Particle Energization

Amitava Bhattacharjee Center for Heliophysics Department of Astrophysical Sciences Princeton Plasma Physics Laboratory Princeton University

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Drivers of space Weather

Space weather refers to the variable conditions on the Sun and in space that can influence performance and reliability of space and ground-based technological systems, and endanger life or health

> Electromagnetic Radiation (R-scale)

Energetic Charged Particles (S-scale)

Magnetic Field (G-scale)

Courtesy: W. Murtagh, this school

Particle acceleration: physical mechanisms

- Shocks
- Wave-particle interactions
- Reconnection and turbulence

Why are shocks important?

→ Collisionless shocks are common in astrophysics

→ Shocks are known to produce strongly non-thermal particle distributions

➔ Note that the concept, and even the existence, of a "collisionless" shock is not obvious

Until the Earth bow shock was detected by spacecraft there was debate as to whether or not a "shock" in the solar wind would exist.

Tycho' s Supernova Remnant



Exploded in 1572 and studied by Tycho Brahe

This is a Chandra X-ray image

Shock heated gas inside 3000 km/s blast wave (filamentary blue)

Blue is nonthermal X-ray emission (synchrotron) from shock accelerated relativistic electrons.

No doubt that TeV electrons are produced by this shock !!

Evidence for TeV ions is less direct but very strong.

http://chandra.harvard.edu/photo/2005/tycho/





Collisionless plasmas :

- Density, ρ, is low enough so particle-particle collisions are rare
 → e.g., in solar wind, particle-particle mean-free-path, Lρ, is on the order of Sun-Earth distance
- 2) Turbulent magnetic field
 - Charged particles pitch-angle scatter in turbulence and have effective mfp, L_B << Lρ</p>

We see "thin" structures in solar wind and interstellar medium : e.g., planetary bow shocks and SNR shocks

The length scale of these structures can be orders of magnitude smaller than the collisional mfp





If B-field is weak enough, and particle flux large enough, particles will distort the field :



Turbulent B

Particles pitch-angle scatter and turn around → can define a collisionless mean free path. This "collision" is nearly elastic in frame of B-field

Shock Solutions



It is easy to show that aside from the trivial solution in which all quantities remain spatially constant, there is a discontinuous solution:

$$\begin{split} \frac{\rho_2}{\rho_1} &= \frac{u_1}{u_2} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2} \\ \text{M}_1 \text{ is the upstream} &\qquad \qquad \frac{p_2}{p_1} = \frac{2\gamma M_1^2}{\gamma+1} - \frac{\gamma-1}{\gamma+1} \\ \frac{T_2}{T_1} &= \frac{[2\gamma M_1^2 - \gamma(\gamma-1)][(\gamma-1)M_1^2+2]}{(\gamma+1)^2 M_1^2} \end{split}$$

Strong Shocks M₁²>>1

In the limit of strong shock fronts these expressions get substantially simpler and one has:

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$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{\gamma + 1}{\gamma - 1}$$
$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2}{\gamma + 1}$$
$$\frac{T_2}{T_1} = \frac{2\gamma(\gamma - 1)}{(\gamma + 1)^2} M_1^2, \quad T_2 = 2\frac{\gamma - 1}{(\gamma + 1)^2} m u_1^2$$

ONE CAN SEE THAT SHOCKS BEHAVE AS VERY EFFICENT HEATING MACHINES IN THAT A LARGE FRACTION OF THE INCOMING RAM PRESSURE IS CONVERTED TO INTERNAL ENERGY OF THE GAS BEHIND THE SHOCK FRONT...

	STROPHYSICAL JOUR INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS	
VOLUME 119	JANUARY 1954	NUMBER 1
	GALACTIC MAGNETIC FIELDS AND THE ORIGIN OF COSMIC RADIATION*	Fermi talks of regions of large field strength with sharp discontinuities
	E. FERMI Institute for Nuclear Studies, University of Chicago Received Settlember 11, 1953	"jaws of the trap"

A particle that finds itself between two such regions will be trapped on the stretch of line of force comprised between them. When this happens, the energy of the particle will change with time at a rate much faster than usual. It will decrease or increase according to whether the jaws of the trap move away from or toward each other.

Recently de Hoffmann and Teller⁵ have discussed the features of magnetohydrodynamic shocks. They show, in particular, that <u>at a shock front</u> sudden variations in direction and intensity of the field are likely to occur. One is tempted to identify the boundaries of many clouds of the galactic diffuse matter with shock fronts. If this is correct, we have a source of magnetic discontinuities.

Fermi postulated that shocks could "trap" particles. This is first-order Fermi acceleration but Fermi did not derive the famous "Universal" power law First-order Fermi acceleration mechanism Also called Diffusive Shock Acceleration (DSA)

Some review papers:

Axford 1981, Drury 1983, Blandford & Eichler 1987, Jones & Ellison 1991, Berezhko & Ellison 1999, Malkov & Drury 2001, Bykov 2004

Fermi 1949 → response to Hannes Alfven's solar model Fermi 1954 → connection to shocks

 Discovery papers for first-order Fermi mechanism in shocks: Krymskii (1976), Axford, Leer & Skadron (1977), Bell (1978), Blandford & Ostriker (1978)



So called "Universal" power law for relativistic particles (in momentum)



Parker 1965: convection-diffusion (aka "confusion-defection") equation

THE TRANSPORT EQUATION APPROACH

INTEGRATION OF THIS SIMPLE EQUATION GIVES:

			<i>/</i>	$-3u_1$
$f_{n}(p) =$	<i>3u</i> ₁	N _{inj}	_ <u>p</u> _	$u_1 - u_2$
$f_0(p) =$	$u_1 - u_2$	$4\pi p_{inj}^2$	p_{inj})

DEFINE THE COMPRESSION FACTOR $r=u_1/u_2 \rightarrow 4$ (strong shock)

THE SLOPE OF THE SPECTRUM IS

 $\frac{3u_1}{u_1-u_2} = \frac{3}{1-1/r} \to 4 \quad \text{if } r \to 4$ Notice that: $N(p)dp = 4\pi p^2 f(p)dp \to N(p) \propto p^{-2}$

- 1. THE SPECTRUM OF ACCELERATED PARTICLES IS A POWER LAW EXTENDING TO INFINITE MOMENTA
- 2. THE SLOPE DEPENDS UNIQUELY ON THE COMPRESSION FACTOR AND IS INDEPENDENT OF THE DIFFUSION PROPERTIES
- 3. INJECTION IS TREATED AS A FREE PARAMETER WHICH DETERMINES THE NORMALIZATION



Quasilinear Theory (Yakimenko 1963; Kennel & Engelmann 1966; Stix 1992)

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \nabla f + \frac{q}{m} \left(\boldsymbol{E} + \frac{1}{c} \, \boldsymbol{v} \times \boldsymbol{B} \right) \cdot \nabla_{\boldsymbol{v}} f = 0 \tag{1}$$

 $f = f_0(x, v, t) + f_1(x, v, t)$ $B = B_0 + B_1(x, t)$ $E = E_1(x, t)$

 B_0 is a uniform background magnetic field.

 f_0 is the background or equilibrium plasma distribution function

 E_1 and B_1 represent a collection of waves, which could be slowly growing or slowly decaying. We're going to treat E_1 and B_1 as known.

 f_1 represents the response of the plasma to these waves

Our goal is to find how f₀ varies over times much longer than the wave periods.

(Yakimenko 1963; Kennel & Engelmann 1966; Stix 1992)

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \nabla f + \frac{q}{m} \left(\boldsymbol{E} + \frac{1}{c} \, \boldsymbol{v} \times \boldsymbol{B} \right) \cdot \nabla_{\boldsymbol{v}} f = 0 \tag{1}$$

 $f = f_0(x, v, t) + f_1(x, v, t)$ $B = B_0 + B_1(x, t)$ $E = E_1(x, t)$

$$\frac{\partial f_0}{\partial t} + \boldsymbol{v} \cdot \nabla f_0 + \frac{q}{mc} \left(\boldsymbol{v} \times \boldsymbol{B}_0 \right) \cdot \nabla_{\boldsymbol{v}} f_0 = 0 \rightarrow f_0(\boldsymbol{x}, \boldsymbol{v}, t) = f_0(\boldsymbol{v}_\perp, \boldsymbol{v}_\parallel)$$

Here, we are using cylindrical coordinates $(v_{\perp}, v_{\parallel}, \theta)$ in velocity space, where the cylindrical axis is aligned with B_0 . Soon, we will set $B_0 \to B_0 \hat{z}$, and v_{\parallel} will become v_z .

(technically, f₀ varies in time over time scales much longer than the wave periods. But here the variable t describes time variations over times comparable to the wave periods, and f₀ doesn't vary on this "fast" time scale.

(Yakimenko 1963; Kennel & Engelmann 1966; Stix 1992)

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \nabla f + \frac{q}{m} \left(\boldsymbol{E} + \frac{1}{c} \, \boldsymbol{v} \times \boldsymbol{B} \right) \cdot \nabla_{\boldsymbol{v}} f = 0 \tag{1}$$

 $f = f_0(x, v, t) + f_1(x, v, t)$ $B = B_0 + B_1(x, t)$ $E = E_1(x, t)$

$$\frac{\partial f_0}{\partial t} + \boldsymbol{v} \cdot \nabla f_0 + \frac{q}{mc} \left(\boldsymbol{v} \times \boldsymbol{B}_0 \right) \cdot \nabla_{\boldsymbol{v}} f_0 = 0 \to f_0(\boldsymbol{x}, \boldsymbol{v}, t) = f_0(\boldsymbol{v}_\perp, \boldsymbol{v}_\parallel)$$

$$\frac{\partial f_1}{\partial t} + \boldsymbol{v} \cdot \nabla f_1 + \frac{q}{mc} \left(\boldsymbol{v} \times \boldsymbol{B}_0 \right) \cdot \nabla_{\boldsymbol{v}} f_1 = -\frac{q}{m} \left(\boldsymbol{E}_1 + \frac{1}{c} \, \boldsymbol{v} \times \boldsymbol{B}_1 \right) \cdot \nabla_{\boldsymbol{v}} f_0$$

Difficult-looking equation. How do we solve this equation for $f_1(\mathbf{x}, \mathbf{v}, t)$ if we know E_1 , B_1 , B_0 , and f_0 ?

Method of characteristics!

(Yakimenko 1963; Kennel & Engelmann 1966; Stix 1992)

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \nabla f + \frac{q}{m} \left(\boldsymbol{E} + \frac{1}{c} \, \boldsymbol{v} \times \boldsymbol{B} \right) \cdot \nabla_{\boldsymbol{v}} f = 0 \tag{1}$$

 $f = f_0(x, v, t) + f_1(x, v, t)$ $B = B_0 + B_1(x, t)$ $E = E_1(x, t)$

$$\frac{\partial f_0}{\partial t} + \boldsymbol{v} \cdot \nabla f_0 + \frac{q}{mc} \left(\boldsymbol{v} \times \boldsymbol{B}_0 \right) \cdot \nabla_{\boldsymbol{v}} f_0 = 0 \to f_0(\boldsymbol{x}, \boldsymbol{v}, t) = f_0(v_{\perp}, v_{\parallel})$$

$$\frac{\partial f_1}{\partial t} + \boldsymbol{v} \cdot \nabla f_1 + \frac{q}{mc} \left(\boldsymbol{v} \times \boldsymbol{B}_0 \right) \cdot \nabla_{\boldsymbol{v}} f_1 = -\frac{q}{m} \left(\boldsymbol{E}_1 + \frac{1}{c} \, \boldsymbol{v} \times \boldsymbol{B}_1 \right) \cdot \nabla_{\boldsymbol{v}} f_0$$

Let $f_1(\boldsymbol{x}, \boldsymbol{v}, t) = f_1(\boldsymbol{x}(t), \boldsymbol{v}(t), t)$, where $d\boldsymbol{x}/dt = \boldsymbol{v}, d\boldsymbol{v}/dt = (q/mc)(\boldsymbol{v} \times \boldsymbol{B}_0)$:

$$\frac{\mathrm{d}f_1}{\mathrm{d}t} = -\frac{q}{m} \left(\boldsymbol{E}_1(\boldsymbol{x}(t), t) + \frac{1}{c} \, \boldsymbol{v}(t) \times \boldsymbol{B}_1(\boldsymbol{x}(t), t) \right) \cdot \nabla_v f_0 \tag{2}$$

Solve for $\boldsymbol{x}(t)$ and $\boldsymbol{v}(t)$; integrate (2) to find f_1 ; plug f_1 into 3^{rd} term in (1); and average.

Single Particle Motion in a Uniform Magnetic Fleld



(Yakimenko 1963; Kennel & Engelmann 1966; Stix 1992)

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \nabla f + \frac{q}{m} \left(\boldsymbol{E} + \frac{1}{c} \, \boldsymbol{v} \times \boldsymbol{B} \right) \cdot \nabla_{\boldsymbol{v}} f = 0, \tag{1}$$

$$f = f_0(v) + f_1(x, v, t)$$
 $B = B_0 + B_1$ $E = E_1$

Solve for f_1 in terms of f_0 , E_1 , B_1 ; plug f_1 into 3^{rd} term in (1); average:

$$\frac{\partial f}{\partial t} = \lim_{V \to \infty} \sum_{n=-\infty}^{\infty} \frac{\pi q^2}{m^2} \int \frac{d^3 k}{(2\pi)^3 V v_{\perp}} G v_{\perp} \delta(\omega_{kr} - k_{\parallel} v_{\parallel} - n\Omega) \psi_{n,k}|^2 G f,$$
$$G \equiv \left(1 - \frac{k_{\parallel} v_{\parallel}}{\omega_{kr}}\right) \frac{\partial}{\partial v_{\perp}} + \frac{k_{\parallel} v_{\perp}}{\omega_{kr}} \frac{\partial}{\partial v_{\parallel}}$$
$$\psi_{n,k} = \frac{1}{\sqrt{2}} \left[E_{k,r} e^{i\phi} J_{n+1}(\sigma) + E_{k,l} e^{-i\phi} J_{n-1}(\sigma) \right] + \frac{v_{\parallel}}{v_{\perp}} E_{kz} J_n(\sigma) \qquad \sigma = k_{\perp} v_{\perp} / \Omega$$

Wave-Particle Resonance Condition



- Consider $\delta \vec{E} = \delta \vec{E}_0 \cos(\vec{k} \cdot \vec{x} \omega t)$
- Let $\vec{x} = \vec{x}' + v_{\parallel}\hat{b}t$, where $\hat{b} = \vec{B}_0/B_0$
- Primed frame moves with particle guiding center
- Consider $\delta \vec{E} = \delta \vec{E}_0 \cos[\vec{k} \cdot \vec{x}' (\omega k_{\parallel} v_{\parallel})t]$, where $k_{\parallel} = \vec{k} \cdot \hat{b}$
- $\omega k_{\parallel} v_{\parallel} =$ Doppler-shifted frequency in guiding center frame
- Wave-particle resonance when $\omega k_{\parallel} v_{\parallel} = n \Omega$



- These are *perpendicular* temperatures inferred from line widths observed at the Sun's limb.
- Protons in the corona and low- β fast-solar-wind streams satisfy $T_{\perp} > T_{\parallel}$

What is Magnetic Reconnection?

If a plasma is perfectly conducting, that is, it obeys the ideal Ohm's law,

 $\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$

B-lines are frozen in the plasma, and no reconnection occurs.







Fig. 1.7. Magnetic field-line conservation: if plasma elements P_1 and P_2 lie on a field line at time t_1 , then they will lie on the same line at a later time t_2 .

Magnetic Reconnection: Mathematical Definition

Departures from ideal behavior, represented by

 $E + v \times B = R$, $B \cdot \nabla \times R = 0$

break ideal topological invariants, allowing field lines to break and reconnect.

In the generalized Ohm's law for weakly collisional or collisionless plasmas, \mathbf{R} contains resistivity, Hall current, electron inertia and pressure.

Magnetic Reconnection







- Topological rearrangement of magnetic field lines
- Magnetic energy => Kinetic energy

The Flaring Sun



Courtesy: The Solar Dynamics Observatory

Magnetic reconnection layers in the magnetosphere



The Sweet-Parker Model for Magnetic Reconnection

Assume;

- 2D
- Steady-state
- Incompressibility
- Classical Spitzer resistivity



$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{\eta}{\mu_0} \nabla^2 \mathbf{B} \quad \Longrightarrow \quad V_{in} B = \frac{\eta_{Spitz}}{\mu_0} \frac{B}{\delta}$$

Mass conservation:







Pressure balance:

 $\frac{1}{2}\rho V_{out}^2 \approx \frac{B^2}{2\mu_0} \Longrightarrow V_{out} \approx V_A$

In solar flares, $\tau_{SP} \sim 1$ year >> τ_{reconn}

S = Lundquist number

Impulsive Reconnection: The Onset/Trigger Problem

Dynamics exhibits an impulsiveness, that is, a sudden change in the time-derivative of the reconnection rate. The magnetic configuration evolves slowly for a long period of time, only to undergo a sudden dynamical change over a much shorter period of time. Dynamics is characterized by the formation of nearsingular current sheets which need to be resolved in computer simulations: a classic multi-scale problem coupling large scales to small. Examples

Magnetospheric substorms Impulsive solar/stellar flares The thin current sheet is explosively stable if we exceed a critical Lundquist number, S_c forming, ejecting, and coalescing a hierarchy of plasmoids.



B. et al. 2009, Huang and B. 2010, Uzdensky et al. 2010

Reconnection Time of 25% of Initial Flux







Multi-scale problem, but with an underlying unity

L~1 km Kinetic



Observations of energetic electrons within magnetic islands [Chen et al., Nature Phys., 2008, PoP 2009]



e bursts & bipolar Bz & Ne peaks ~10 islands within 10 minutes



Post CME Current Sheet



Courtesy: Lijia Guo

Turbulent Region Broadens as Instabilities Evolve



• In fully developed turbulent state, approximately 70% of turbulence energy is in y = [-0.01, 0.01].

0.05

Plasmoid-Induced Turbulent Reconnection









Recent development: 3D Magnetic Reconnection



Daughton et al, Nature Physics, 2011, Guo et al. 2014 PRL, 2015 ApJ



- 2D and 3D kinetic simulations for relativistic magnetic reconnection show that the reconnection layer is dominated by development of flux ropes, and generates strong particle acceleration.
- Despite turbulence in the reconnection layer, nonthermal particles are efficiently generated and form power-law distributions.
- Using a number of diagnostics, we show the contributions from different acceleration mechanism. For anti-parallel case, the acceleration is dominated by Fermi acceleration, and this leads to power-law distribution. Acceleration by parallel electric field is important for reconnection with a strong guide field.
- The acceleration mechanism and power-law formation are quite robust and general.