+)^2/y =$ constant, $z^+ \propto \rho^{1/4}$. Waves are "stuck to the plasma" and do work on the expanding flow. Amplitude decreases as waves propagate into lower density. Like adiabatic cooling.

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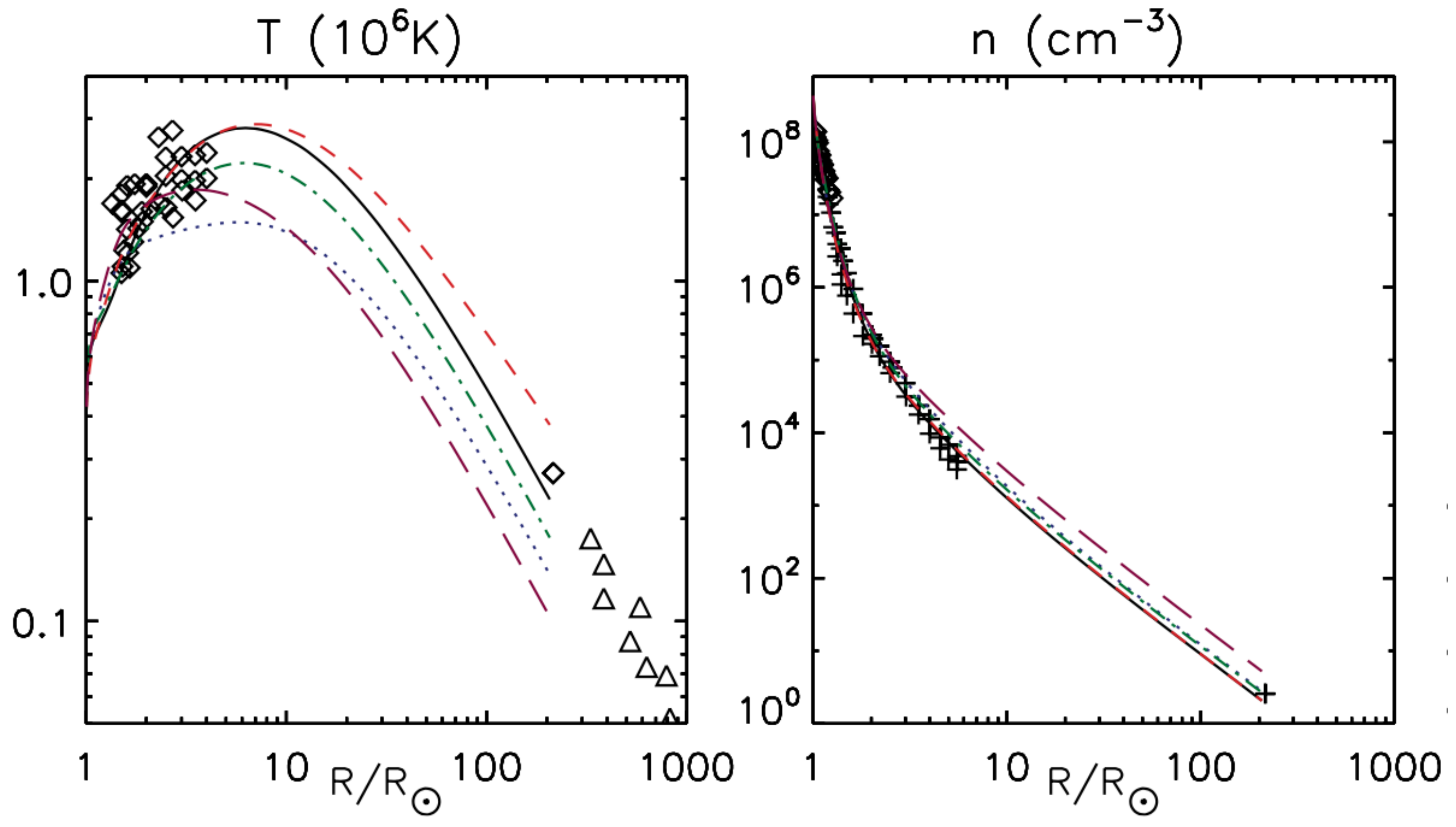
This is a key point — leads to strong, extended turbulent heating out to the Alfvén critical point.

Two Key Modeling Results for Reflection-Driven Alfvén-Wave Turbulence

- Heating by reflection-driven Alfvén-wave turbulence generically causes the region inside the Alfvén critical point to become approximately isothermal, but the region outside the Alfvén critical point does not become approximately isothermal.
  (Chandran, J. Plasm. Phys., **87**, 905870304 (2021).)



(Verdini et al 2010)

- Between 50% and 70% of the outward-propagating Alfvén-wave power at the coronal base dissipates before it reaches the Alfvén critical point.
  (Perez, Chandran, Klein, & Martinovic, J. Plasm. Phys., **87**, 905870218 (2021).)

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Possible Starting Point: Isothermal Solar-Wind Model

$$U\frac{\mathrm{d}U}{\mathrm{d}r} = -\frac{c_{\mathrm{s}}^2}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}r} - \frac{v_{\mathrm{esc}}^2 R_\odot}{2r^2} \quad (1)$$

(momentum equation divided by ρ)

$$p = \rho c_{\mathrm{s}}^2 \qquad \frac{\mathrm{d}p}{\mathrm{d}r} = c_{\mathrm{s}}^2\frac{\mathrm{d}\rho}{\mathrm{d}r} \qquad v_{\mathrm{esc}}^2 = \frac{2GM_\odot}{R_\odot}$$

$$\dot{M} = 4\pi r^2 U\rho = \text{ constant} \longrightarrow \frac{1}{U}\frac{\mathrm{d}U}{\mathrm{d}r} + \frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}r} + \frac{2}{r} = 0 \quad (2)$$

$$\frac{(c_{\mathrm{s}}^2 - U^2)}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}r} = \frac{2U^2}{r} - \frac{v_{\mathrm{esc}}^2 R_\odot}{2r^2} \quad (3)$$

$$U_{\mathrm{c}} = c_{\mathrm{s}} \qquad \frac{r_{\mathrm{c}}}{R_\odot} = \frac{v_{\mathrm{esc}}^2}{4c_{\mathrm{s}}^2} \quad (4)$$

$$\frac{U^2}{2} + c_{\mathrm{s}}^2 \ln\left(\frac{\rho}{\rho_{\mathrm{b}}}\right) - \frac{v_{\mathrm{esc}}^2 R_\odot}{2r} = \text{ constant} = -\frac{v_{\mathrm{esc}}^2}{2} \quad (5)$$

$$\ln\left(\frac{\rho_{\mathrm{c}}}{\rho_{\mathrm{b}}}\right) = -\frac{v_{\mathrm{esc}}^2}{2c_{\mathrm{s}}^2} + \frac{3}{2} \quad (6)$$

$$\dot{M} = 4\pi r_{\mathrm{c}}^2 \rho_{\mathrm{c}} U_{\mathrm{c}} = \frac{\pi R_\odot^2 v_{\mathrm{esc}}^4 \rho_{\mathrm{b}}}{4c_{\mathrm{s}}^3}\exp\left(-\frac{v_{\mathrm{esc}}^2}{2c_{\mathrm{s}}^2} + \frac{3}{2}\right) \quad (7)$$

(Hansteen & Velli 2012)

Possible Starting Point: Isothermal Solar-Wind Model

$$U\frac{\mathrm{d}U}{\mathrm{d}r} = -\frac{c_\mathrm{s}^2}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}r} - \frac{v_\mathrm{esc}^2 R_\odot}{2r^2} \quad (1)$$

$$\dot{M} = 4\pi r^2 U\rho = \text{ constant} \longrightarrow \frac{1}{U}\frac{\mathrm{d}U}{\mathrm{d}r} + \frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}r} + \frac{2}{r} = 0 \quad (2)$$

$$\frac{(c_\mathrm{s}^2 - U^2)}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}r} = \frac{2U^2}{r} - \frac{v_\mathrm{esc}^2 R_\odot}{2r^2} \quad (3)$$

$$U_\mathrm{c} = c_\mathrm{s} \qquad \frac{r_\mathrm{c}}{R_\odot} = \frac{v_\mathrm{esc}^2}{4c_\mathrm{s}^2} \quad (4)$$

$$\frac{U^2}{2} + c_\mathrm{s}^2 \ln\left(\frac{\rho}{\rho_\mathrm{b}}\right) - \frac{v_\mathrm{esc}^2 R_\odot}{2r} = \text{ constant} = -\frac{v_\mathrm{esc}^2}{2} \quad (5)$$

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$$\dot{M} = 4\pi r_\mathrm{c}^2 \rho_\mathrm{c} U_\mathrm{c} = \frac{\pi R_\odot^2 v_\mathrm{esc}^4 \rho_\mathrm{b}}{4c_\mathrm{s}^3}\exp\left(-\frac{v_\mathrm{esc}^2}{2c_\mathrm{s}^2} + \frac{3}{2}\right) \quad (7)$$

- Plasma at r_c expands at speed c_s
- As c_s decreases, density scale height in corona decreases, and ρ_c and $\dot{M}$ decrease exponentially
- Formula has little predictive value unless we can determine c_s

(Note: observations $\longrightarrow c_\mathrm{s} \simeq v_\mathrm{esc}/6$)

(Hansteen & Velli 2012)

Approximate Analytic Solution to the Coupled Problems of Coronal Heating and Solar-Wind Acceleration

(Chandran, J. Plasm. Phys., **87**, 905870304 (2021))

- Central approximation: "sub-Alfvénic region" (at $r < r_{\mathrm{A}}$, where $U < v_{\mathrm{A}}$) has a uniform temperature
- Presented in two steps:
  1. A back-of-the-envelope calculation of $\dot{M}$, U_∞, and c_{s}
  2. More sophisticated analytic treatment

Isothermal mass loss rate: $$\dot{M} = 4\pi R_\odot^2 \rho_{\rm b} v_{\rm Ab}\, c_1 \exp\left(-\frac{v_{\rm esc}^2}{2c_{\rm s}^2}\right)$$ (Hansteen & Velli 2012)

$$c_1 \equiv \left(\frac{e^{3/2} v_{\rm esc}^4}{16 v_{\rm Ab} c_{\rm s}^3}\right)$$

'b' subscript indicates quantity is evaluated at the coronal base, i.e., at $r = r_{\rm b} \simeq R_\odot$

Isothermal mass loss rate:

$$\dot{M} = 4\pi R_\odot^2 \rho_{\rm b} v_{\rm Ab}\, c_1 \exp\left(-\frac{v_{\rm esc}^2}{2c_{\rm s}^2}\right)$$

Internal-energy balance of sub-Alfvénic region:

$$\dot{M} \underbrace{c_{\rm s}^2 \ln\left(\frac{\rho_{\rm b}}{\rho_{\rm A}}\right)} \simeq \overbrace{P_{\rm AW}(r_{\rm b})}^{\text{Alfvén-wave power at coronal base}}$$

$c_{\rm s}^2 \ln\left(\frac{\rho_{\rm b}}{\rho_{\rm A}}\right)$ is the heating cost per unit mass to transit the quasi-isothermal sub-Alfvénic region

($\rho_{\rm A}$ = density at r=$r_{\rm A}$, where U = $v_{\rm A}$)

Isothermal mass loss rate: $$\dot{M} = 4\pi R_\odot^2 \rho_{\rm b} v_{\rm Ab}\, c_1 \exp\left(-\frac{v_{\rm esc}^2}{2c_{\rm s}^2}\right)$$

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A(r) = cross-sectional area of flow

($\rho_{\rm A}$ = density at $r = r_{\rm A}$, where $U = v_{\rm A}$)

$$\frac{\rho U A}{BA} = \text{constant}$$

Isothermal mass loss rate:

$$\dot{M} = 4\pi R_\odot^2 \rho_{\rm b} v_{\rm Ab}\, c_1 \exp\left(-\frac{v_{\rm esc}^2}{2c_{\rm s}^2}\right)$$

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$$\frac{\rho U \cancel{A}}{B \cancel{A}} = \text{constant} \qquad \frac{\rho^{1/2} U}{v_{\rm A}} = \text{constant} = \rho_{\rm A}^{1/2}$$

$$\left(v_{\rm A} = \frac{B}{\sqrt{4\pi\rho}}\right)$$

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Internal-energy balance of sub-Alfvénic region:

$$\dot{M} c_\mathrm{s}^2 \ln\left(\frac{\rho_\mathrm{b}}{\rho_\mathrm{A}}\right) \simeq P_\mathrm{AW}(r_\mathrm{b})$$

Flux and mass conservation:

$$\dot{M} = 4\pi R_\odot^2 \rho_\mathrm{b} v_\mathrm{Ab} \left(\frac{\rho_\mathrm{A}}{\rho_\mathrm{b}}\right)^{1/2}$$

$$\frac{\rho U \cancel{A}}{B \cancel{A}} = \text{ constant} \qquad \frac{\rho^{1/2} U}{v_\mathrm{A}} = \text{ constant} = \rho_\mathrm{A}^{1/2} \qquad U = v_\mathrm{A} \left(\frac{\rho_\mathrm{A}}{\rho}\right)^{1/2}$$

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In an AW-driven solar wind, the heating cost per unit mass to transit the sub-Alfvénic region is ~ $(v_\mathrm{esc})^2$

Flux and mass conservation: $$\dot{M} = 4\pi R_\odot^2 \rho_\mathrm{b} v_\mathrm{Ab} \boxed{\left(\frac{\rho_\mathrm{A}}{\rho_\mathrm{b}}\right)^{1/2}}$$

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- increasing c_s^2 leads to an exponential increase in $\dot{M}$ and the solar-wind density and an exponential reduction of $\rho_\mathrm{b}/\rho_\mathrm{A}$, leaving $c_\mathrm{s}^2 \ln(\rho_\mathrm{b}/\rho_\mathrm{A})$ approximately unchanged.

Isothermal mass loss rate:

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$$\cancel{\ln(c_1)} - \frac{v_{\rm esc}^2}{2c_{\rm s}^2} = \frac{1}{2}\ln\frac{\rho_{\rm A}}{\rho_{\rm b}} \quad \longrightarrow \quad v_{\rm esc}^2 \simeq c_{\rm s}^2 \ln\left(\frac{\rho_{\rm b}}{\rho_{\rm A}}\right)$$

$$\dot{M} \simeq \frac{P_{\rm AWb}}{c_{\rm s}^2 \ln(\rho_{\rm b}/\rho_{\rm A})} \simeq \frac{P_{\rm AWb}}{v_{\rm esc}^2}$$

Isothermal mass loss rate:

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In an AW-driven solar wind, the heating cost per unit mass to transit the sub-Alfvénic region is ~ $(v_{\rm esc})^2$

Internal-energy balance of sub-Alfvénic region:

$$\dot{M} c_{\rm s}^2 \ln\left(\frac{\rho_{\rm b}}{\rho_{\rm A}}\right) \simeq P_{\rm AW}(r_{\rm b})$$

Flux and mass conservation:

$$\dot{M} = 4\pi R_\odot^2 \rho_{\rm b} v_{\rm Ab} \left(\frac{\rho_{\rm A}}{\rho_{\rm b}}\right)^{1/2}$$

$$\cancel{\ln(c_1)} - \frac{v_{\rm esc}^2}{2c_{\rm s}^2} = \frac{1}{2}\ln\frac{\rho_{\rm A}}{\rho_{\rm b}} \quad \longrightarrow \quad v_{\rm esc}^2 \simeq c_{\rm s}^2 \ln\left(\frac{\rho_{\rm b}}{\rho_{\rm A}}\right)$$

$$\dot{M} \simeq \frac{P_{\rm AWb}}{c_{\rm s}^2 \ln(\rho_{\rm b}/\rho_{\rm A})} \simeq \frac{P_{\rm AWb}}{v_{\rm esc}^2}$$

Energy conservation:

$$\frac{1}{2}\dot{M} U_\infty^2 \simeq P_{\rm AWb} - \frac{1}{2}\dot{M} v_{\rm esc}^2 \longrightarrow U_\infty \simeq v_{\rm esc}$$

Like a mirror in energy space: energy per unit mass goes from $-(v_{\rm esc})^2/2$ to $+(v_{\rm esc})^2/2$

Isothermal mass loss rate: $$\dot{M} = 4\pi R_\odot^2 \rho_\mathrm{b} v_\mathrm{Ab}\, c_1 \exp\left(-\frac{v_\mathrm{esc}^2}{2c_\mathrm{s}^2}\right)$$

Internal-energy balance of sub-Alfvénic region: $$\dot{M} c_\mathrm{s}^2 \ln\left(\frac{\rho_\mathrm{b}}{\rho_\mathrm{A}}\right) \simeq P_\mathrm{AW}(r_\mathrm{b})$$

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$$\boxed{\dot{M} \simeq \frac{P_\mathrm{AWb}}{c_\mathrm{s}^2 \ln(\rho_\mathrm{b}/\rho_\mathrm{A})} \simeq \frac{P_\mathrm{AWb}}{v_\mathrm{esc}^2}}$$

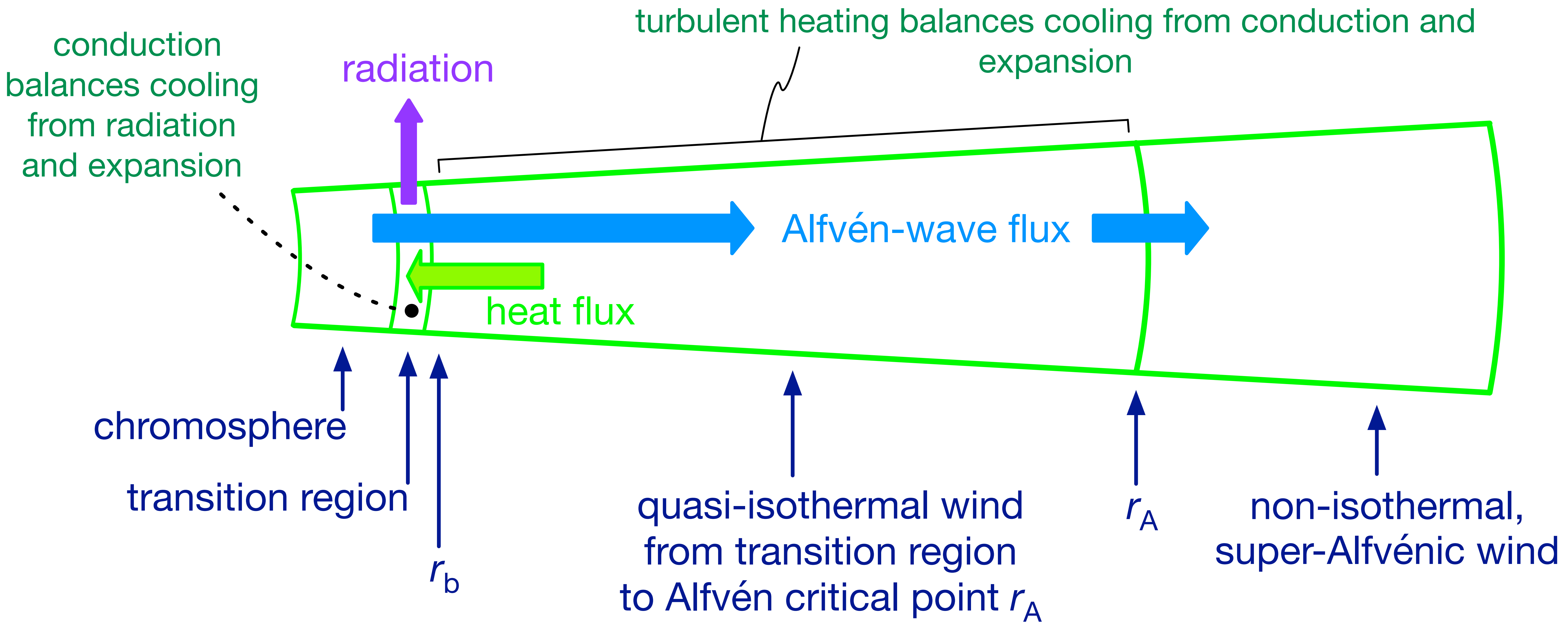
$$\boxed{\text{also,}\quad T \simeq \frac{m_\mathrm{p} v_\mathrm{esc}^2}{8k_\mathrm{B} \ln(v_\mathrm{esc}/\delta v_\mathrm{b})}}$$

Energy conservation: $$\frac{1}{2}\dot{M} U_\infty^2 \simeq P_\mathrm{AWb} - \frac{1}{2}\dot{M} v_\mathrm{esc}^2 \longrightarrow \boxed{U_\infty \simeq v_\mathrm{esc}}$$

$(v_\mathrm{esc} = 618\ \mathrm{km\ s^{-1}})$

Analytic Model of Coronal Heating and Solar-Wind Acceleration

(Chandran, JPP, accepted, arXiv:2101.04156)



- c_s is taken to be independent of r in sub-Alfvénic region
- r_b is the radius at which $Q(r) = R(r)$.
- Neglect radiative cooling at $r > r_b$, and neglect turbulent heating at $r < r_b$

Main Mathematical Elements of Model

- Simple turbulence model (the decay length for the wave action flux is ~ twice the local density scale height divided by an adjustable parameter $\sigma \sim 0.1 - 0.5$)
- Continuity equation (integrated)
- Momentum equation with wave pressure force (integrated)
- Critical point conditions
- Internal-energy equation for the transition region (integrated)
- Internal-energy equation for the sub-Alfvenic region (integrated)

Yields 3 simultaneous transcendental equations in 3 unknowns. I'll present asymptotic solutions in two parameter regimes as well as numerical solutions.

Internal-Energy Balance of Sub-Alfvénic Region

$$\cancel{\nabla \cdot \left(\frac{\boldsymbol{v} \rho c_{\mathrm{s}}^2}{\gamma - 1} \right)} = -\rho c_{\mathrm{s}}^2 \nabla \cdot \boldsymbol{v} - \nabla \cdot \boldsymbol{q} + Q - \cancel{R}$$

$c_{\mathrm{s}} = \text{ constant}$ $\qquad 0 = \nabla \cdot (\rho \boldsymbol{v})$ $\qquad -\rho \nabla \cdot \boldsymbol{v} = \boldsymbol{v} \cdot \nabla \rho = U \frac{\mathrm{d}\rho}{\mathrm{d}r}$

$$-c_{\mathrm{s}}^2 U \frac{\mathrm{d}\rho}{\mathrm{d}r} = -\frac{1}{A} \frac{\mathrm{d}}{\mathrm{d}r} (A q_r) + Q$$

$$-c_{\mathrm{s}}^2 \int_{r_{\mathrm{b}}}^{r_{\mathrm{A}}} A U \rho \frac{1}{\rho} \frac{\mathrm{d}\rho}{\mathrm{d}r} \mathrm{d}r = -A q_r \Big|_{r_{\mathrm{b}}}^{r_{\mathrm{A}}} + \int_{r_{\mathrm{b}}}^{r_{\mathrm{A}}} A Q \mathrm{d}r$$

$$\boxed{\dot{M} c_{\mathrm{s}}^2 \ln \left(\frac{\rho_{\mathrm{b}}}{\rho_{\mathrm{A}}} \right) = A_{\mathrm{b}} q_{\mathrm{r}}(r_{\mathrm{b}}) + \int_{r_{\mathrm{b}}}^{r_{\mathrm{A}}} A Q \mathrm{d}r}$$

Determining the Heat Flux at the Coronal Base

$$\nabla \cdot \left[\boldsymbol{v} \left(\frac{p}{\gamma - 1} \right) \right] = -p\nabla \cdot \boldsymbol{v} - \nabla \cdot \boldsymbol{q} + Q - R$$

Solve in transition region,
setting Q = 0

- Rosner et al (1978): $p =$ constant, $q_r = -\alpha T^{5/2} \frac{\mathrm{d}T}{\mathrm{d}r}$, $\Lambda(T) = c_{\mathrm{R}} T^{-1/2}$
- Rosner et al (1978): $\nabla \cdot \boldsymbol{q} = \frac{\mathrm{d}q_r}{\mathrm{d}r} = \frac{\mathrm{d}q_r}{\mathrm{d}T} \frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{q_r}{\alpha T^{5/2}} \frac{\mathrm{d}q_r}{\mathrm{d}T}$
- Schwadron & McComas (2003): $\nabla \cdot \boldsymbol{v} = -(\boldsymbol{v}/\rho) \cdot \nabla \rho = (U/T)\mathrm{d}T/\mathrm{d}r = -U q_r / (\alpha T^{7/2})$
- $\longrightarrow$ first-order nonlinear ODE for $q(T)$

$$\boxed{q_{\mathrm{b}} = -q_{r\mathrm{b}} = -\frac{a_1}{a_2} \left[1 + W_{-1} \left(-e^{-(1+a_3)} \right) \right]}$$

lower branch of Lambert W function

q_{b} balances internal-energy losses from radiation, $p\mathrm{d}V$ work, and advection in TR. These are determined by ρ_{b}, T_{b}, and U_{b}.

$$a_1 = \frac{\alpha p^2 c_{\mathrm{R}}}{4 k_{\mathrm{B}}^2} \qquad a_2 = \left(\frac{\gamma}{\gamma - 1} \right) \frac{pU}{T} \qquad a_3 = \frac{a_2^2 T_{\mathrm{b}}}{a_1}$$

Low-Mach-number limit: $q_{\mathrm{b}} \simeq 0.15 \rho_{\mathrm{b}} c_{\mathrm{s}}^3$

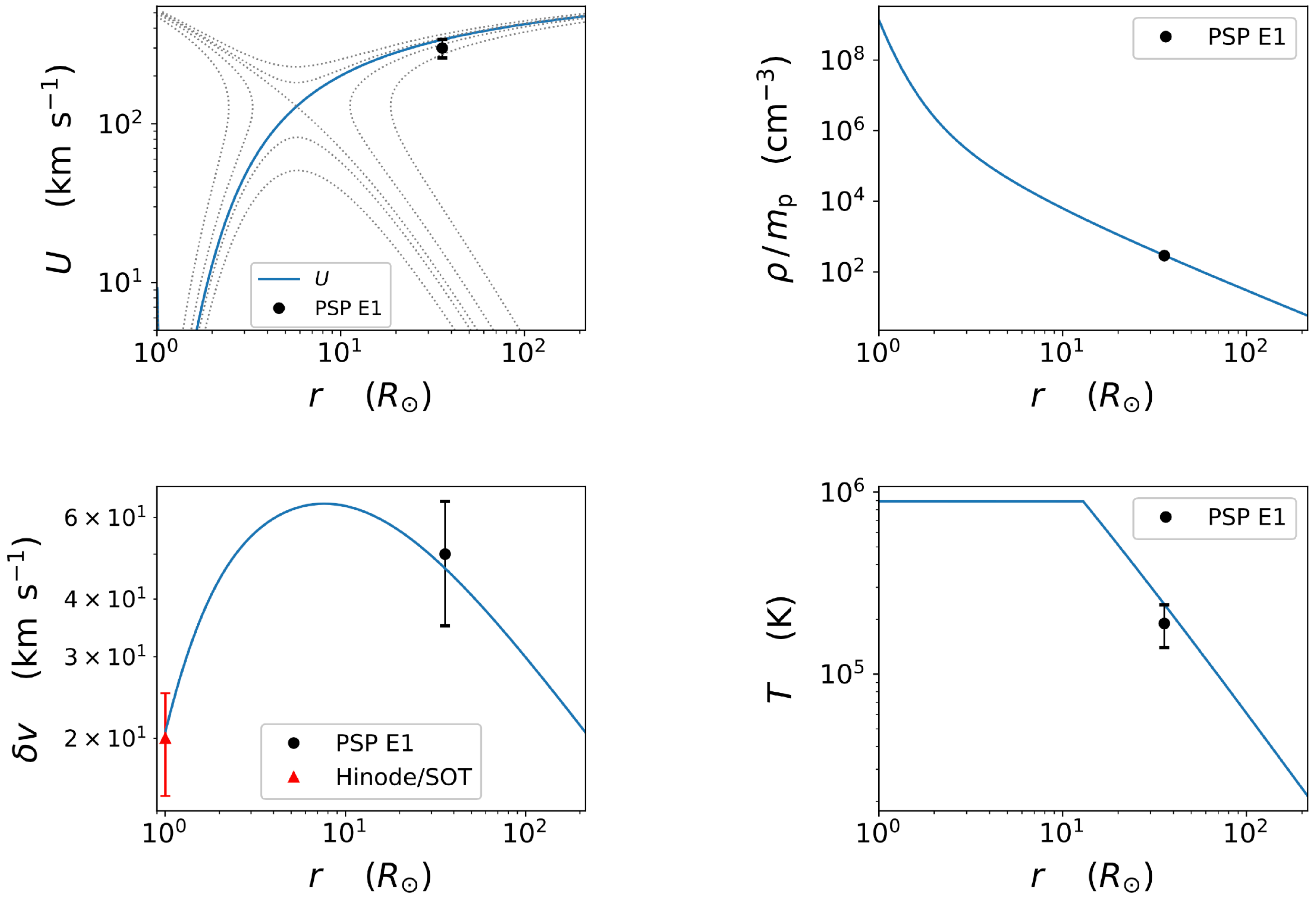
For the Numerical Examples to Follow

- Parker Solar Probe $\longrightarrow$ $|B_r| = \left(2.2 \times 10^{-5}\ \mathrm{G}\right)\left(\dfrac{1\ \mathrm{au}}{r}\right)^2 \eta(r)$, where the "super-radial expansion factor" $\eta(r)$ approaches 1 at $r \gg R_\odot$.

- $\eta(r) = 1$ at $r \geq r_\mathrm{c}$, and $\eta_\mathrm{b} = \eta(r_\mathrm{b})$ is inferred from observations or guessed.

- I set $P_\mathrm{AW}(r_\mathrm{b}) = \dfrac{1}{2}\dot{M}\left(U_\infty^2 + v_\mathrm{esc}^2\right)$ and determine right-hand side using Ulysses measurements (Schwadron & McComas 2008)

Comparison to Measurements from Parker Solar Probe

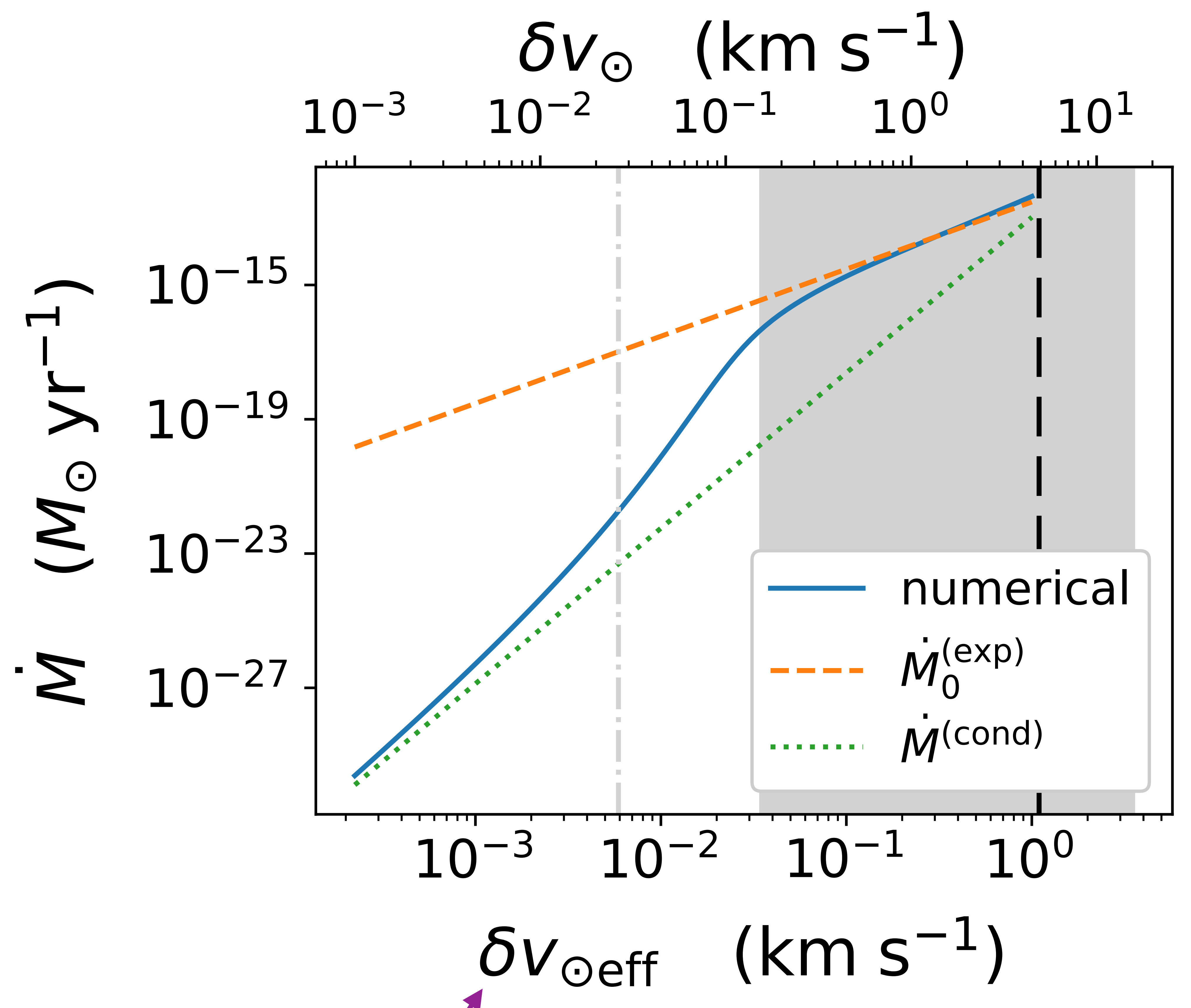


PSP data from Kasper et al (2019), Bale et al (2019), Chen et al (2020)

Hinode data from DePontieu et al (2007)

$$\sigma = 0.5, \quad \eta_\mathrm{b} = 100$$

Expansion-Dominated and Conduction-Dominated Regimes



$$\dot{M}_0^{(\mathrm{cond})} = \frac{R_\odot^2 B_{\mathrm{ref}}^2}{v_{\mathrm{esc}}} \frac{1}{I_1^{1/14} I_2} \left[\epsilon_\odot^{14-4\sigma} (\eta_{\mathrm{b}} B_*)^{-4\sigma} \tilde{l}_{\mathrm{b}}^{3\sigma} \tilde{\rho}_\odot^{7-2\sigma} \right]^{1/(7-7\sigma)}$$

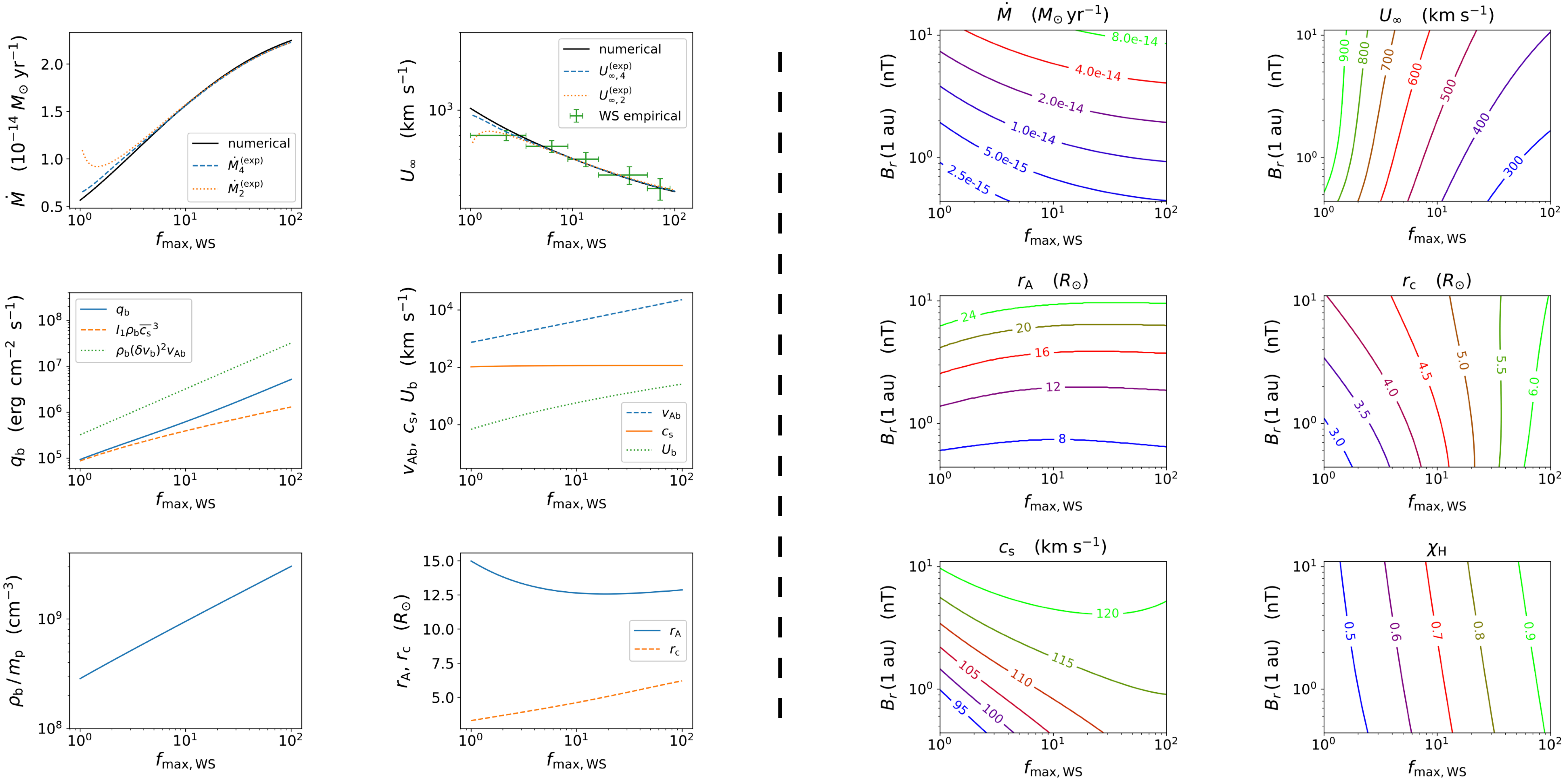
$$\dot{M}_0^{(\mathrm{exp})} = \epsilon_\odot \overline{B} R_\odot^2 \sqrt{4\pi\rho_\odot} = \frac{P_{\mathrm{AW}}(r_{\mathrm{b}})}{v_{\mathrm{esc}}^2}$$

In expansion-dominated regime, leading-order analytic solution reproduces the back-of-the-envelope calculation

Another label for Alfven-wave power:

$$P_{\mathrm{AW}}(r_{\mathrm{b}}) \equiv \frac{A_\odot B_\odot}{\sqrt{4\pi\rho_\odot}} \rho_\odot (\delta v_{\odot\mathrm{eff}})^2 \qquad \epsilon_{\odot\mathrm{eff}} = \left(\frac{\delta v_{\odot\mathrm{eff}}}{v_{\mathrm{esc}}} \right)^2 \qquad \delta v_{\odot\mathrm{eff}} \simeq 0.2 \delta v_\odot$$

And Much More…. (Advertisement for Paper)



$$f_{\mathrm{max,WS}} = \eta_\mathrm{b}/5.75,\ \ \sigma = 5.7\times 10^{-2}\,\eta_\mathrm{b}^{0.34}$$

Possible Applications to Other Astrophysical Outflows

- Analytic theory of reflection-driven AW turbulence also holds in general relativistic MHD. (Chandran, Foucart & Tchekhovskoy, JPP, 2018)
    - WKB evolution causes amplitudes to increase with distance in sub-Alfvénic region and decrease with distance in super-Alfvénic region.
    - A substantial fraction of the local Alfvén-wave action flux decays within each Alfvén-speed scale height
- May cause sub-Alfvénic region to become quasi-isothermal in relativistic wave-driven outflows.

Conclusion

- Heating cost per unit mass to transit sub-Alfvénic region is

$$c_{\mathrm{s}}^2 \ln\left(\frac{\rho_{\mathrm{b}}}{\rho_{\mathrm{A}}}\right) \simeq v_{\mathrm{esc}}^2 \quad \longrightarrow \quad \dot{M} \simeq \frac{P_{\mathrm{AW}}(r_{\mathrm{b}})}{v_{\mathrm{esc}}^2}$$

- Energy conservation:

$$\frac{1}{2}\dot{M}U_\infty^2 \simeq P_{\mathrm{AWb}} - \frac{1}{2}\dot{M}v_{\mathrm{esc}}^2 \quad \longrightarrow \quad U_\infty \simeq v_{\mathrm{esc}}$$

- Higher-order corrections describe effects of Alfvén wave power at $r > r_{\mathrm{A}}$, conductive losses, and wave momentum deposition inside r_{c}.

- Coming decade should be very interesting both for what we learn from Parker Solar Probe and Solar Orbiter, and for what our growing understanding of the solar wind teaches us about other astrophysical plasmas.