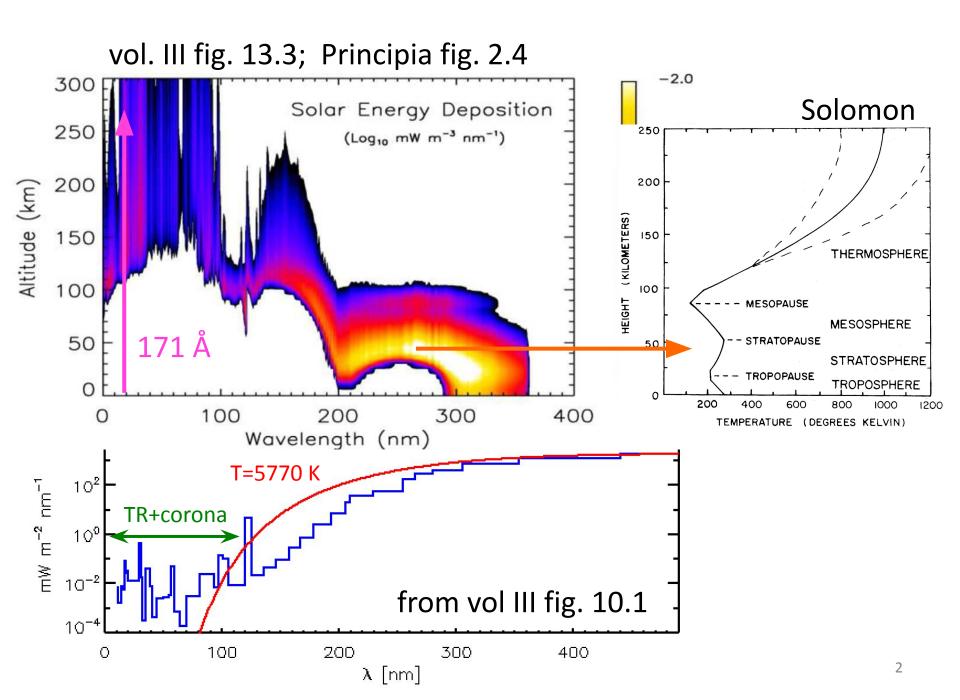
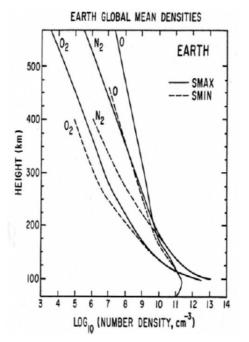
Q: Why do the Earth & planets have ionospheres? magnetospheres?

Dana Longcope Montana State University

Review & Activities



Activity 17 (p. 32): The fact that concentrations of atomic nitrogen are not shown in Fig. 2.5 should make you wonder given that molecular nitrogen is the most common species in the troposphere. Why is atomic nitrogen rare in the upper atmosphere? Hint: compare the molecular binding energies of nitrogen, oxygen, and water.





Bond-dissociation energy:

https://en.wikipedia.org/wiki/Bond-dissociation_energy

	O-H in H ₂ O	O=O in O ₂	C=O in CO ₂	N≡N in N ₂
eV/bond	5.15	5.15	5.51	9.79

Activity 9 (p. 21): Compute scale heights H_p in the Earth's atmosphere for molecular nitrogen (the dominant component) at a range of temperatures, and compare these with the value H_p for the atomic hydrogen-dominated gas in the solar photosphere, and for the CO_2 -rich atmospheres of Venus and Mars. Use the data in Tables 2.1 and 2.3. Consider how the value of H_p/R_{\odot} contributes to the appearance of the Sun as having a well-defined surface. Also, consider why neutral, atomic hydrogen dominates in the solar photosphere

$$H = \frac{\kappa I}{mg}$$

Earth:

T = 300 K g = 10^{3} cm/s² particles = N₂ m = 28 m_p = 3 × 10^{-23} g

 $H_{p} = 9 \text{ km}$

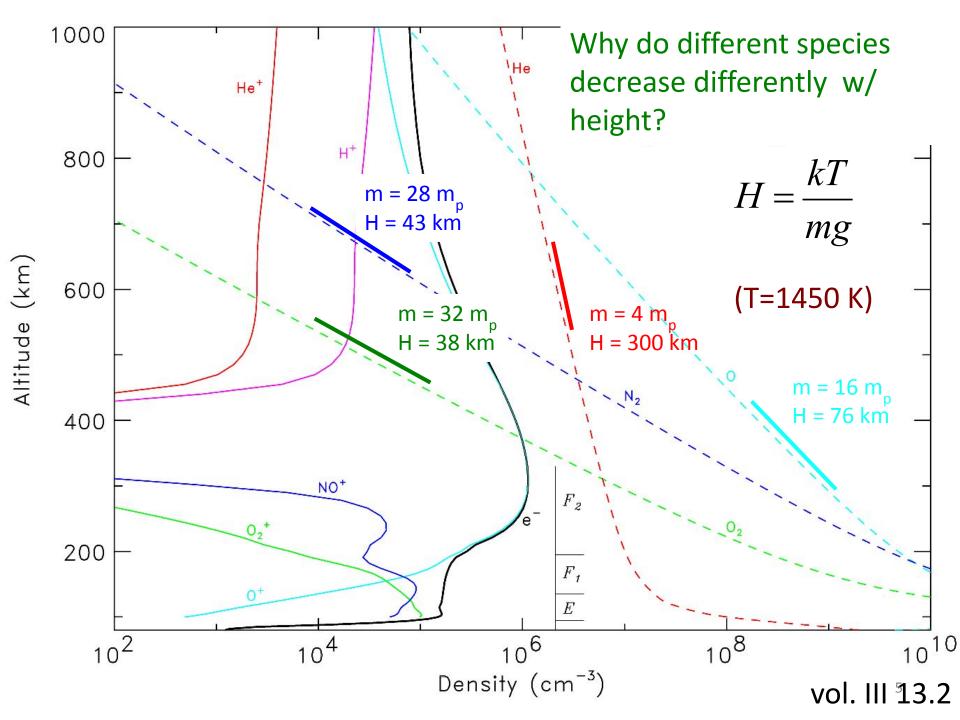
Venus: T = 740 K g = 900 cm/s² particles = CO₂ m = 44 m_p = 7 × 10⁻²³ g

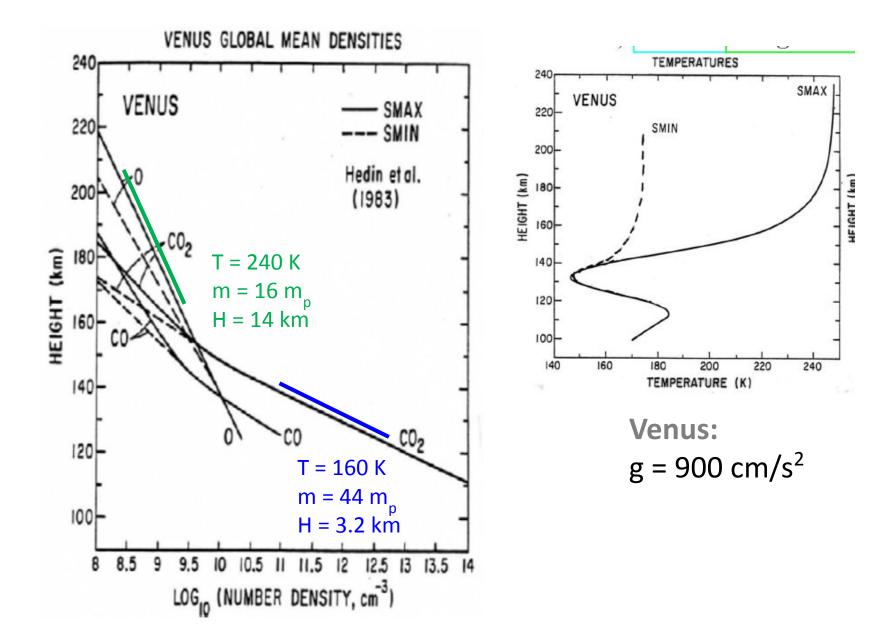
 $H_{p} = 15 \text{ km}$

Solar photosphere: T = 6000 K $g = 3 \times 10^4 \text{ cm/s}^2$ particles = H atoms $m = m_p = 2 \times 10^{-24} \text{ g}$

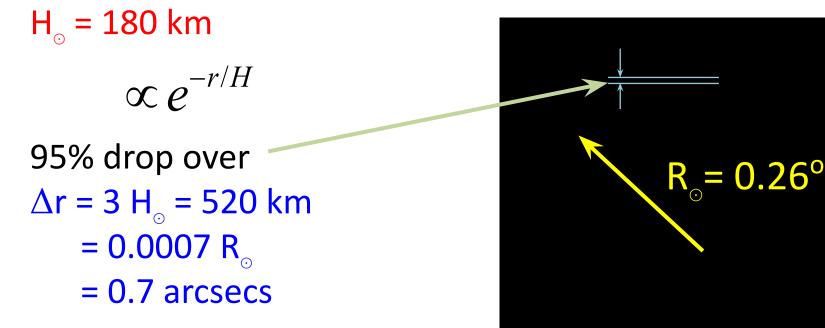
 $H_{p} = 180 \text{ km}$

T= 1450 K \Box H_p = 43 km

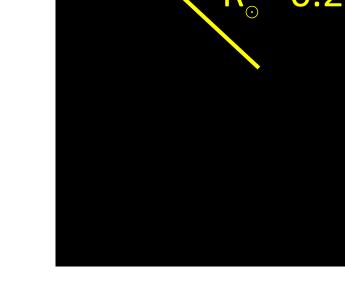


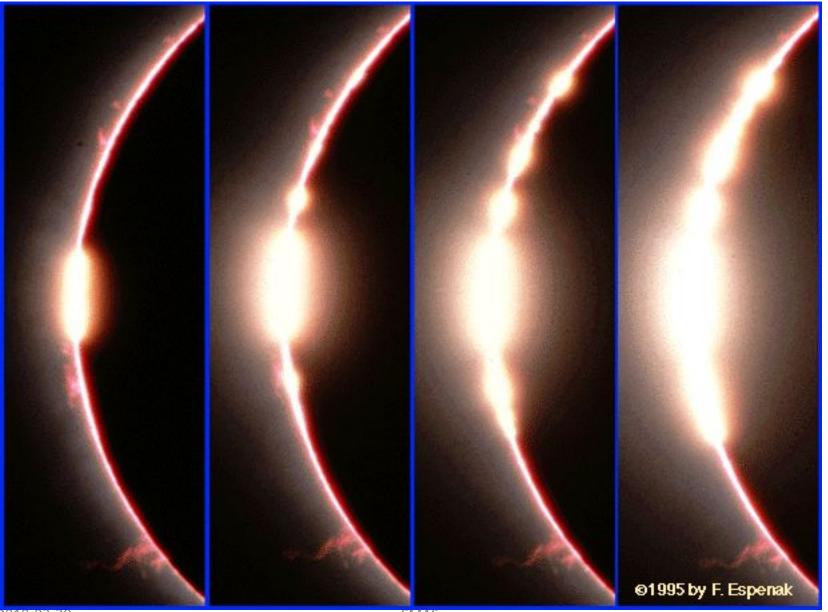


How fuzzy is the solar limb

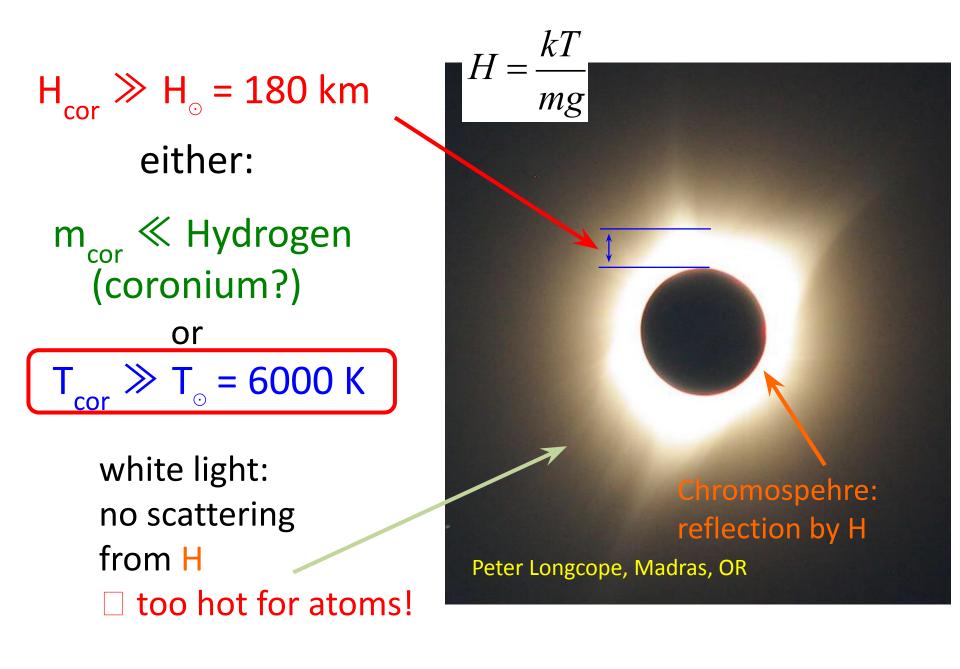


<u>Resolving power</u>: Human eye (good): ~60 arcsec Telescope on Earth: ~1 arcsec





2018-03-30



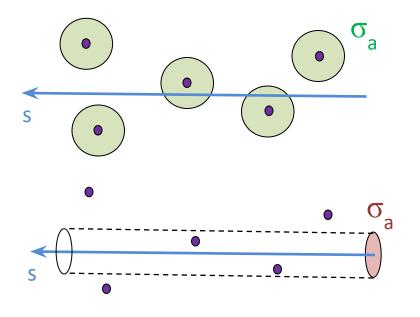
Activity 19 (p. 33): You can think of the optical depth as the mean number of absorbers within the cross-section along a photon's path from infinity to height h. The probability of suffering zero absorptions, and thus making it to h, is $exp(-\tau)$. The intensity at h is then an integral from infinity over the expected number of absorptions along the way. Combine that with Eq. (2.18) to derive Eqs. (2.19) and (2.20).

show:
$$au = \sigma_{a} n(h) \frac{H_{p}(h)}{\cos(\chi)},$$
 (2.19)

use:
$$q = \sigma_{a}I(h)n(h)\eta_{i},$$
 (2.18)

.

to derive:
$$q(h) = I_{\infty} \exp\left[-\sigma_{a}n(h)\frac{H_{p}(h)}{\cos(\chi)}\right]\eta_{i}\sigma_{a}n(h).$$



Compute number N of absorbers (cross section σ_a) the line passes through

same answer as:

Compute number N of centers included in a cylinder w/ cross section σ_a

local # density of centers

Answer on average:
$$\langle N \rangle = \int \sigma_a n(s) ds = \tau$$

Poisson: the probability of including **exactly N** centers

$$p_N = \frac{\tau^N}{N!} e^{-\tau} .$$

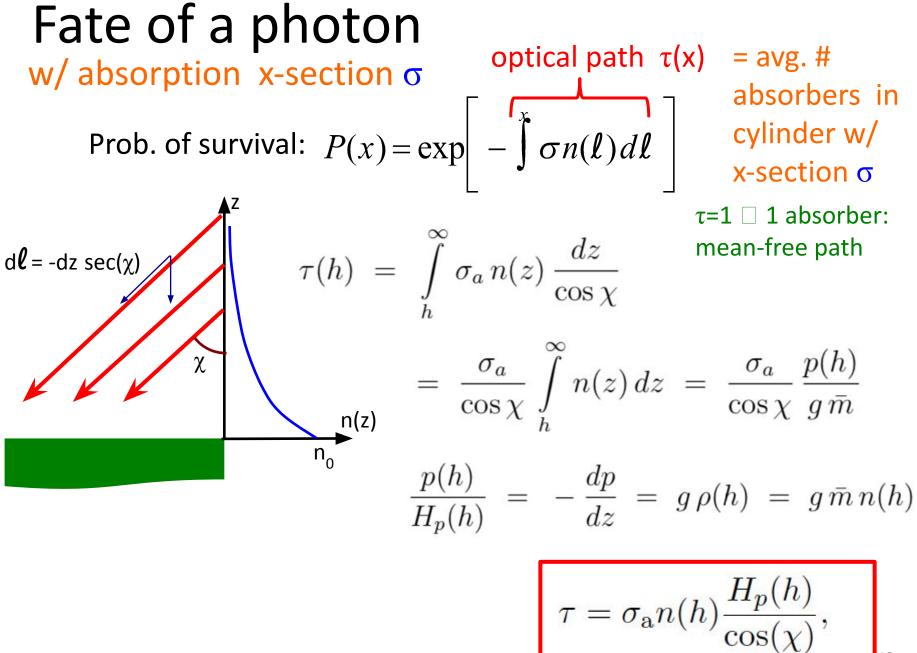
survival \iff N=0 absorptions This occurs with probability $p_0 = e^{-\tau}$ Activity 19 (p. 33): You can think of the optical depth as the mean number of absorbers within the cross-section along a photon's path from infinity to height h. The probability of suffering zero absorptions, and thus making it to h, is $exp(-\tau)$. The intensity at h is then an integral from infinity over the expected number of absorptions along the way. Combine that with Eq. (2.18) to derive Eqs. (2.19) and (2.20).

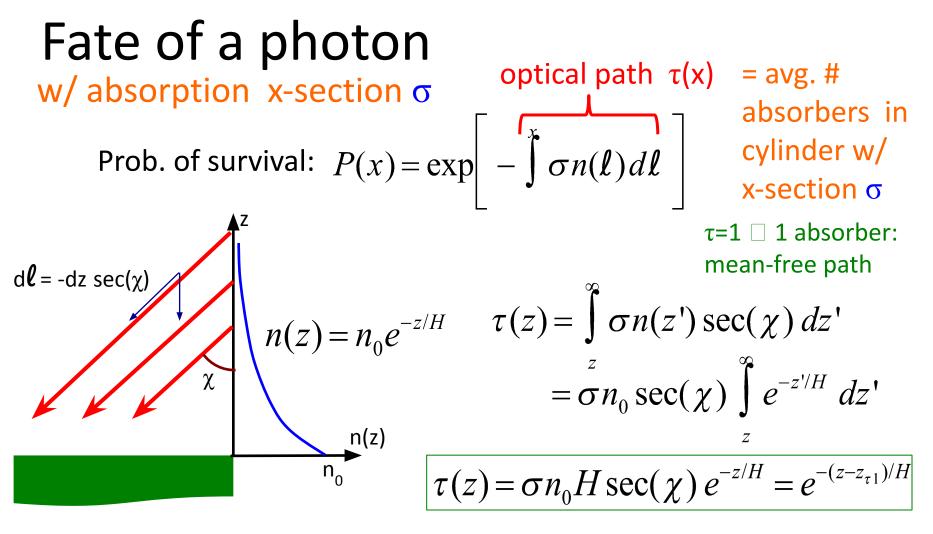
show:
$$au = \sigma_{a} n(h) \frac{H_{p}(h)}{\cos(\chi)},$$
 (2.19)

use:
$$q = \sigma_{a}I(h)n(h)\eta_{i},$$
 (2.18)

.

to derive:
$$q(h) = I_{\infty} \exp\left[-\sigma_{a}n(h)\frac{H_{p}(h)}{\cos(\chi)}\right]\eta_{i}\sigma_{a}n(h).$$

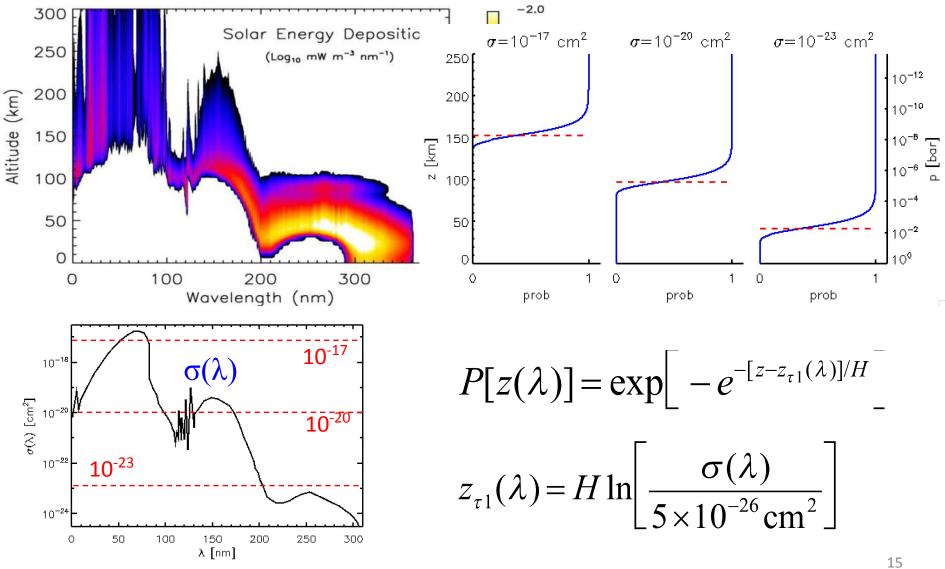




height of
$$\tau = 1$$
: $z_{\tau 1} = H \ln [\sigma n_0 H \sec(\chi)]$

Prob. of survival:

$$P(z) = e^{-\tau(z)} = \exp\left[-e^{-(z-z_{\tau 1})/H}\right]$$



Prob. of surviving to h: $P(h) = e^{-\tau(h)}$

$$\tau(h) = \int_{h}^{\infty} \sigma_a \, n(s) \, ds$$

Energy flux @ h: ∝ # surviving

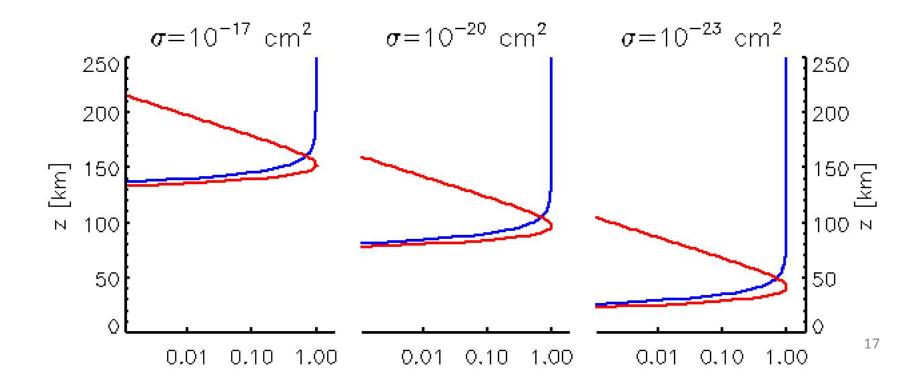
$$I(h) = I_{\infty} e^{-\tau(h)}$$

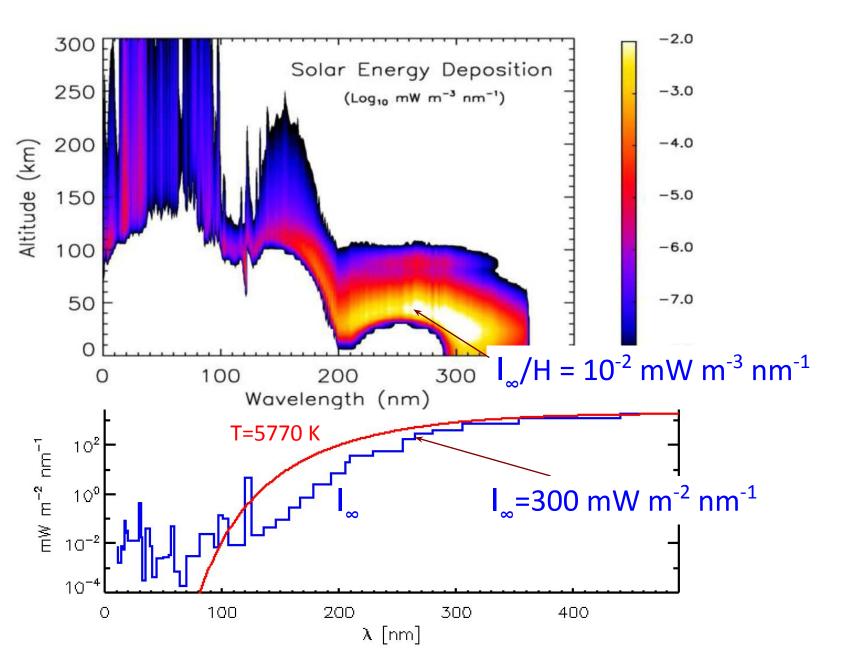
$$q(h) = \sigma_a n(h) \eta_i I(h) = \sigma_a n(h) \eta_i I_{\infty} e^{-\tau(h)}$$
$$\tau = \sigma_a n(h) \frac{H_p(h)}{\cos(\chi)},$$

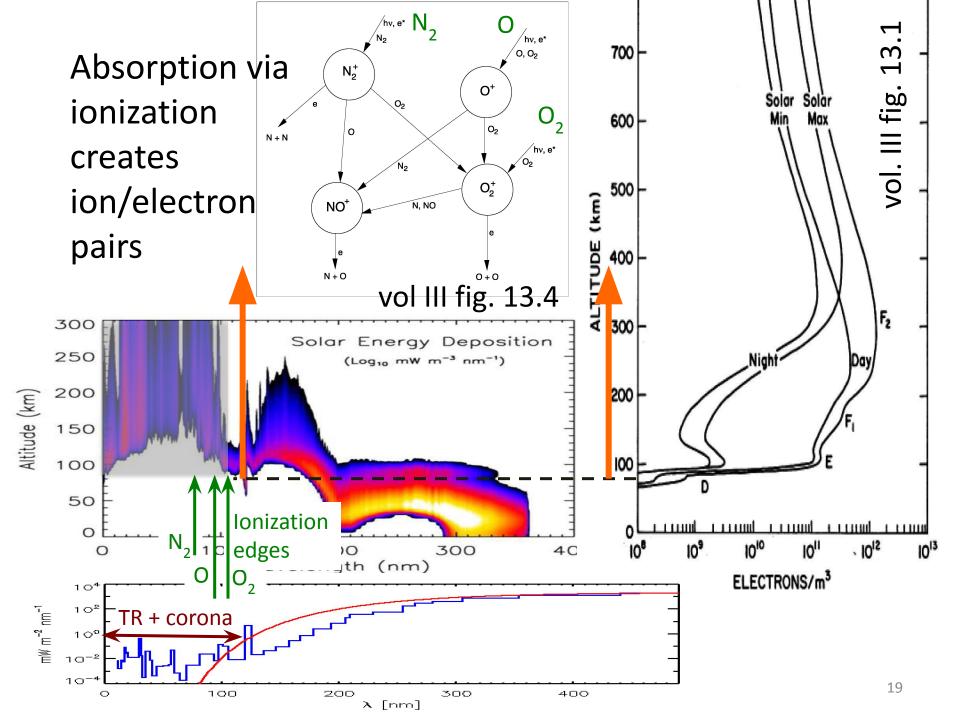
$$q(h) = I_{\infty} \exp\left[-\sigma_{\mathbf{a}} n(h) \frac{H_p(h)}{\cos(\chi)}\right] \eta_{\mathbf{i}} \sigma_{\mathbf{a}} n(h).$$

Radiation intensity & heating

$$q(h) = I_{\infty} \exp\left[-\sigma_{a}n(h)\frac{H_{p}(h)}{\cos(\chi)}\right]\eta_{i}\sigma_{a}n(h).$$
$$= \frac{I_{\infty}}{H_{p}} \exp\left[-e^{-(z-z_{1})/H_{p}} - \frac{(z-z_{1})}{H_{p}}\right]\eta_{i} \quad \begin{array}{c} \text{Chapman}\\ \text{layer} \end{array}$$







Common Physics

 $\tau(h)$

optical depth

$$= \int_{h}^{\infty} \sigma_a \, n(z) \, \frac{dz}{\cos \chi}$$

Planetary atmospheres:

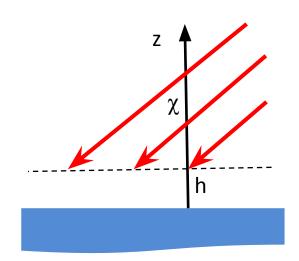
 $e^{-\tau(h)}$ = probability of photon surviving **to** height z = *h* after **originating** at z = ∞

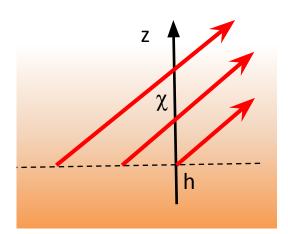
 $\tau=$ 1 identifies average point of death

Stellar atmospheres:

 $e^{-\tau(h)}$ = probability of photon surviving **to** z = ∞ after **originating** at height z = *h*

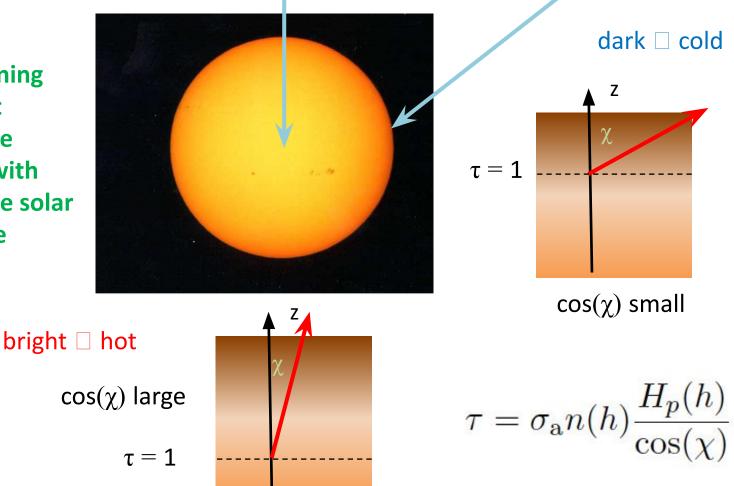
 $\tau = 1$ identifies average point of **birth** (among photons reaching ∞)





Activity 18 (p. 33): Optical depth is an integral over absorption along a line of sight, and thus as useful for incoming as for outgoing radiation. Explain why the layers contributing most to the light from the solar photosphere are geometrically higher as you look away from disk center. What can you infer about the stratification of the solar atmosphere from the fact that the Sun (emitting close to black-body radiation over much of the optical spectrum) is brightest near disk center, darkening towards the limb?

Limb darkening tells us that temperature decreases with height in the solar atmosphere



Shocks

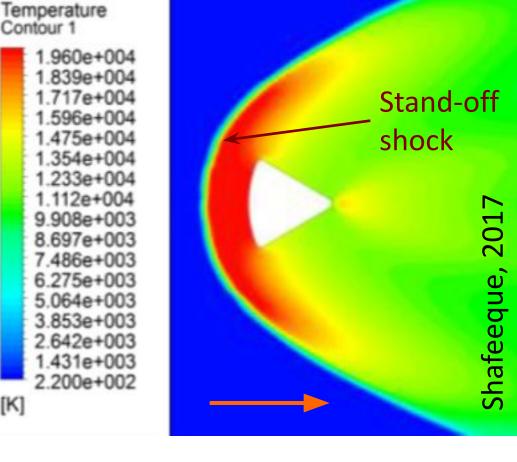


Supersonic $(u_1/c_{s,1}>1)$ flow encounters obstacle:

1. shock slows flow ($u_2 < u_1$)

[K]

2. shock heats fluid ($c_{s,2} > c_{s,1}$) - flow kinetic energy is partly converted to thermal energy □ Makes flow **subsonic** $(u_2/c_{s,2}<1)$ so it can go around obstacle



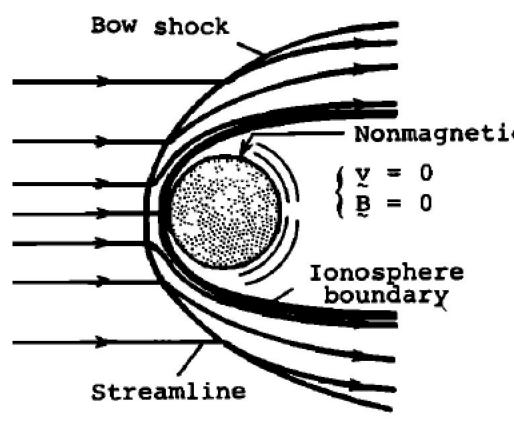
$$u_1 = v_{esc} = 11 \text{ km/s}$$

M = $u_1/c_{s,1} = 33$

Venus or Mars

- No dynamo no B
- Ionosphere
 conducting bdry
- SW– w/ B can't penetrate
- Supersonic flow deflected by obstacle
- Bow shock forms





Simple picture of bow shock

- Ignore pressure from SW **B**
- SW: $u_{\omega}/c_{s,\omega}$, ρ_{ω} , $M_{\omega} \gg 1$
- Standing shock ~ sphere radius= R_s
- Post-shock flow
 - v. ubsonic M \ll 1
 - \Box ~ incompressible w/ u_r(R) = 0

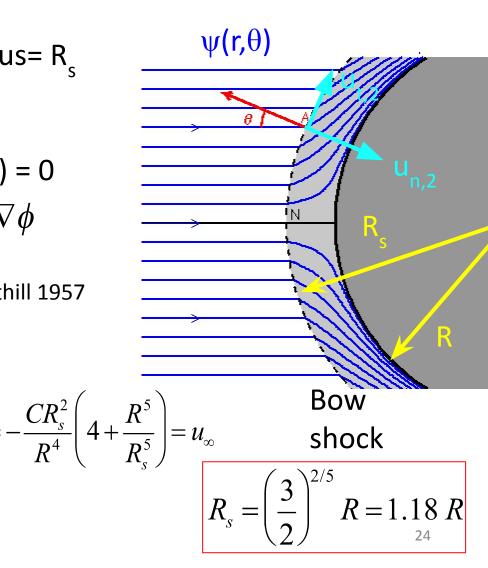
$$\mathbf{u} = \nabla \psi \times \nabla \phi$$

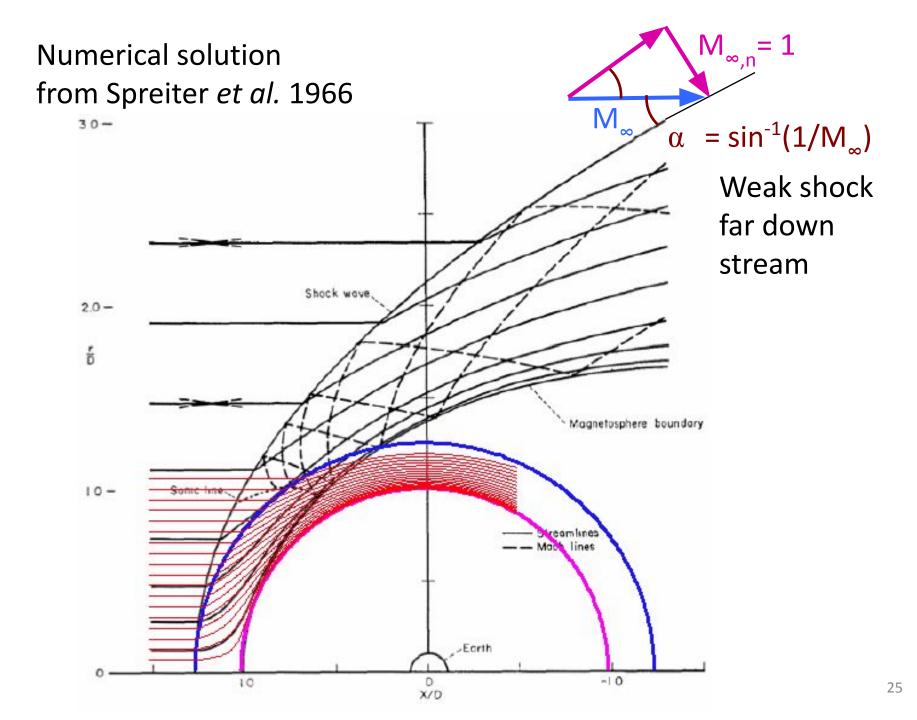
$$\Psi(r,\theta) = C \left(\frac{r^4}{R^4} - \frac{R^2}{r^2} \right) \sin^2 \theta \quad \text{Lighthill 1957}$$

• $\Psi = \Psi / 4$, $\Psi = \Psi$

•
$$u_{n,2} = u_{n,1}/4$$
, $u_{t,2} = u_{t,1}$

$$\frac{u_{r,2}}{\cos\theta} = 2\frac{CR_s^2}{R^4} \left(1 - \frac{R^5}{R_s^5}\right) = -\frac{1}{4}u_{\infty} \quad \frac{u_{\theta,2}}{\sin\theta} =$$





Activity 65 (p. 134): Use Eqs. (5.2) and (5.8-5.12) to show that in the case of a strong shock (in which the thermal energy of the solar wind upstream of the bow shock can be ignored) the temperature just downwind of the bow shock is given by $(3m_p/32k)v_{sw}^2$ for a wind speed of v_{sw} and that the density contrast across the shock is a factor of 4 (show that is true anywhere along the shock).

Use this to estimate the angle from the upwind direction out to which the flow remains supersonic just inside the shock front (remembering that the transverse component of the velocity is unaffected by the shock).

$$\{\rho \mathbf{v} \cdot \hat{\mathbf{e}}_{\perp}\} = 0 , \qquad (5.2)$$

$$\left\{\rho v_{\perp}^{2} + p + \frac{B_{\parallel}^{2}}{8\pi}\right\} = 0$$
(5.8)

$$\left\{\rho\mathbf{v}_{\parallel}v_{\perp} - \frac{\mathbf{B}_{\parallel}B_{\perp}}{4\pi}\right\} = 0 \tag{5.9}$$

$$\left\{ \left(\frac{1}{2}\rho v^2 + \frac{\gamma p}{\gamma - 1} + \frac{B^2}{4\pi}\right) v_{\perp} - (\mathbf{v} \cdot \mathbf{B}) \frac{B_{\perp}}{4\pi} \right\} = 0$$
 (5.10)

$$\{B_{\perp}\} = 0 \tag{5.11}$$

 $\{\mathbf{v}_{\perp} \times \mathbf{B}_{\parallel} + \mathbf{v}_{\parallel} \times \mathbf{B}_{\perp}\} = \mathbf{0} . \qquad (5.12)$

Activity 65 (p. 134): Use Eqs. (5.2) and (5.8-5.12) to show that in the case of a strong shock (in which the thermal energy of the solar wind upstream of the bow shock can be ignored) the temperature just downwind of the bow shock is given by $(3m_p/32k)v_{sw}^2$ for a wind speed of v_{sw} and that the density contrast across the shock is a factor of 4 (show that is true anywhere along the shock).

$$\left\{\rho v_{\perp}^{2} + p + \frac{B_{\perp}^{2}}{8\pi}\right\}^{\beta} \ge 1$$

$$(5.8)$$

$$\rho_{\rm sw} v_{\rm sw}^2 + p_{\rm sw} = \rho_2 v_2^2 + p_2 = \rho_2 v_2^2 + \frac{k_b}{(m_p/2)} \rho_2 T_2$$

$$T_2 = \frac{m_p}{2k_b} \left[\frac{\rho_{\rm sw}}{\rho_2} v_{\rm sw}^2 - v_2^2 \right] = \frac{m_p}{2k_b} \left[\frac{1}{4} v_{\rm sw}^2 - \frac{1}{16} v_{\rm sw}^2 \right] = \frac{3m_p}{32k_b} v_{\rm sw}^2$$

Use this to estimate the angle from the upwind direction out to which the flow remains supersonic just inside the shock front (remembering that the transverse component of the velocity is unaffected by the shock).

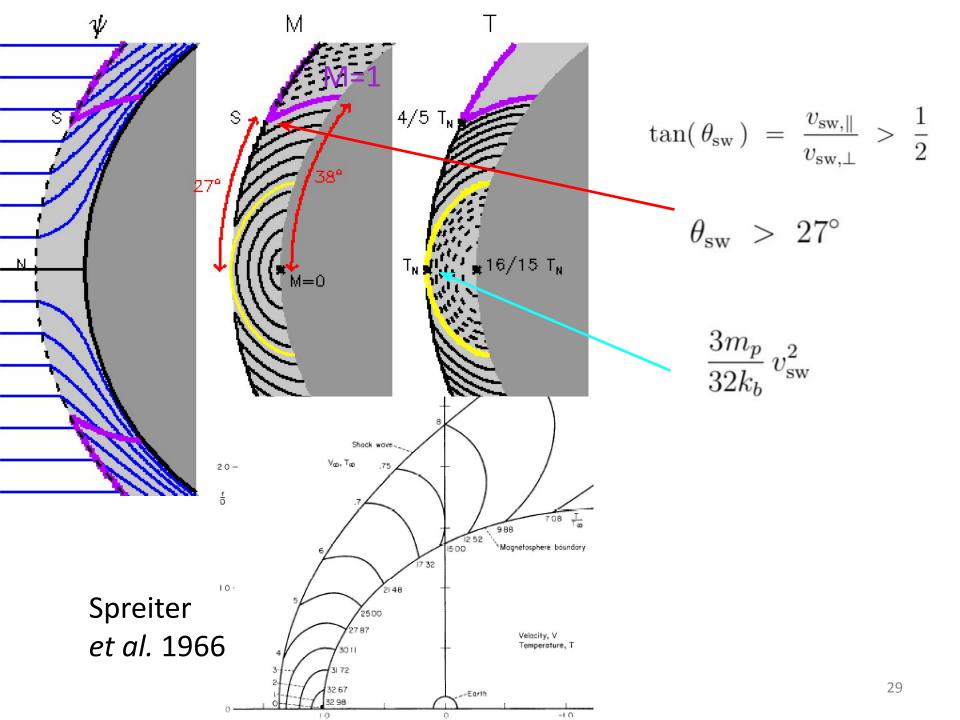
$$\frac{k_b T_2}{m_p} = \frac{3}{32} v_{\text{sw},\perp}^2$$

$$c_{s,2}^2 = \frac{5k_b T_2}{3(m_p/2)} = \frac{5}{16} v_{\text{sw},\perp}^2 = 5 v_{2,\perp}^2$$

remains
$$v_{
m sw,\parallel}^2 = v_{2,\parallel}^2 > \frac{4}{5}c_{s,2}^2 = 4v_{2,\perp}^2 = \frac{1}{4}v_{
m sw,\perp}^2$$
 supersonic:

$$\tan(\theta_{\rm sw}) = \frac{v_{\rm sw,\parallel}}{v_{\rm sw,\perp}} > \frac{1}{2}$$

$$\theta_{\rm sw}~>~27^{\circ}$$



Activity 66 (p. 135): What is the expression for the temperature of the gas at the stagnation point on the magnetopause assuming that the flow continues adiabatically after the shock (i.e., that it conserves the sum of bulk kinetic and thermal energies)? What is the value for $v_{sw} = 800$ km/s.

$$\frac{1}{2}v^{2} + \frac{5}{2}\frac{p}{\rho} = \frac{1}{2}v^{2} + \frac{5}{2}\frac{k_{b}}{(m_{p}/2)}T = \text{const.}$$

$$= \frac{1}{32}v_{\text{sw}}^{2} + 5\frac{k_{b}}{m_{p}}T_{N} = 0 + 5\frac{k_{b}}{m_{p}}T_{s}$$

$$\frac{1}{32}v_{\text{sw}}^{2} + 5\frac{k_{b}}{m_{p}}T_{N} = 0 + 5\frac{k_{b}}{m_{p}}T_{s}$$

$$\frac{4/5}{10}T_{N}$$

$$\frac{4/5}{10}T_{N}$$

$$\frac{4}{5}T_{N}$$

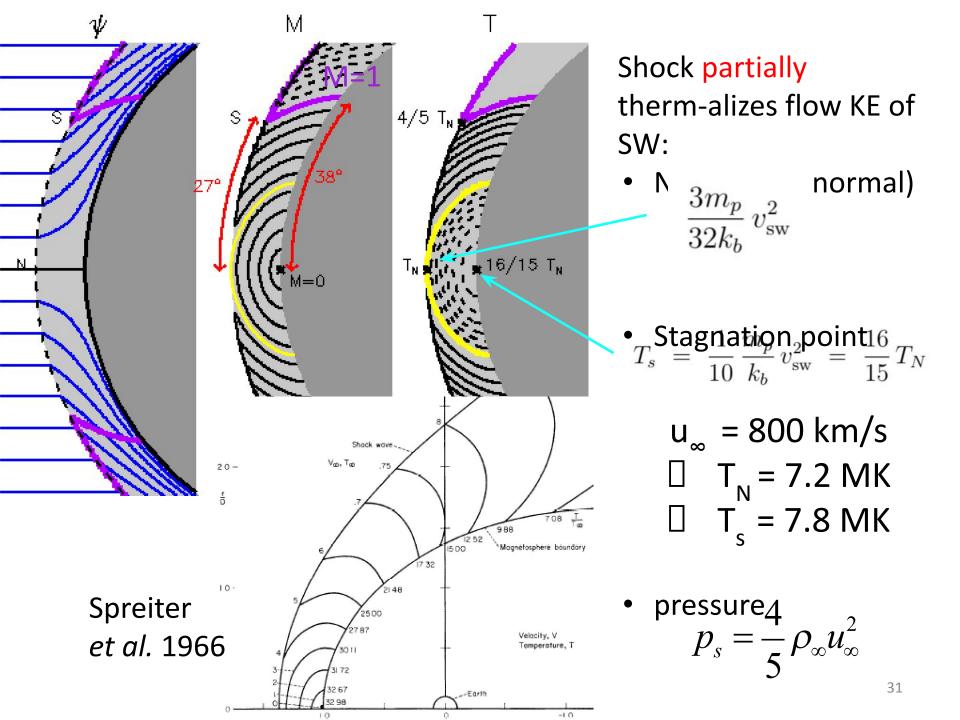
$$\frac{1}{32}v_{\text{sw}}^{2} + 5\frac{3}{32}v_{\text{sw}}^{2} = \frac{1}{2}v_{\text{sw}}^{2}$$

$$T_{s} = \frac{1}{10}\frac{m_{p}}{k_{b}}v_{\text{sw}}^{2} = \frac{16}{15}T_{N}$$

$$\frac{3m_{p}}{32k_{b}}v_{\text{sw}}^{2}$$

$$T_{N} = \frac{16}{16}(15)T_{N}$$

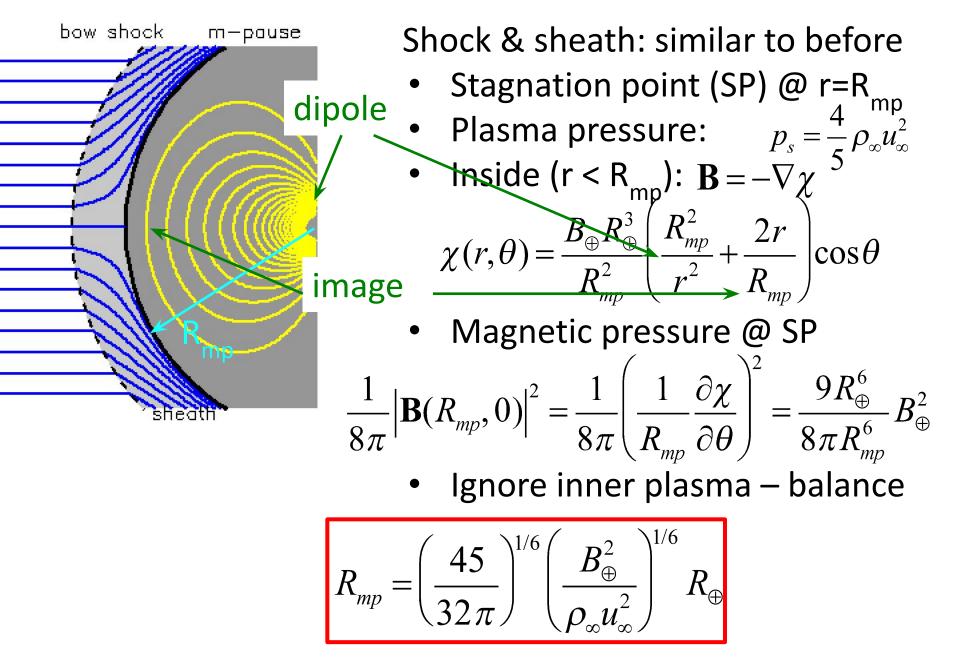
$$\frac{16}{15}T_{N}$$



Wind @ Magnetized Planets Earth, Jupiter, Saturn, ...

Bow Shock Magnetopause Planetary B prevents SW from Magnetotail reaching Solar Wind ionosphere Current Sheet () (*) (.) SW deflected by magnetosphere "squishy" obstacle Magnetosheath

Hughes (*cf.* vol. I fig. 10.1) 32



Chapman-Ferraro Distance

$$R_{\rm mp} = \frac{(\xi\mu_{\rm p})^{1/3}}{(8\pi\rho_{\rm sw}v_{\rm sw}^2)^{1/6}}$$
(5.22)

$$R_{mp} = \left(\frac{45}{32\pi}\right)^{1/6} \left(\frac{B_{\oplus}^2}{\rho_{\infty} u_{\infty}^2}\right)^{1/6} R_{\oplus}$$

Earth:
$$B_{\oplus} = 0.3 G$$

steady slow wind: $\rho_{sw} = 10^{-23} \text{ g/cm}^3$, $u_{sw} = 400 \text{ km/s}$ $R_{mp} = 0.875 (0.09 / 1.6 \times 10^{-8})^{1/6} R_{\oplus} = 11.7 R_{\oplus}$ steady fast wind: $\rho_{sw} = 10^{-23} \text{ g/cm}^3$, $u_{sw} = 800 \text{ km/s}$

$$R_{mp} = 0.875 (0.09 / 6.4 \times 10^{-8})^{1/6} R_{\oplus} = 9.3 R_{\oplus}$$

Acitivity 68 (p. 135): With the fastest recorded solar-wind gusts at $v_{sw} = 2500$ km/s, what is the required plasma density to push the magnetopause to within geosynchronous orbit according to Eq. (5.22)?

$$R_{\rm mp} = \frac{(\xi\mu_{\rm p})^{1/3}}{(8\pi\rho_{\rm sw}v_{\rm sw}^2)^{1/6}}$$
(5.22)
$$R_{mp} = \left(\frac{45}{32\pi}\right)^{1/6} \left(\frac{B_{\oplus}^2}{\rho_{\infty}u_{\infty}^2}\right)^{1/6} R_{\oplus} \qquad \rho_{\infty} = \frac{45}{32\pi} \frac{B_{\oplus}^2}{u_{\infty}^2} \left(\frac{R_{\oplus}}{R_{\rm mp}}\right)^6$$

geosynchronous orbit (P=24 h *vs.* 1.5 h @ R_{\oplus}) R_{gs} = (24/1.5)^{2/3} R_{\oplus} = 6.3 R_{\oplus}

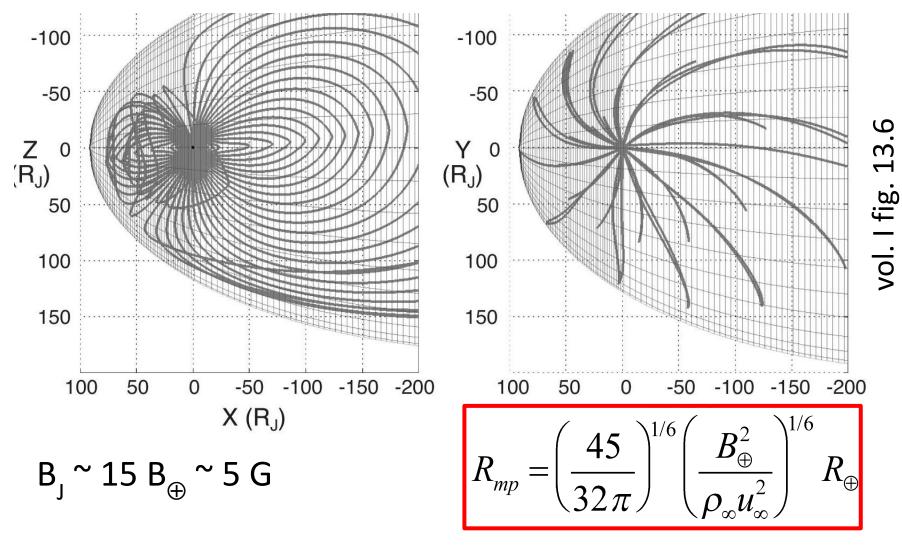
u_{sw} = 2500 km/s from statement

4 /0

$$\rho_{sw} = 0.45 (0.09 / 6.2 \times 10^{16}) (6.3)^{-6}$$

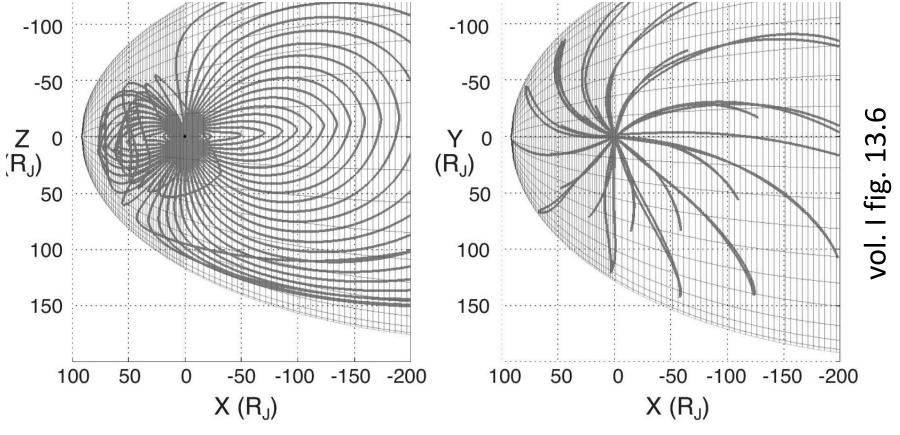
= 10⁻²³ g/cm³ = m_p × 6 cm⁻³

Other planets... same story

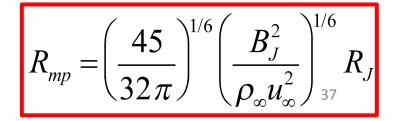


Q: how do u_{∞} & ρ_{∞} @ Jupiter compare to @ Earth?

Other planets... same story

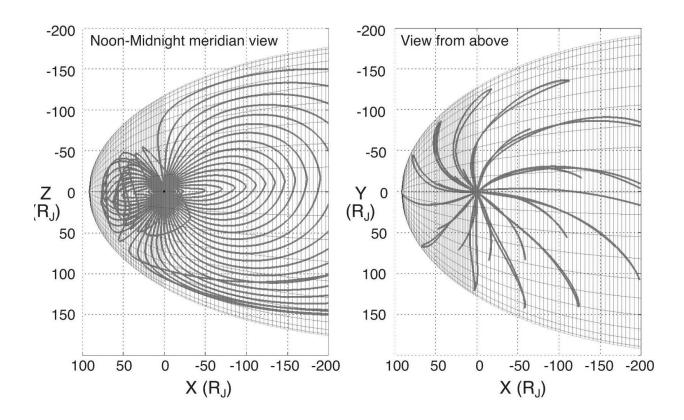


B_J ~ 15 B_⊕ ~ 5 G ; ρ_{∞} ~ 0.04 $\rho_{\infty, \oplus}$ □ Jupiter's magnetopause: R_{mp,J} ~ 50 R_J = 3.5 × 10¹¹ cm



Activity 74 (p. 145): Many so-called 'hot Jupiters' have been found among the exoplanet population: giant planets that orbit very close to their parent stars. What would the estimated magnetopause distance R_{CF;hJ} be if Jupiter were orbiting the present-day Sun at 0.05AU?

For a younger Sun (see Ch. 12) the solar wind would have been stronger, pushing R_{CF;hJ} to below the orbital radius of Ganymede; describe what that would mean for this 'hot-Ganymede' moon?



Activity 74 (p. 145): Many so-called 'hot Jupiters' have been found among the exoplanet population: giant planets that orbit very close to their parent stars. What would the estimated magnetopause distance R_{CF;hJ} be if Jupiter were orbiting the present-day Sun at 0.05AU?

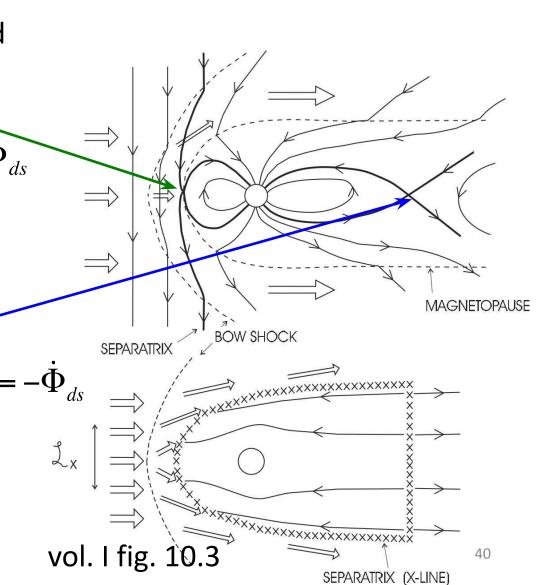
$$R_{mp} = \left(\frac{45}{32\pi}\right)^{1/6} \left(\frac{B_J^2}{\rho_{\infty} u_{\infty}^2}\right)^{1/6} R_J$$

$$B_{J} \sim 15 B_{\oplus} \sim 5 G$$
; $\rho_{\infty} \sim (20)^{2} \rho_{\infty, \oplus} \sim 4 \times 10^{-21} \text{ g/cm}^{3}$
 $u_{\infty} \sim 400 \text{ km/s} = 4 \times 10^{7} \text{ cm/s}$
 $R_{mp} = 0.875 (25 / 6.4 \times 10^{-6})^{1/6} R_{\oplus} = 11 R_{J}$

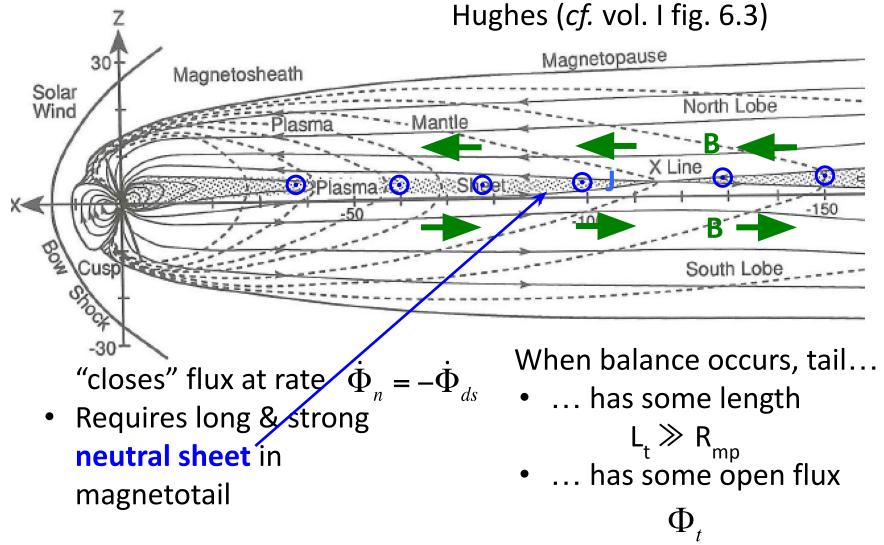
But not all of Earth's field stays confined to m-sphere

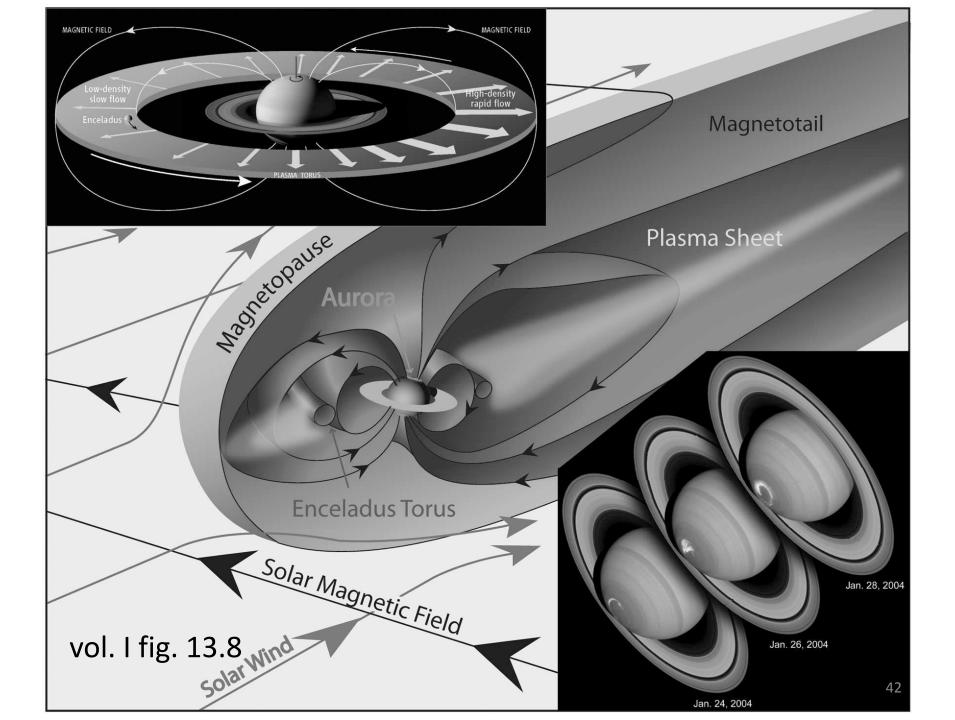
Reconnection with SW field (consider southward IMF)

- Creates "open" flux connected to poles @ $\dot{\Phi}_{ds}$
- SW sweeps flux downstream – into magnetotail
- Steady state only when reconnection in tail "closes" flux at rate $\dot{\Phi}_n = -\dot{\Phi}_{ds}$
- Requires long & strong neutral sheet in magnetotail

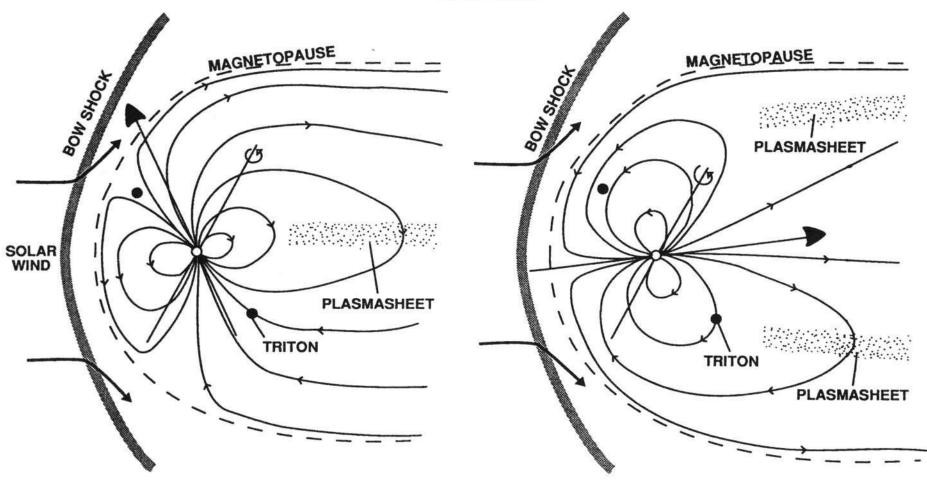


But not all of Earth's field stays confined to m-sphere



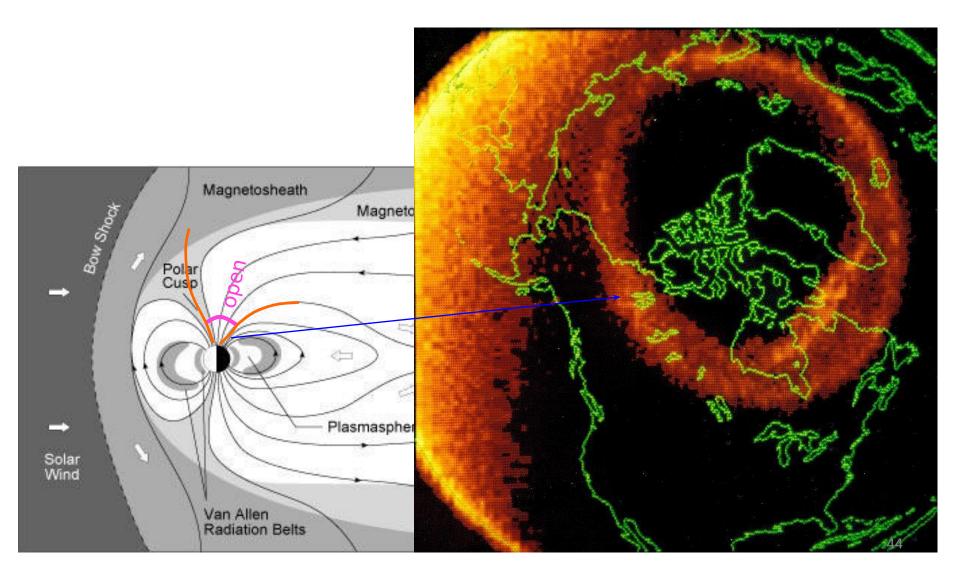


NEPTUNE



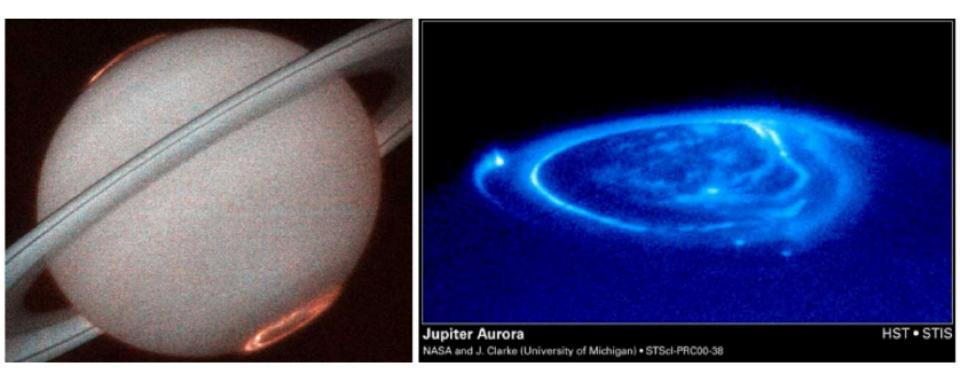
vol. I fig. 13.10

closed/open boundary maps down to "auroral oval"



$$\begin{split} & \int_{0}^{4} \int_{0}^{4}$$

Other auroral ovals



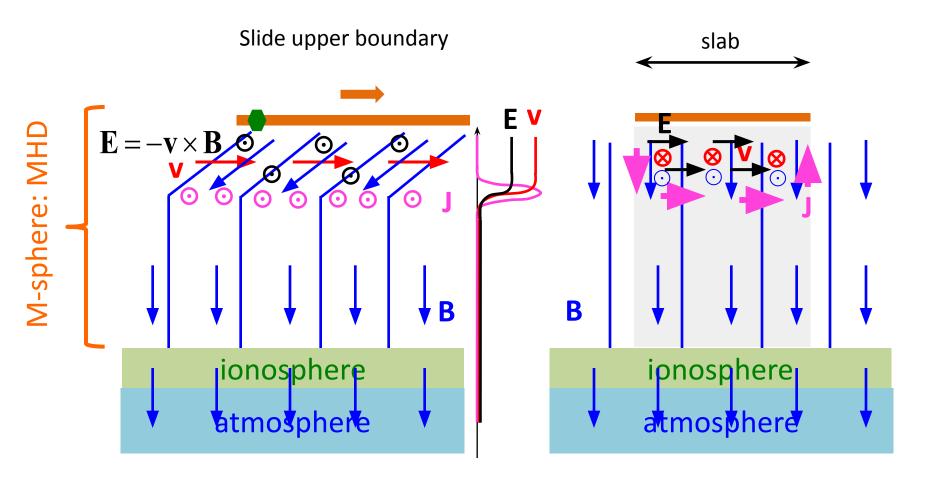
vol. I fig. 2.9

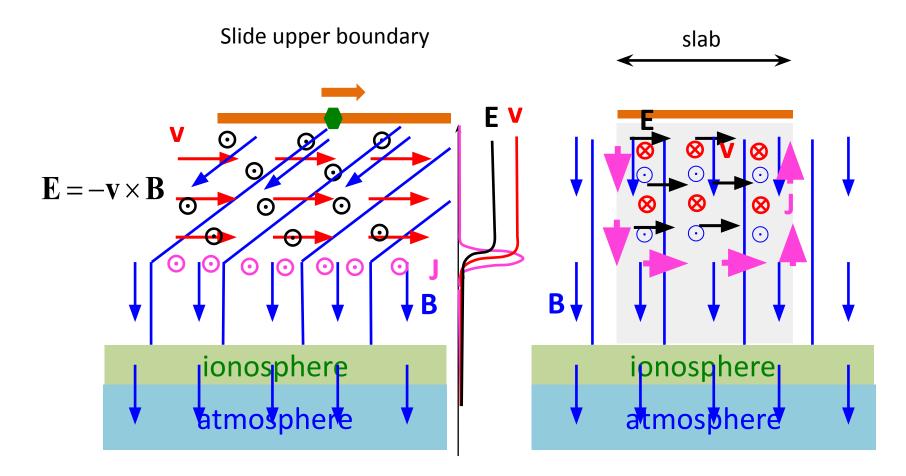
Convection: magnetosphere meets ionosphere

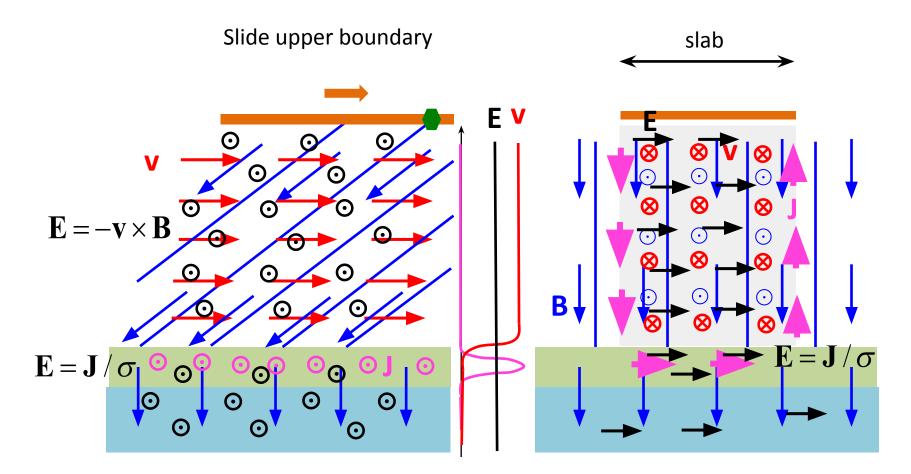
field lines are frozen to M-spheric plasma. motion sweeps filed lines back

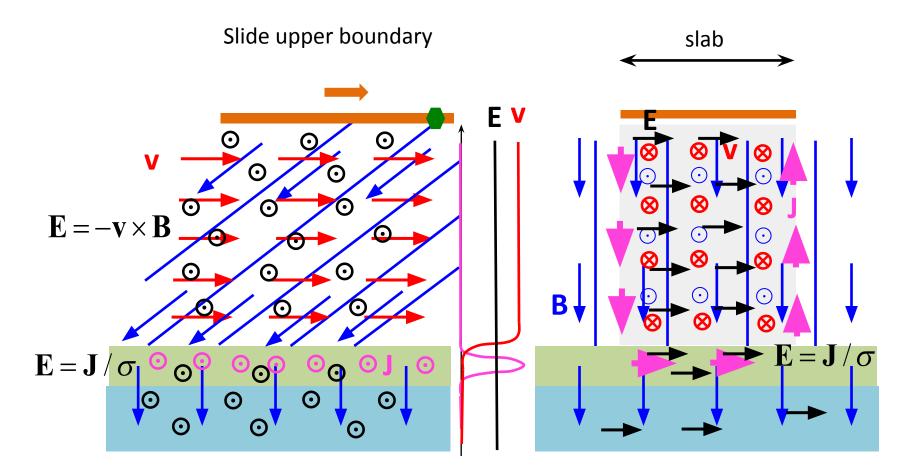
> BUT: atmosphere & solid crust are insulators – field lines are imaginary there

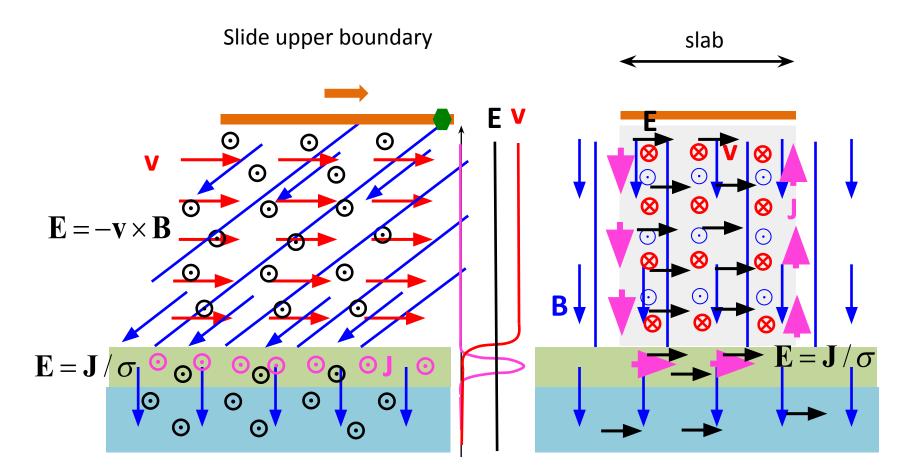
Objection: field lines are also frozen into liquid core – ends cannot be moved

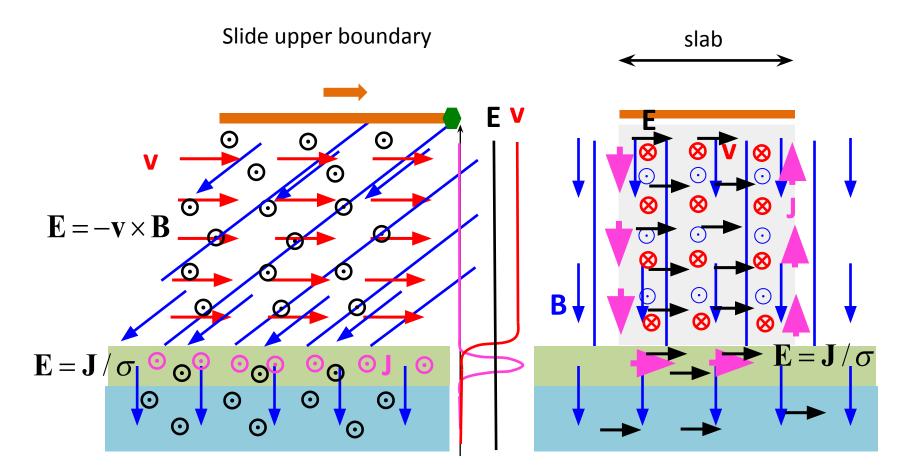


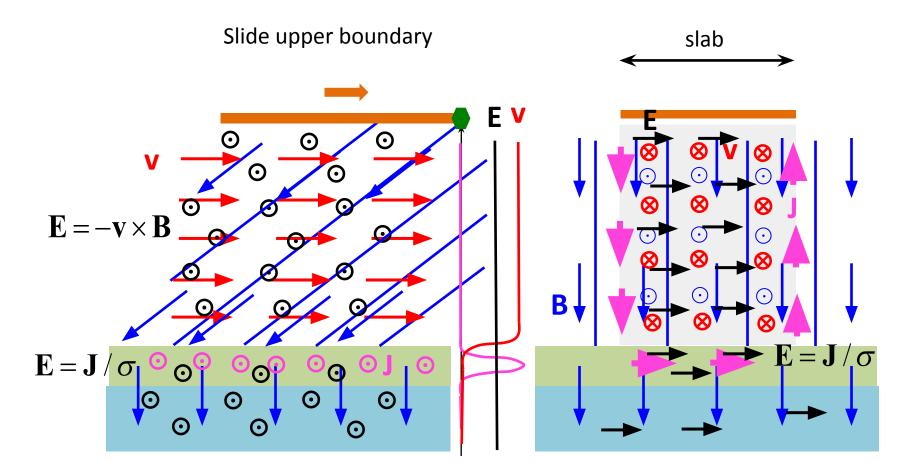


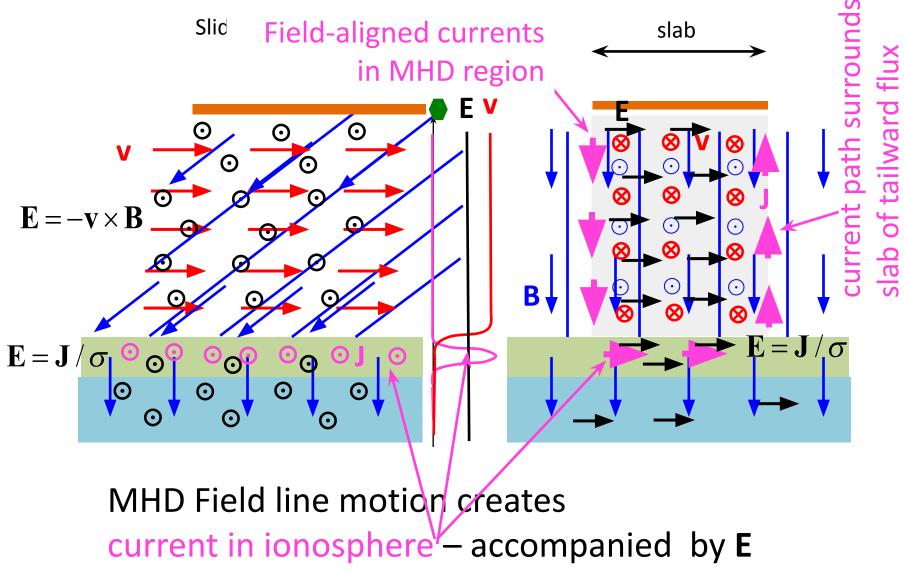












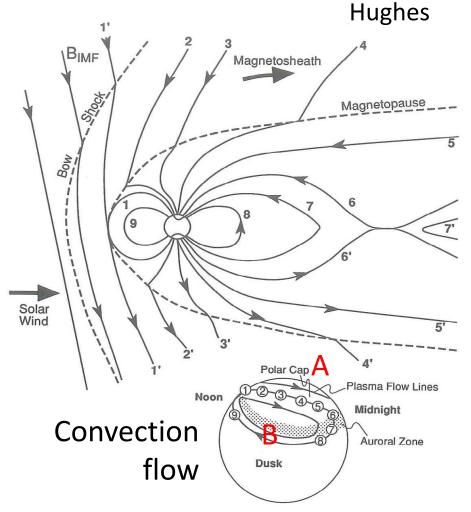
Convection: magnetosphere meets ionosphere

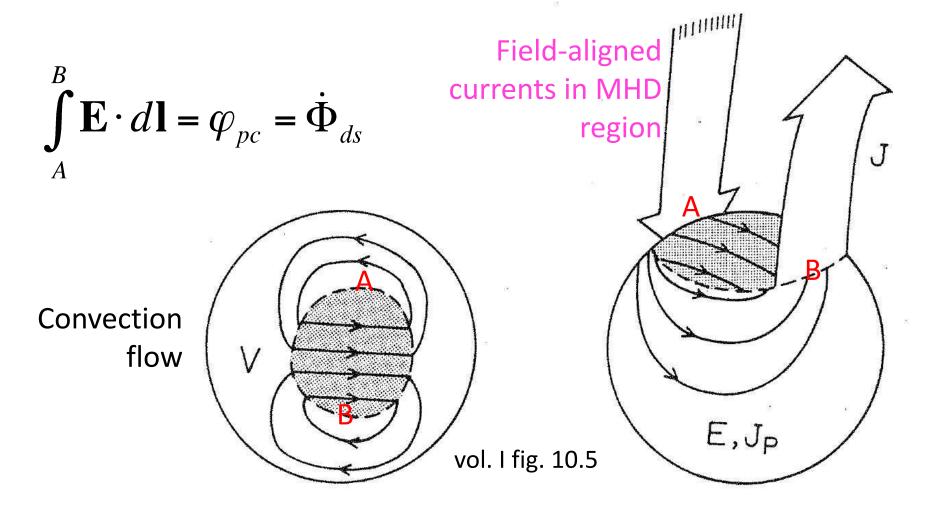
MHD motions drag footpoints across polar caps and back around to day side

Integrate* **E** across polar cap:

$$\int_{A}^{B} \mathbf{E} \cdot d\mathbf{l} = \varphi_{pc} = \dot{\Phi}_{ds}$$

Really an EMF – but called "cross polar cap potential"





$$\varphi_{pc} = 50 \text{ kV}$$

= 5 × 10¹² Mx/s recycle in Φ_t in ~ 5 hours

Summary

- Ionospheres created by EUV & X-rays from Sun's TR and corona
- Diminish during night lower during solar minimum
- SW deflected by ionospheres of unmagnetized planets (Venus & Mars)
- SW deflected by magnetospheres
- Magnetotail created by reconnection with solar wind magnetic field