Q: Why do the Sun and planets have magnetic fields?

Dana Longcope
Montana State University

w/ liberal “borrowing” from Bagenal, Stanley, Christensen, Schrijver, Charbonneau, …

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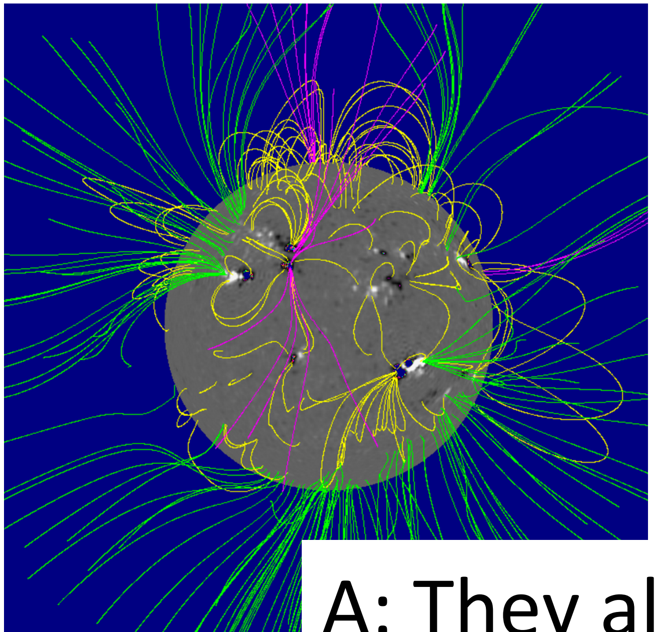
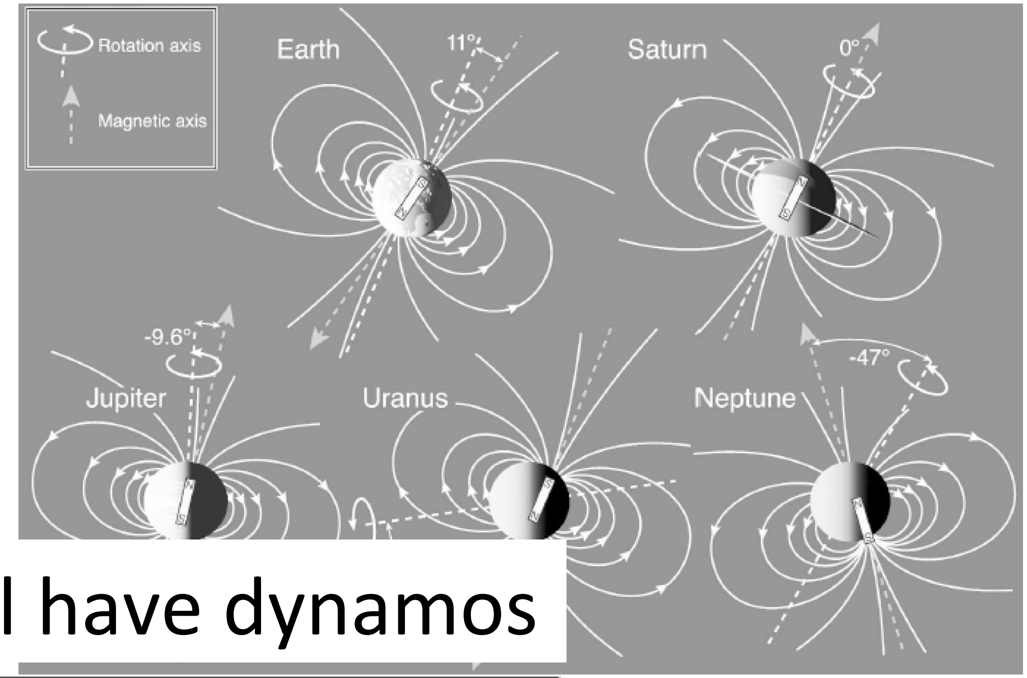
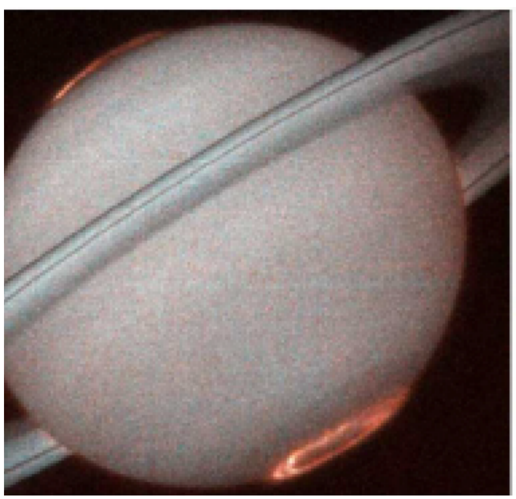
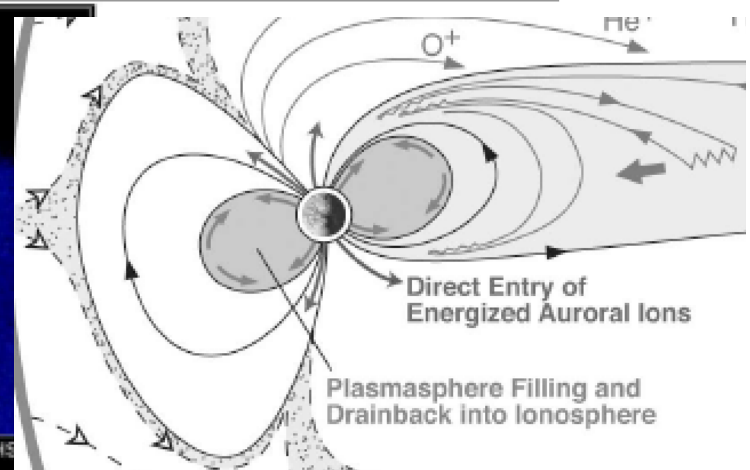


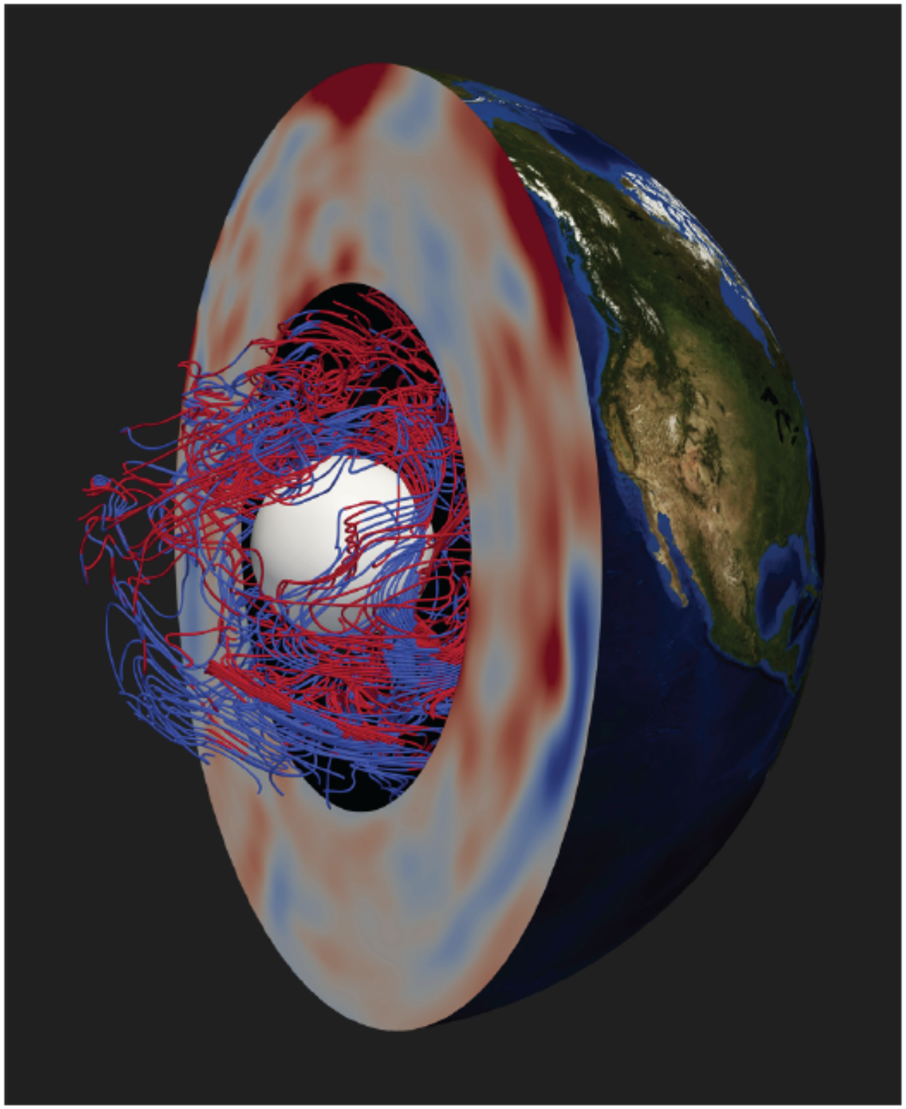


A: They all have dynamos





DYNAMO INGREDIENTS



From Stanley 2013

(1)electrically conducting fluid

- Plasma (stars)
- Liquid iron (terrestrial planets)
- Metallic hydrogen (gas giants)
- Ionized water (ice giants)

(2) fluid must have complex motions

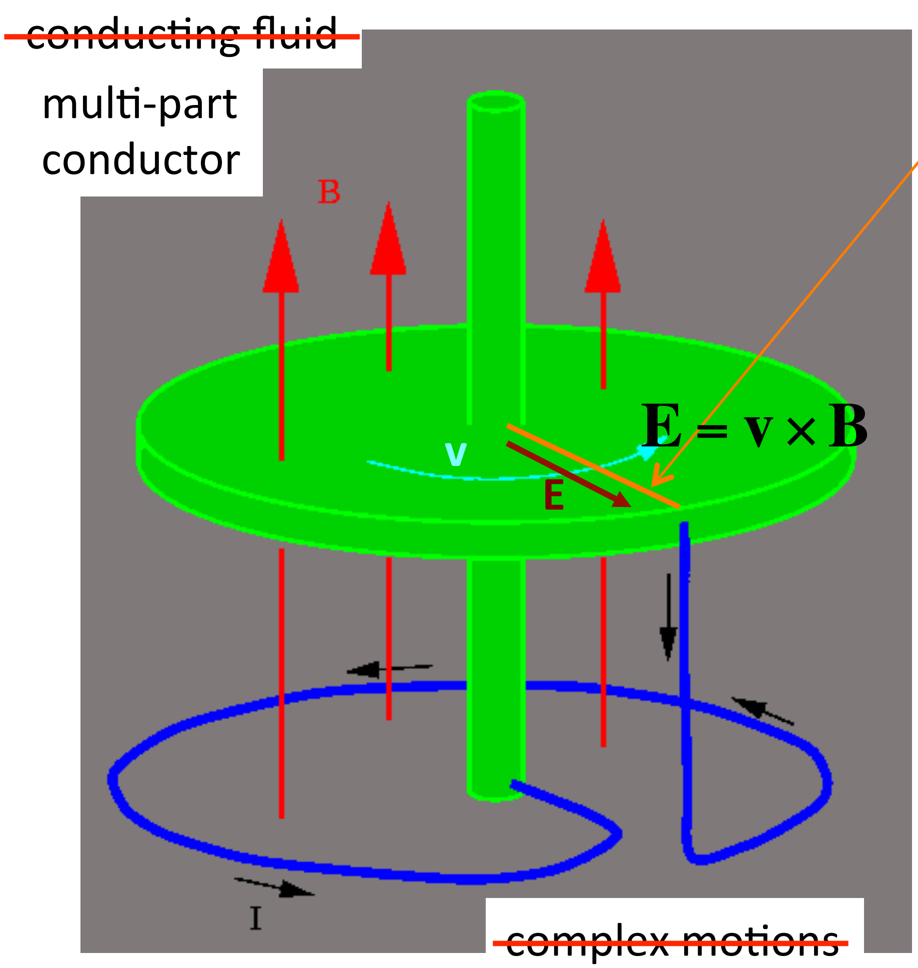
- Complex turbulent flows
- Rotation: breaks mirror-symmetry not required, but needed for large-scale, organized fields

(3) motions must be vigorous enough

- Figure of merit: Magnetic Reynold's #

Rm = velocity × size × conductivity

A Toy w/ all ingredients



lack of mirror symmetry

differential motion of parts

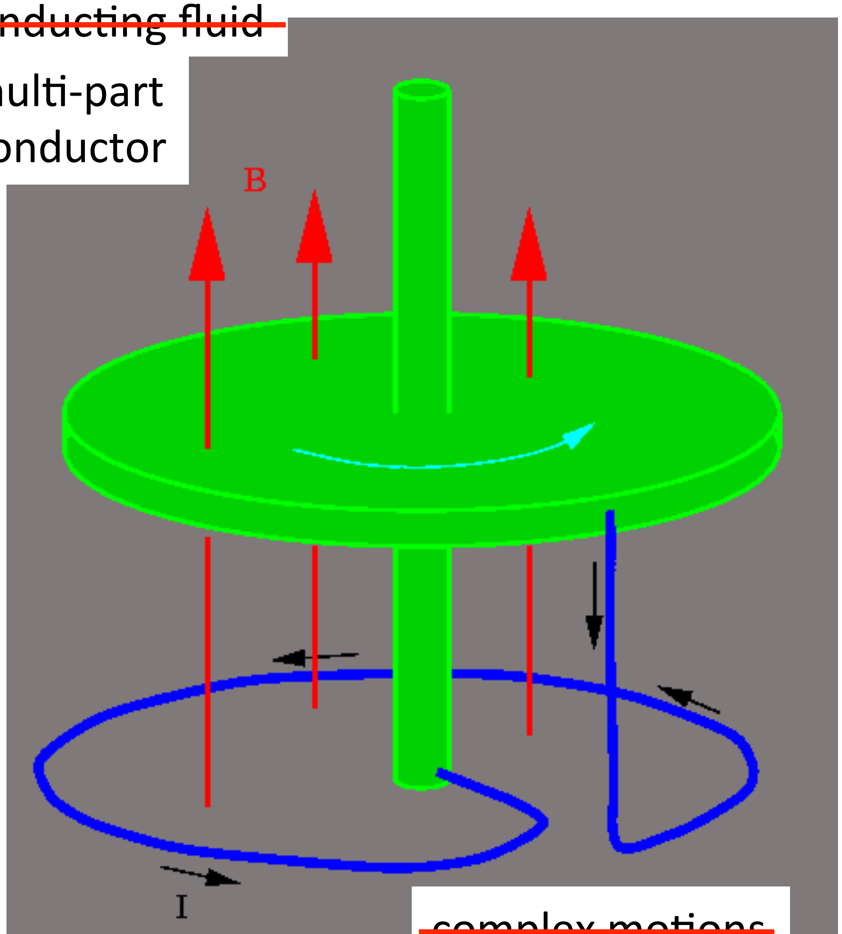
$$V_{\text{disk}} = \int_0^\ell vB\,dr = \int_0^\ell (\Omega r) B\,dr = \tfrac{1}{2\pi}\Omega\Phi_{\text{disk}}$$

- No magnets
- No batteries
- No (net) charge

A Toy w/ all ingredients

~~conducting fluid~~

multi-part conductor



~~complex motions~~

lack of mirror symmetry

differential motion of parts

$$V_{\text{disk}} = \int_0^{\ell} vB\,dr = \int_0^{\ell} (\Omega r)\,B\,dr = \tfrac{1}{2\pi}\Omega\Phi_{\text{disk}}$$

$$IR = \underbrace{\frac{\Omega}{2\pi}\Phi_{\text{disk}}}_{\text{motional EMF}} - \underbrace{L\frac{dI}{dt}}_{\text{back EMF}} = \frac{\Omega}{2\pi}M_{w,d}I - L\frac{dI}{dt}$$

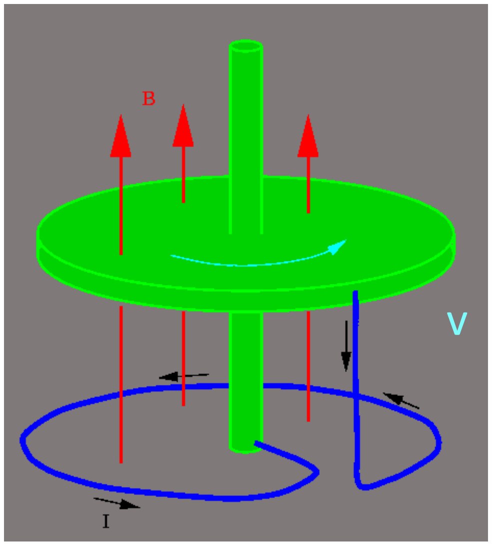
$$\frac{dI}{dt} = \Bigg(\underbrace{\frac{\Omega}{2\pi}\frac{M_{w,d}}{L}}_{\text{generation}} - \underbrace{\frac{R}{L}}_{\text{dissipation}}\Bigg)I = \gamma I \qquad I(t) = I_0 e^{\gamma t}$$

Growth: $\gamma > 0 \;\Leftrightarrow\; \Omega > 2\pi \dfrac{R}{M_{w,d}}$

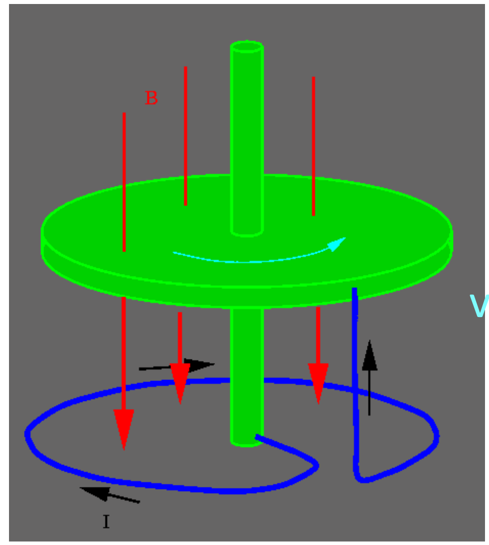
$$\frac{v}{\ell} > 2\pi \frac{1/\sigma\ell}{\mu_0\ell} = \frac{2\pi}{\mu_0\sigma\ell^2}$$

Growth: $Rm = \mu_0\sigma\ell v \;>\; 2\pi$

Toy dynamo amplifies fields of either sign: two attracting states



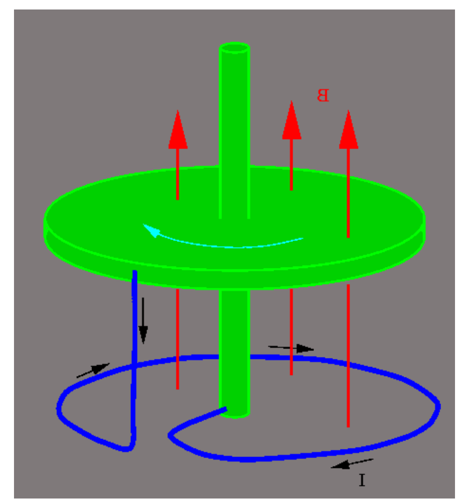
$I_0 > 0$



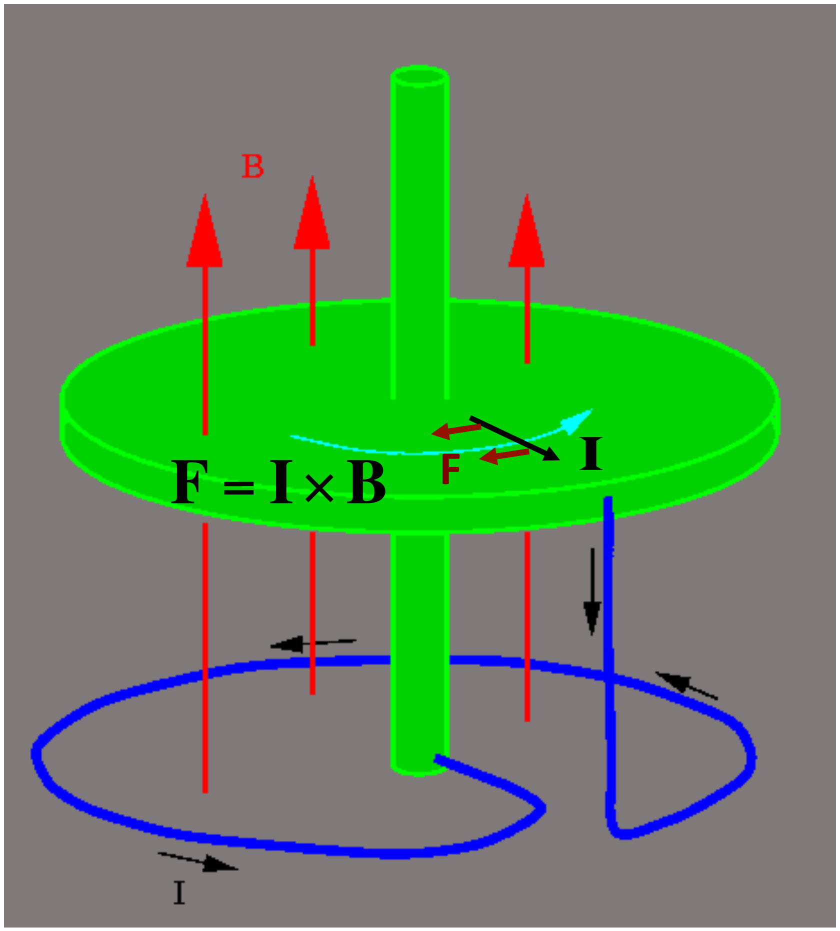
$I_0 < 0$

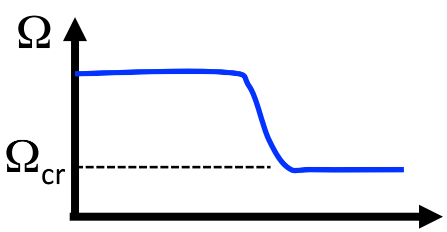
$$I(t) = I_0 e^{\gamma t}$$

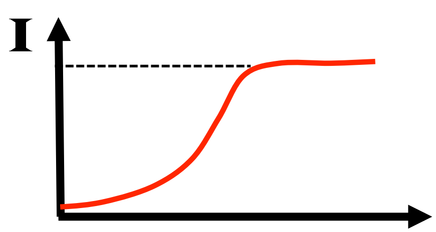
- Reverse velocity AND reflect in mirror ➔ still amplifies
- Do one and not the other ➔ no amplification



Will $\mathbf{I}$ grow forever?







Torque on disk carrying current:

$$\tau = \int_0^\ell F r\,dr = \int_0^\ell I B r\,dr = \tfrac{1}{2\pi} I \Phi_{\text{disk}} = \frac{M_{w,d}}{2\pi} I^2$$

Power needed to turn disk:

$$P_\Omega = \Omega\tau = \frac{\Omega}{2\pi} M_{w,d} I^2$$

Subtracting Ohmic losses

$$P_\Omega - I^2 R = \left(\frac{\Omega}{2\pi} M_{w,d} - R\right) I^2 = \gamma L I^2$$

$$= \frac{d}{dt}\left(\tfrac{1}{2} L I^2\right)$$

Stored in energy of **B**

Reality: Conducting fluid – MHD

Effect of **B** on conducting fluid

$$\mathbf{J} = \frac{1}{\mu_0}\nabla \times \mathbf{B}$$

Fluid dynamics

$$\begin{cases} \dfrac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v}) \\ \rho \dfrac{\partial \mathbf{v}}{\partial t} + \rho(\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla p + \rho \mathbf{g} + \nabla \cdot \overleftrightarrow{\sigma} + \boxed{\mathbf{J} \times \mathbf{B}} \\ \rho c_v \left(\dfrac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right) = -\frac{2}{3} T \nabla \cdot \mathbf{v} + \nabla \mathbf{v} : \overleftrightarrow{\sigma} + \dot{Q} + \boxed{\dfrac{1}{\sigma}|\mathbf{J}|^2} \end{cases}$$

Lorentz force

Ohmic heat

Faraday's + Ohm's laws

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} = \nabla \times \left[\mathbf{v} \times \mathbf{B} - \frac{1}{\sigma}\mathbf{J} \right]$$

$$\nabla \times \left[-\frac{1}{\sigma}\mathbf{J} \right] = \nabla \times \left[-\frac{1}{\sigma \mu_0} \nabla \times \mathbf{B} \right] = \eta \nabla^2 \mathbf{B} \qquad \eta = \frac{1}{\mu_0 \sigma}$$

magnetic diffusivity

Reality: Conducting fluid – MHD

If **B** is weak: kinematic equations

Fluid dynamics

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v})$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho(\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla p + \rho \mathbf{g} + \nabla \cdot \vec{\vec{\sigma}}$$

$$\rho c_v \left(\frac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla T \right) = -\tfrac{2}{3} T \nabla\cdot\mathbf{v} + \nabla\mathbf{v} : \vec{\vec{\sigma}} + \dot{Q}$$

Traditional (neutral) fluid – solve first

Faraday's + Ohm's laws

$$\frac{\partial \mathbf{B}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{B} = (\mathbf{B}\cdot\nabla)\mathbf{v} - \mathbf{B}(\nabla\cdot\mathbf{v}) + \eta\nabla^2\mathbf{B}$$

Linear equation **for** **B**(**x**,t) – solve w/ known **v**(**x**,t)

Dynamo action in MHD

$$\frac{D\mathbf{B}}{Dt} \qquad \mathbf{B}\cdot[\nabla\mathbf{v} - \mathbf{I}(\nabla\cdot\mathbf{v})] = \mathbf{B}\cdot\mathbf{M}$$

$$\underbrace{\frac{\partial\mathbf{B}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{B}}_{} = \underbrace{(\mathbf{B}\cdot\nabla)\mathbf{v} - \mathbf{B}(\nabla\cdot\mathbf{v})}_{} + \eta\nabla^2\mathbf{B}$$

If **M** has a positive eigenvalue $\lambda > 0$

B can grow exponentially: **DYNAMO ACTION**

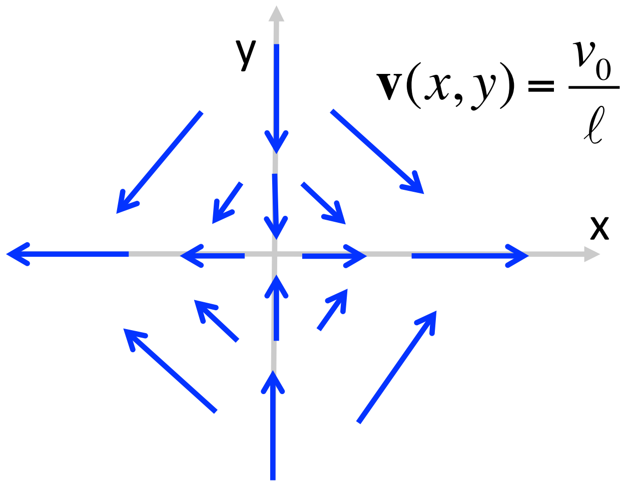
- **B** → -**B** : same e-vector ➔ same λ
- Reverse velocity AND reflect in mirror ➔ $\lambda \rightarrow \lambda$
- Do one and not the other ➔ $\lambda \rightarrow -\lambda$

λ → $\frac{v}{\ell}$, $\eta\nabla^2$ → $\frac{\eta}{\ell^2}$

$$\gamma \sim \frac{v}{\ell} - \frac{\eta}{\ell^2} = \frac{v}{\ell}\left(1 - \frac{\eta}{\ell v}\right)$$

Growth: $Rm = \frac{\ell v}{\eta} = \mu_0 \ell v \sigma > 1$

Q: What kind of flow has λ > 0?

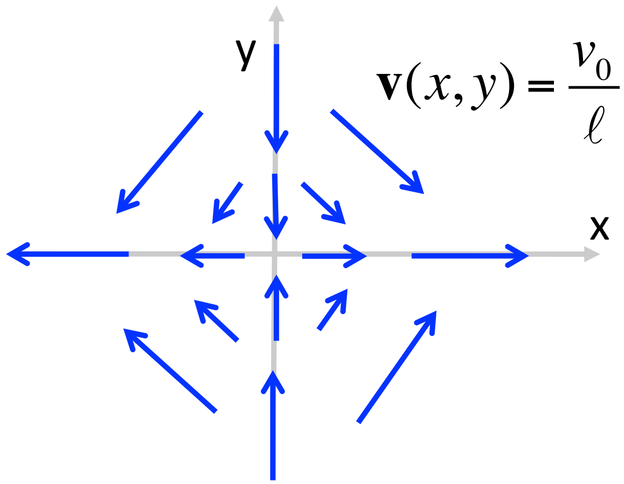


$$\mathbf{v}(x,y) = \frac{v_0}{\ell}\left(x\hat{\mathbf{x}} - y\hat{\mathbf{y}}\right)$$

$$\mathrm{M} = \begin{bmatrix} \partial v_x / \partial x & \partial v_x / \partial y \\ \partial v_y / \partial x & \partial v_y / \partial y \end{bmatrix} = \frac{v_0}{\ell}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\mathrm{M}\cdot\begin{bmatrix} B_0 \\ 0 \end{bmatrix} = \frac{v_0}{\ell}\cdot\begin{bmatrix} B_0 \\ 0 \end{bmatrix} \qquad \lambda = +\frac{v_0}{\ell}$$

Q: What kind of flow has λ > 0?



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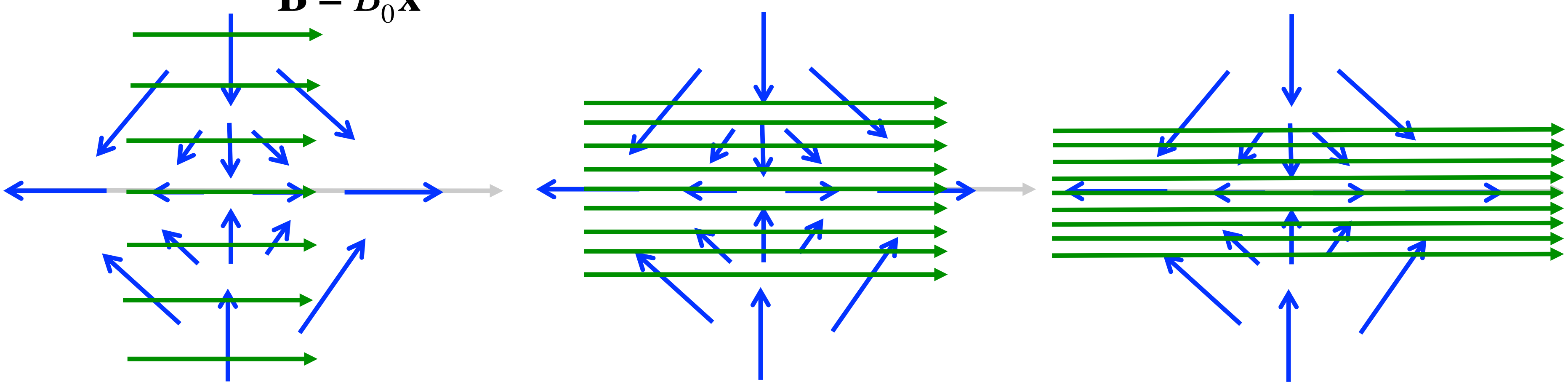
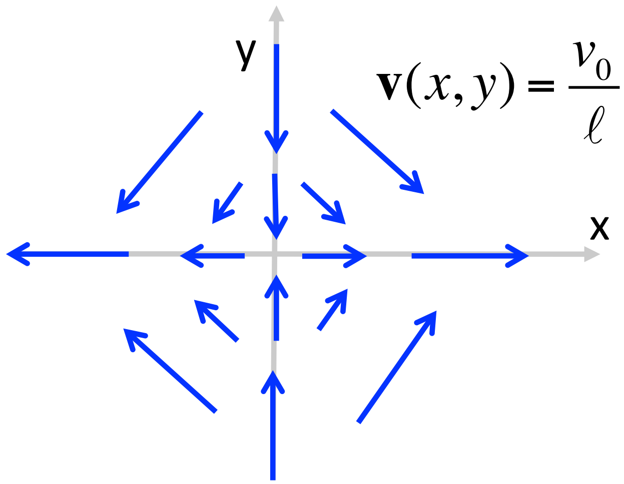
$$\mathbf{B} = B_0\hat{\mathbf{x}}$$



A: stretching flow

Q: What kind of flow has λ > 0?



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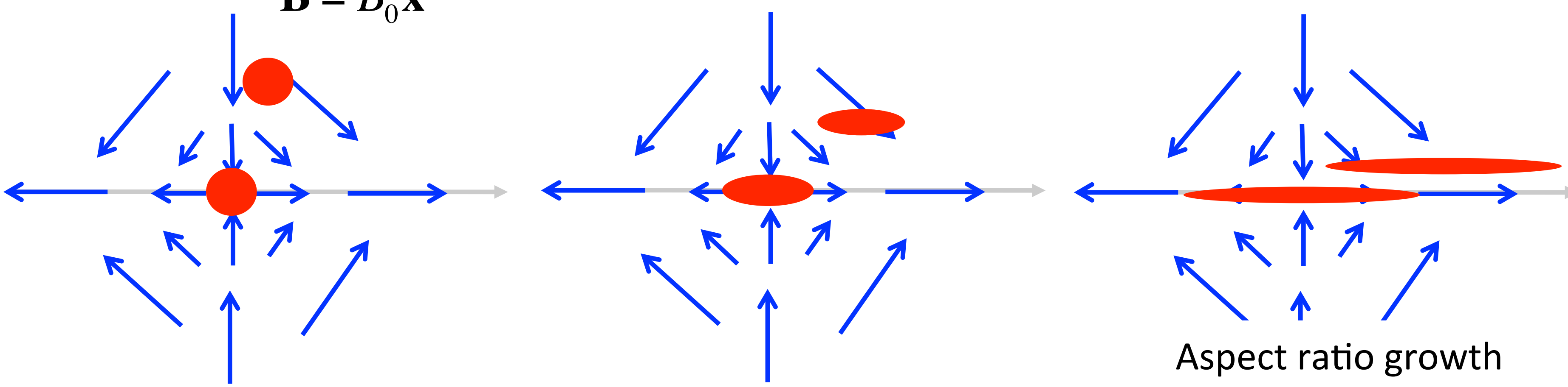


Aspect ratio growth
~ Lyapunov exponent

A: stretching flow

Q: What kind of flow has $\lambda > 0$?

- Turbulent flows have pos. Lyapunov exponent: $\lambda > 0$
  - tend to stretch balls into strands 
  - tend to amplify fields
- Conditions for turbulence:
  - driving: e.g. Rayleigh-Taylor instability
  - viscosity fights driving – must be small

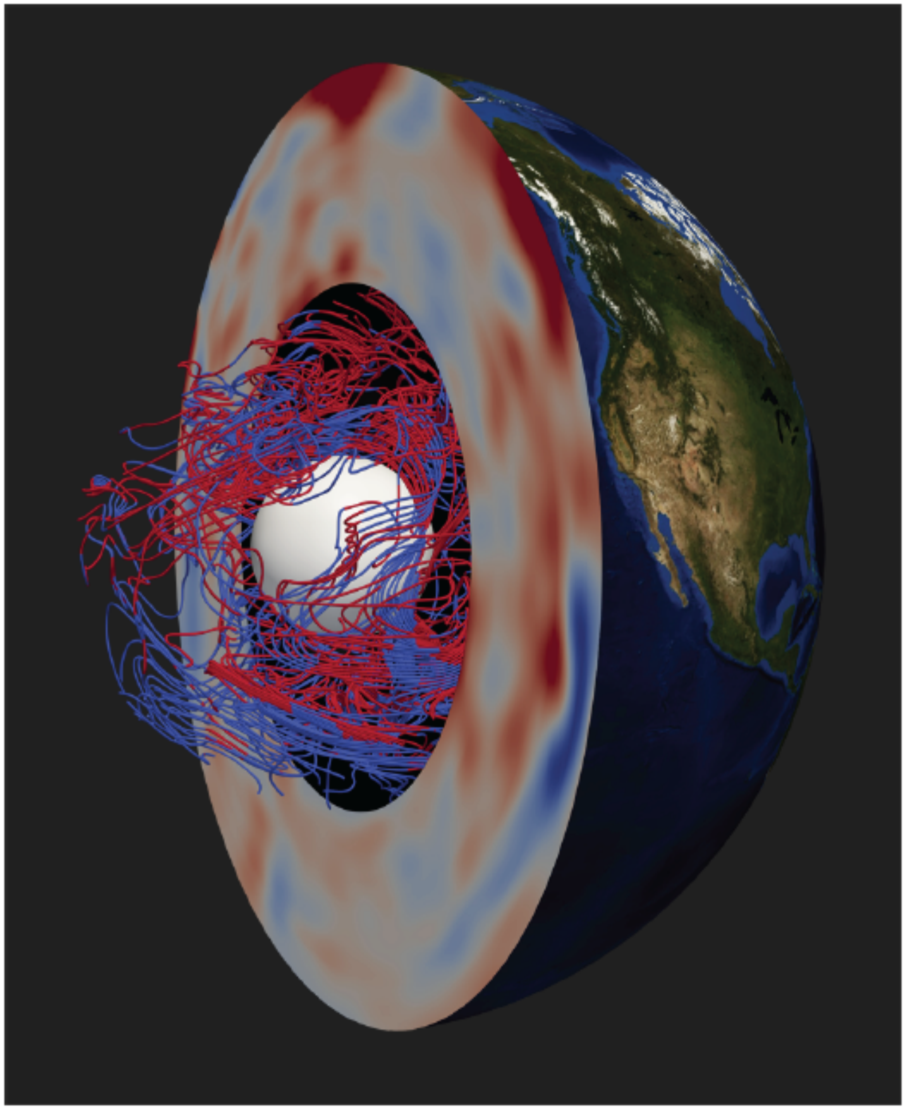
$$\mathrm{Re} = \frac{\ell v}{\upsilon} >> 1$$

- Rotation can organize turbulence: align stretching direction → azimuthal (toroidal) – known as Ω-effect – must be significant w.r.t. fluid motion

$$\mathrm{R}o = \frac{v}{\ell \Omega} << 1$$

	η [m²/s]	υ [m²/s]	L [m]	v [m/s]	Ω [rad/s]	Rm	Re	Ro
Sun (CZ)	1	10^{-2}	10^{8}	1	10^{-6}	10^{8}	10^{10}	10^{-2}
Earth (core)	1	10^{-5}	10^{6}	10^{-4}	10^{-4}	10^{2}	10^{7}	10^{-6}

Dynamo Ingredients



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- Complex turbulent flows
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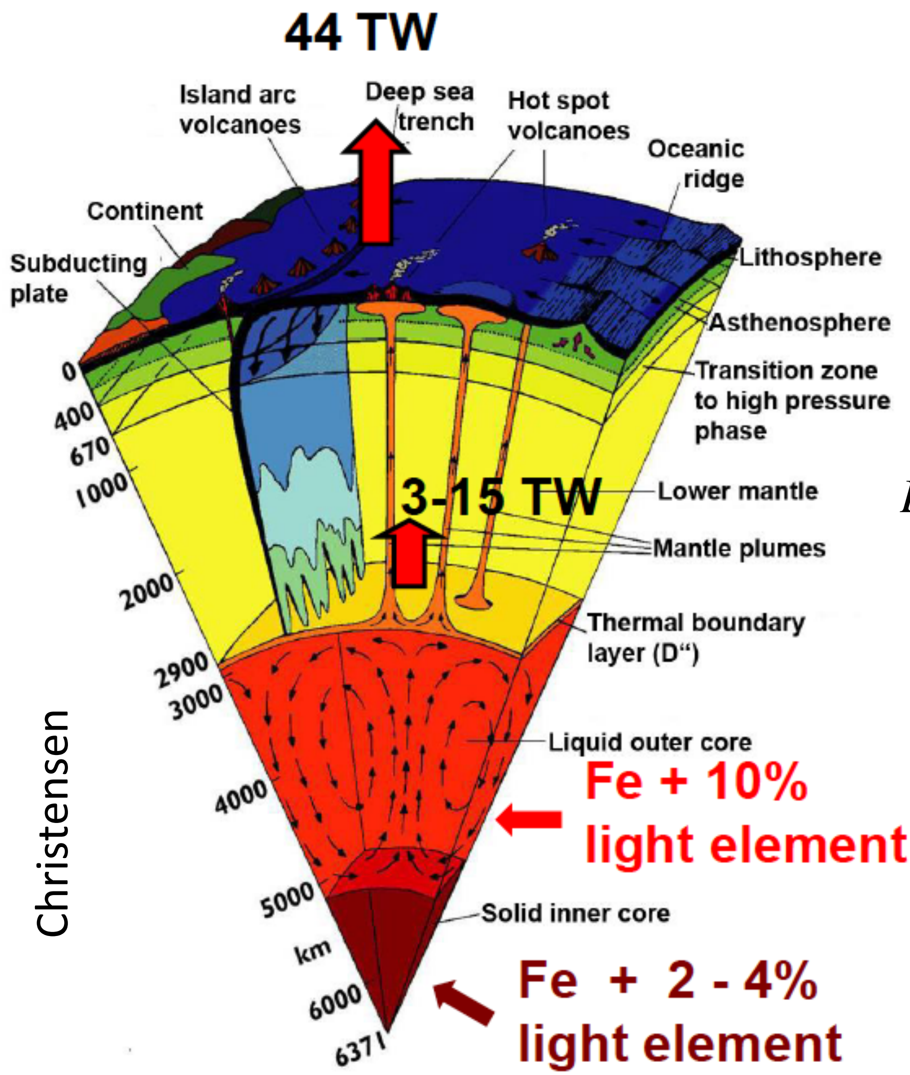
(3) motions must be vigorous enough

- Figure of merit: Magnetic Reynold's #

Rm = velocity × size × conductivity

How this works for Earth



Non-conducting mantle

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} = 0$$

$$\mathbf{B} = -\nabla \chi \qquad \nabla \cdot \mathbf{B} = -\nabla^2 \chi = 0$$

$$\chi(r,\theta,\phi) = \sum_{\ell,m} \tilde{g}_{\ell,m}\, Y_\ell^m(\theta,\phi) \left(\frac{R_\oplus}{r}\right)^{\ell+1}$$

$$B_r(r,\theta,\phi) = -\frac{\partial \chi}{\partial r} = \sum_{\ell,m} (\ell+1)\tilde{g}_{\ell,m}\, Y_\ell^m(\theta,\phi) \left(\frac{R_\oplus}{r}\right)^{\ell+2}$$

simplifies w/ increasing r

Turbulent conducting fluid:

DYNAMO

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \neq 0$$

Complex flows – complex field

A Spherical Harmonic Refresher

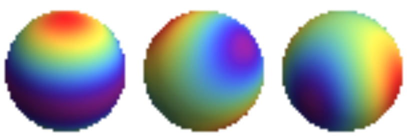
ℓ = 1
dipole

$$Y_1^0(\theta,\varphi) \sim \cos\theta \qquad Y_1^{\pm 1}(\theta,\varphi) \sim \sin\theta\, e^{\pm i\varphi}$$

$$\tilde{g}_{1,\pm 1} \sim g_{1,1} \mp i h_{1,1} \qquad (g_{1,0}, g_{1,1}, h_{1,1}) \leftrightarrow \vec{\mu}$$ dipole moment

ℓ = 2
quadrupole

$$Y_2^0(\theta,\varphi) \sim \tfrac{3}{4}\cos 2\theta + \tfrac{1}{4} \qquad Y_2^{\pm 2}(\theta,\varphi) \sim \sin\theta\, e^{\pm 2i\varphi}$$

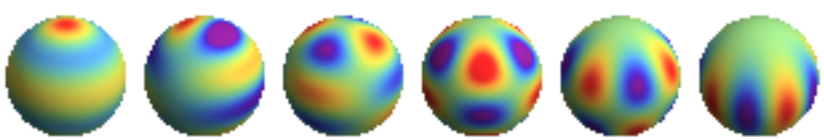
ℓ=2 

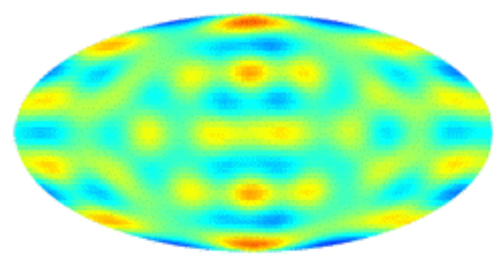
$$(g_{2,0}, g_{2,1}, h_{2,1}, g_{2,2}, h_{2,2}) \leftrightarrow \overleftrightarrow{Q}$$ quadrupole tensor

higher ℓ:

$$Y_\ell^0(\theta,\varphi) \sim \cos \ell\theta + \cdots \qquad Y_\ell^{\pm\ell}(\theta,\varphi) \sim \sin\theta\, e^{\pm \ell i\varphi}$$

Finer scale: ℓ periods around circle
More components: 2ℓ +1 real coefficients

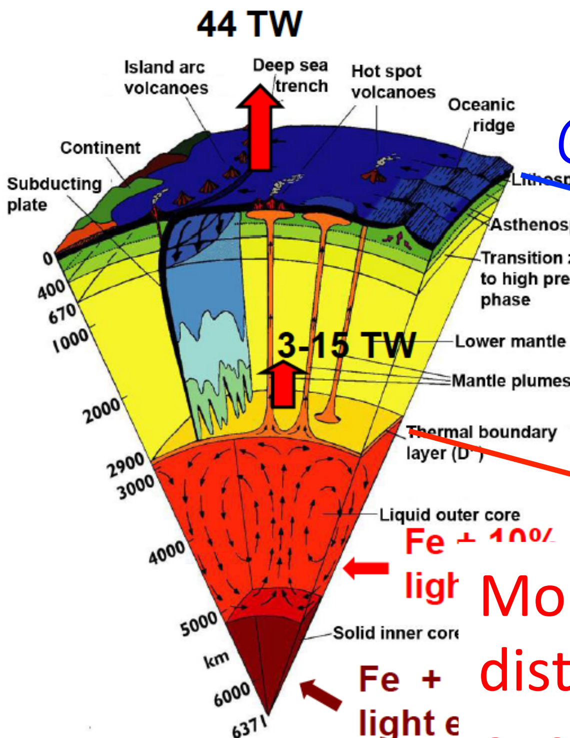
ℓ=5 

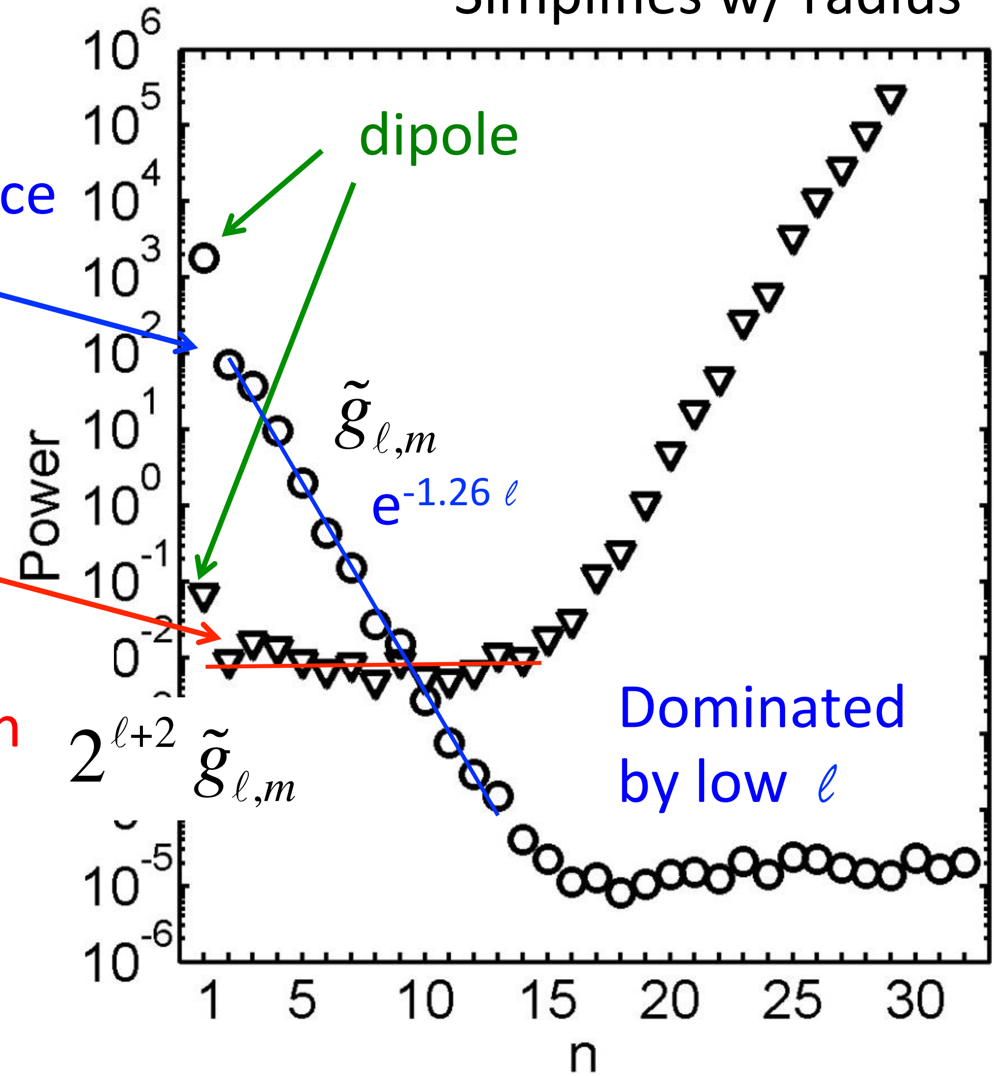


$\ell = 9$

$$B_r(r,\theta,\phi) = \sum_{\ell,m} (\ell+1)\tilde{g}_{\ell,m}\, Y_\ell^m(\theta,\phi) \left(\frac{R_\oplus}{r}\right)^{\ell+2}$$

Simplifies w/ radius





@ surface

More even distribution over ℓ

@ core-mantle boundary

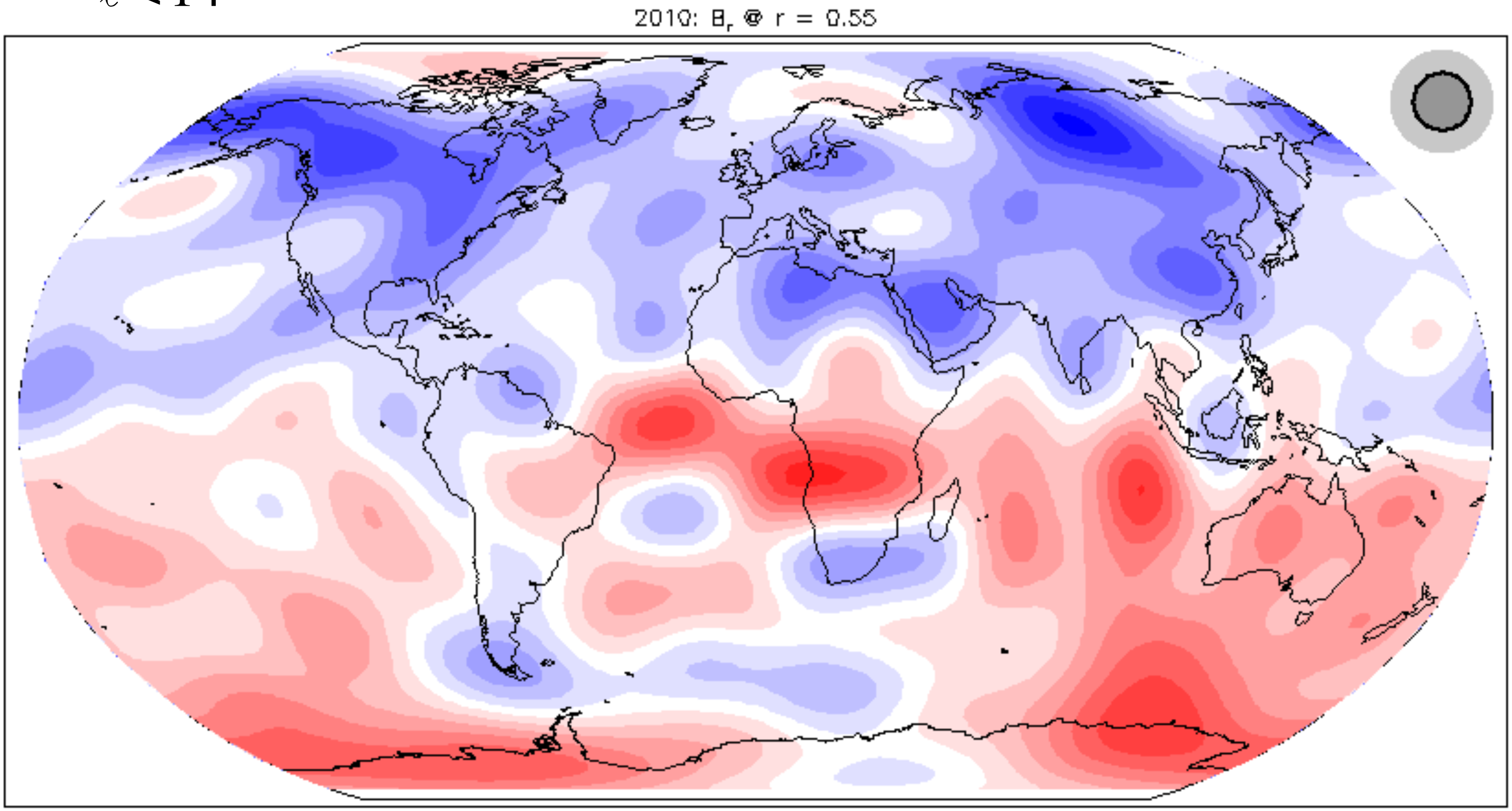
Dominated by low ℓ

Christensen

Observation: what B_r looks like today

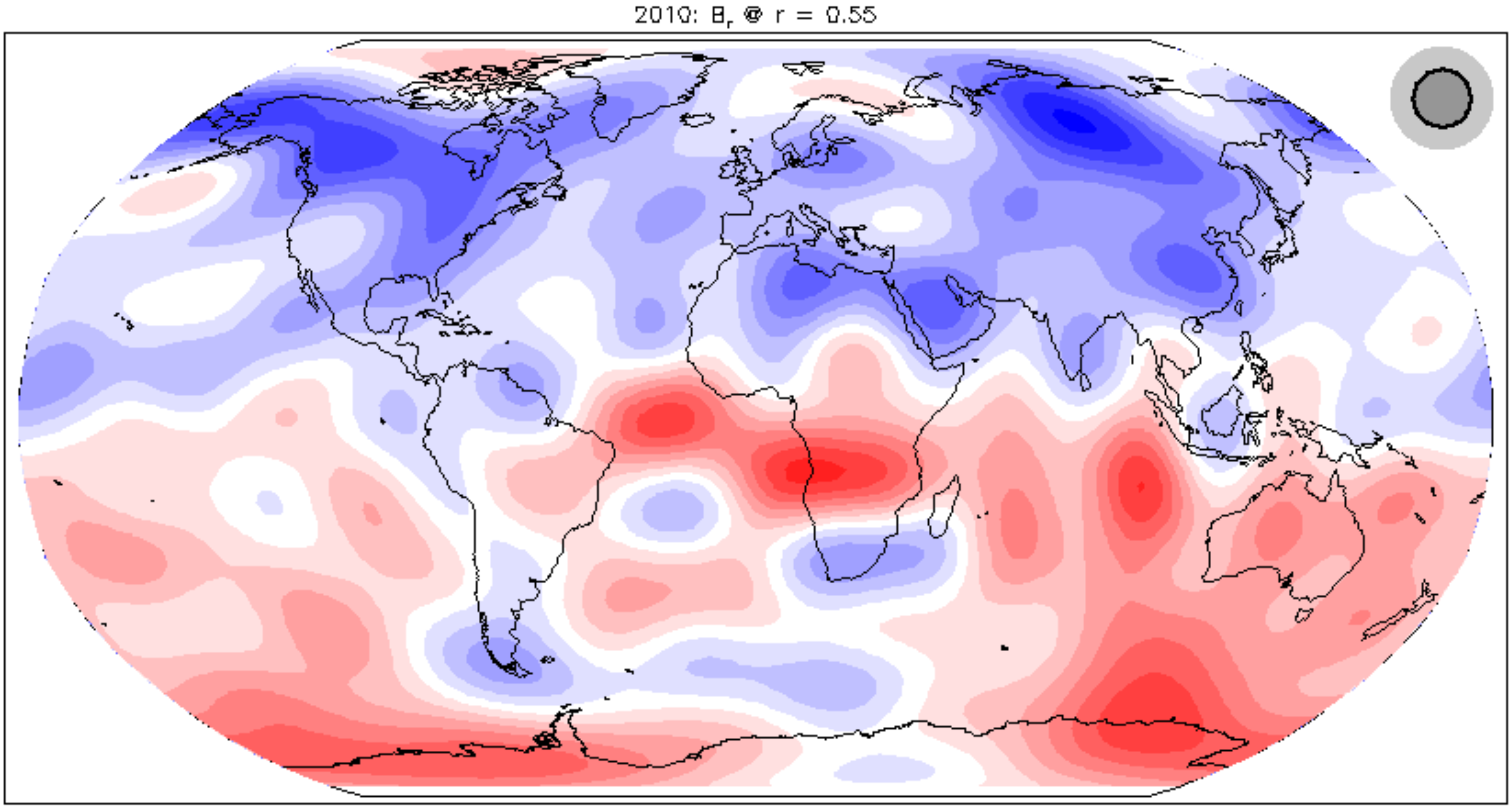
@ core-mantle boundary: lower boundary of potential region

$\ell < 14$



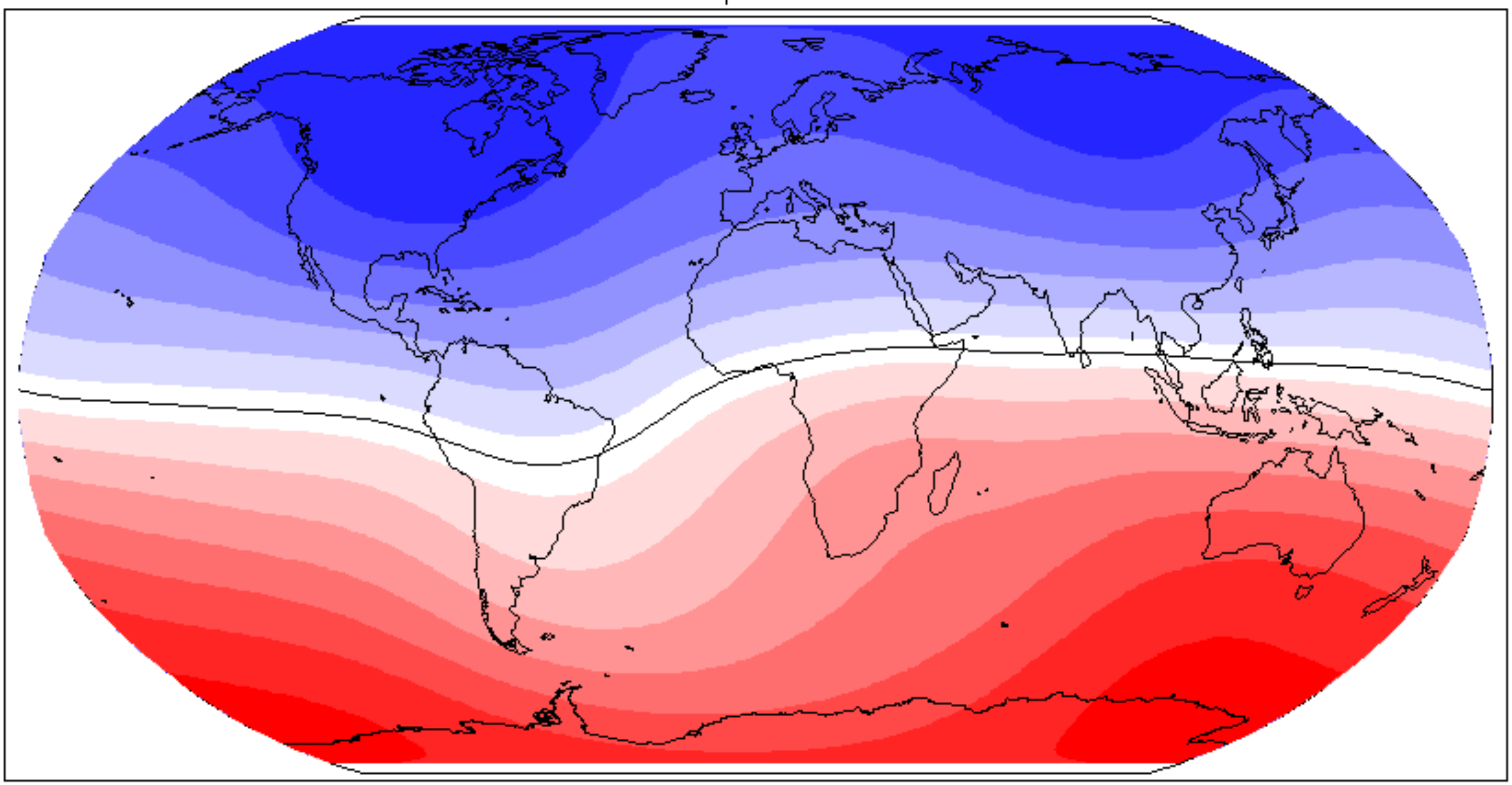
Simplifies w/ increasing r

$$B_r(r,\theta,\phi) = -\frac{\partial\chi}{\partial r} = \sum_{\ell,m}(\ell+1)\tilde{g}_{\ell,m}\, Y_\ell^m(\theta,\phi)\left(\frac{R_\oplus}{r}\right)^{\ell+2}$$



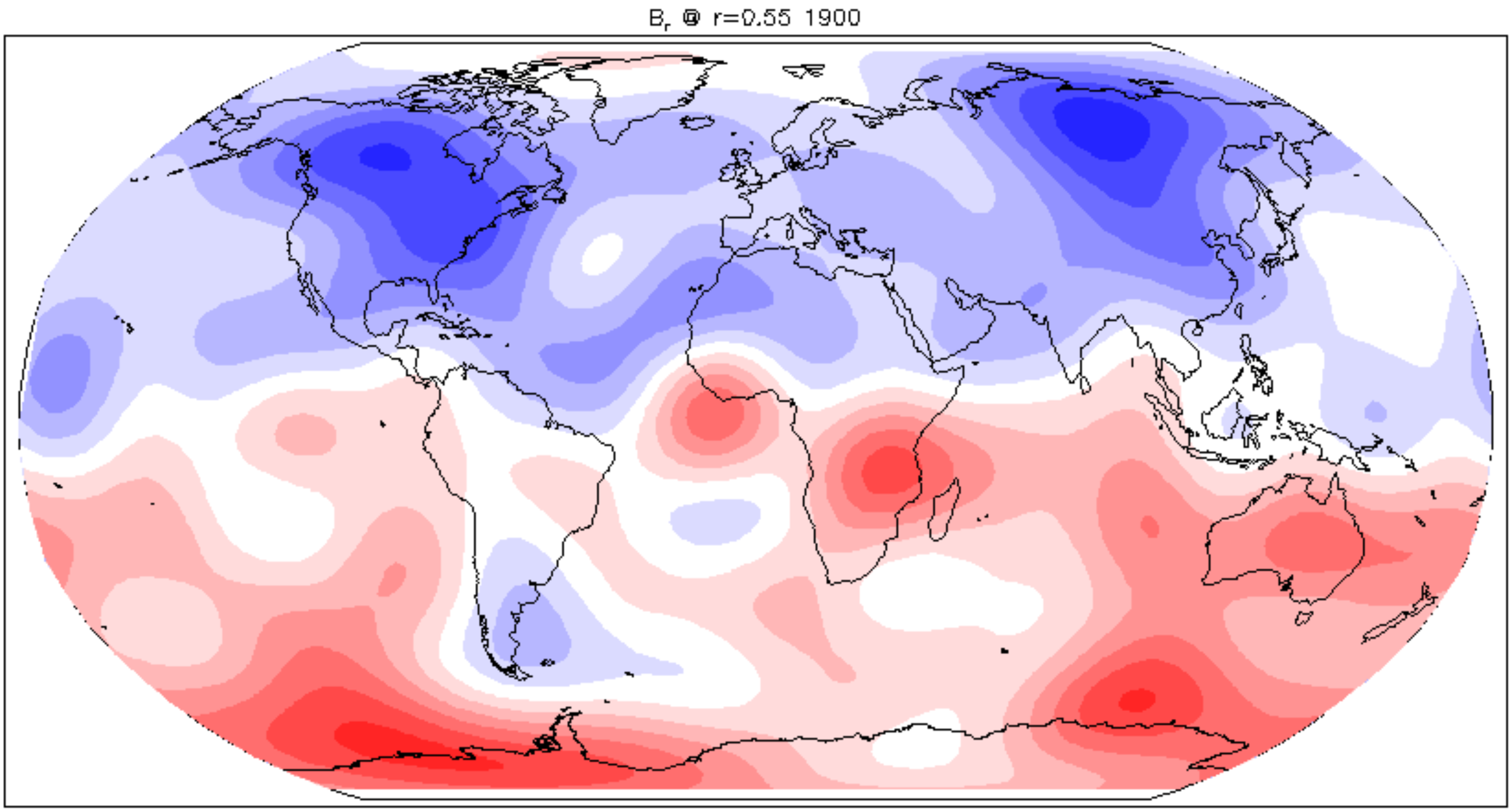
Evolution of field

@ surface for 100 years

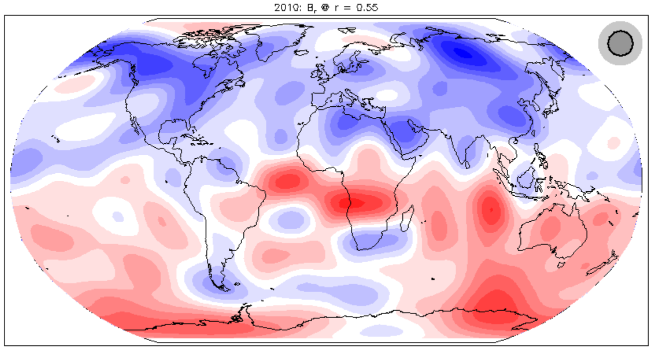


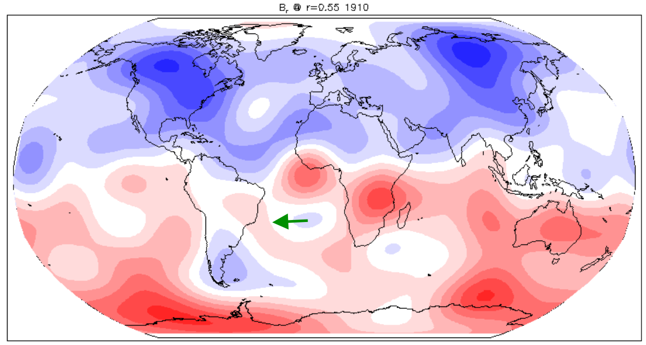
Evolution of field

@ core-mantle boundary for 100 years

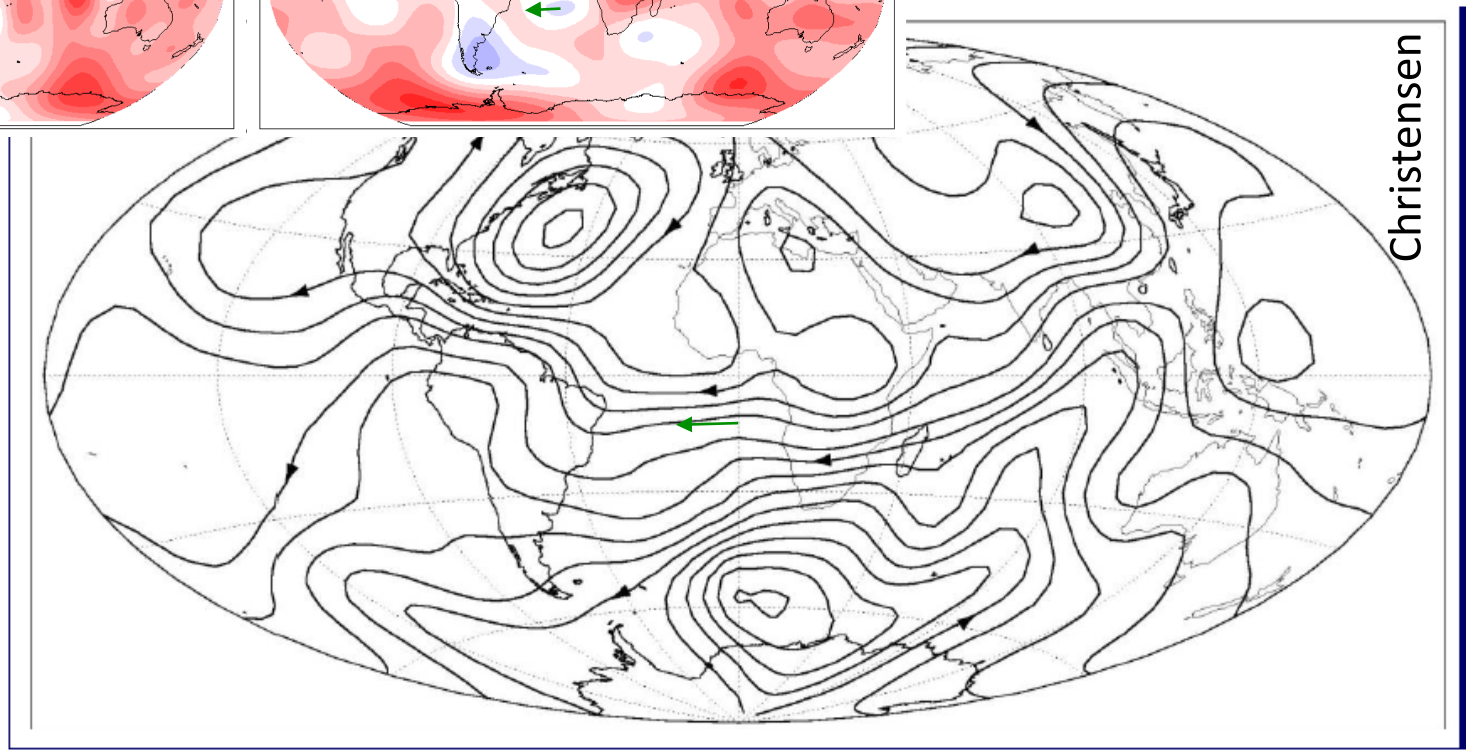


Use evolution to infer fluid velocity





v ~ 3 X 10^{-4} m/s
➔ Δx = 1,000 km
➔ in 100 years



Subsonic flow,
Ignore stratification

$$\nabla \cdot \mathbf{v} = \mathbf{0}$$

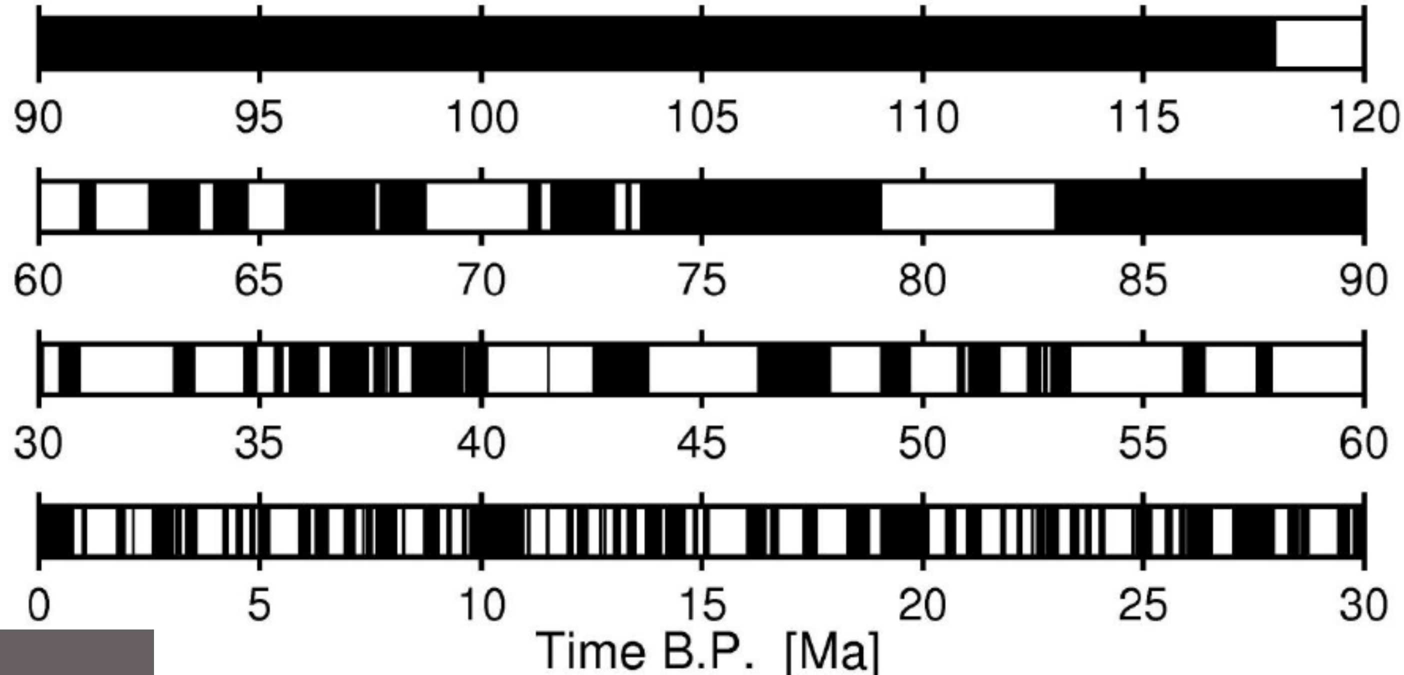
Ignore diffusion

$$\frac{\partial \mathbf{B}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{B} = (\mathbf{B} \cdot \nabla)\mathbf{v} - \mathbf{B}(\nabla \cdot \mathbf{v}) + \eta \nabla^2 \mathbf{B}$$

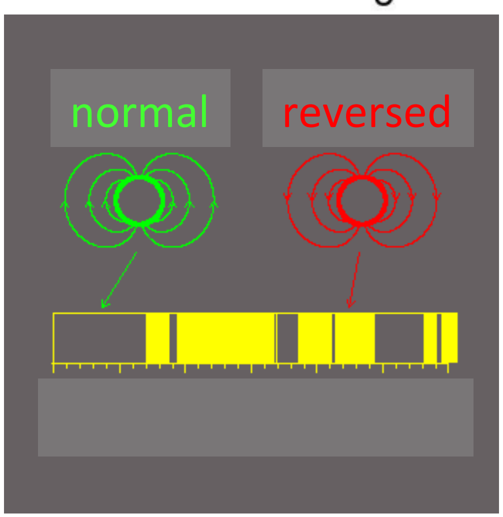
known

$$\frac{\partial B_r}{\partial t} + \nabla \cdot (\mathbf{v} B_r) = 0$$

Longer-term evolution

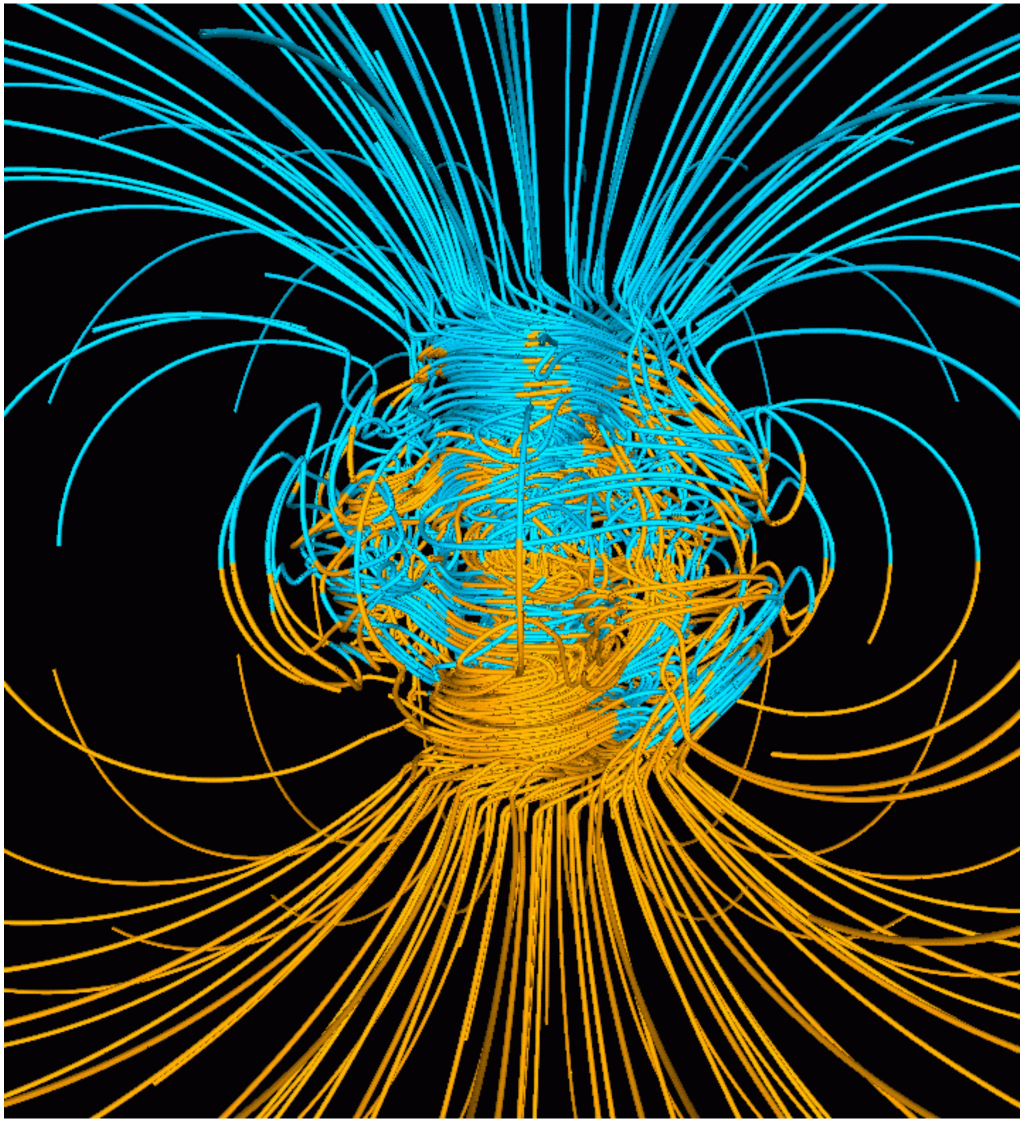


Christensen



Like toy dynamo, Earth works in 2 modes. Flips between them seemingly at random

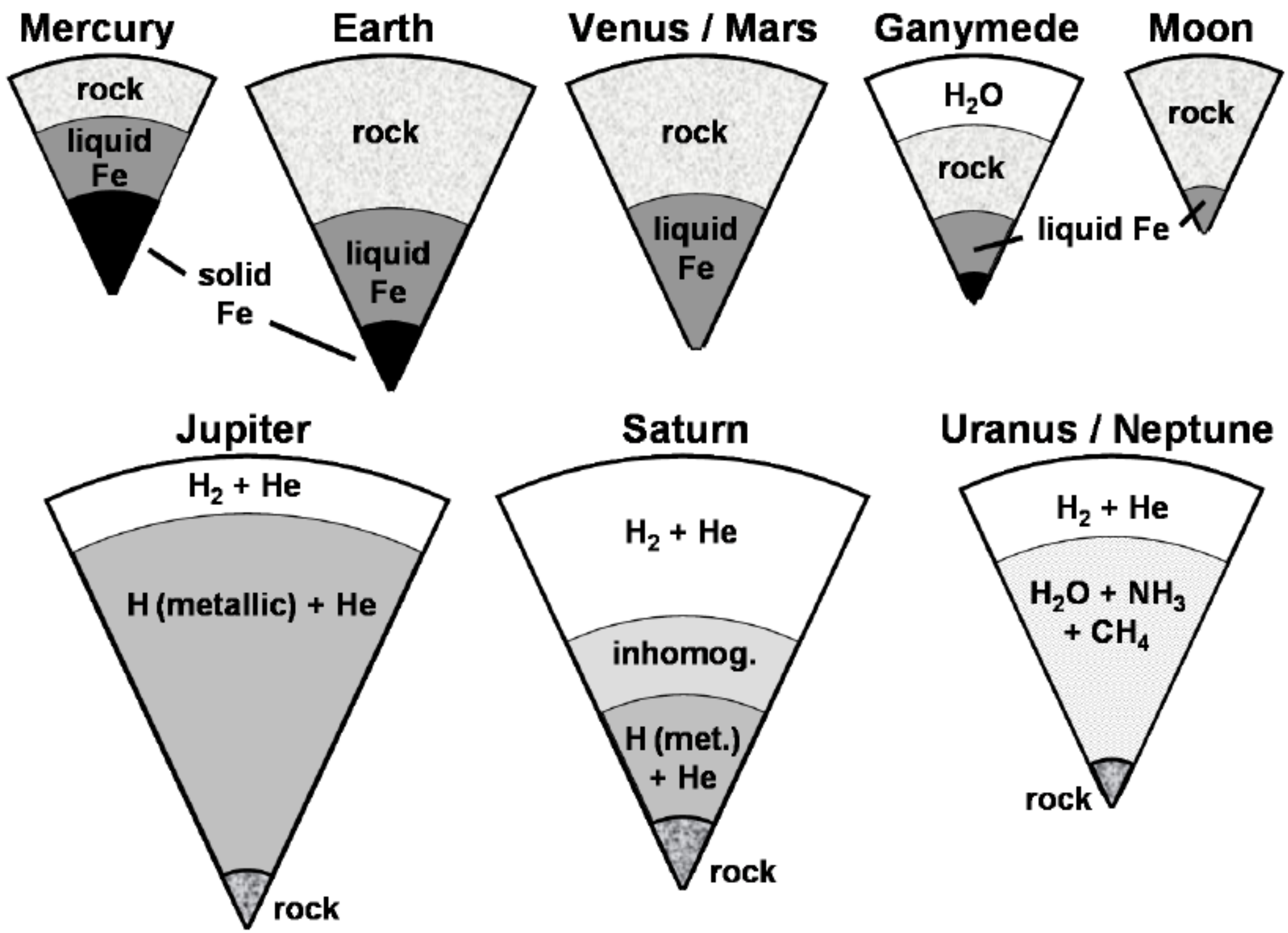
Model geodynamo



http://www.es.ucsc.edu/~glatz/geodynamo.html

- Glatzmaier & Roberts 1995
- Numerical solution of MHD
- Toroidal structure inside convecting core

Other planets



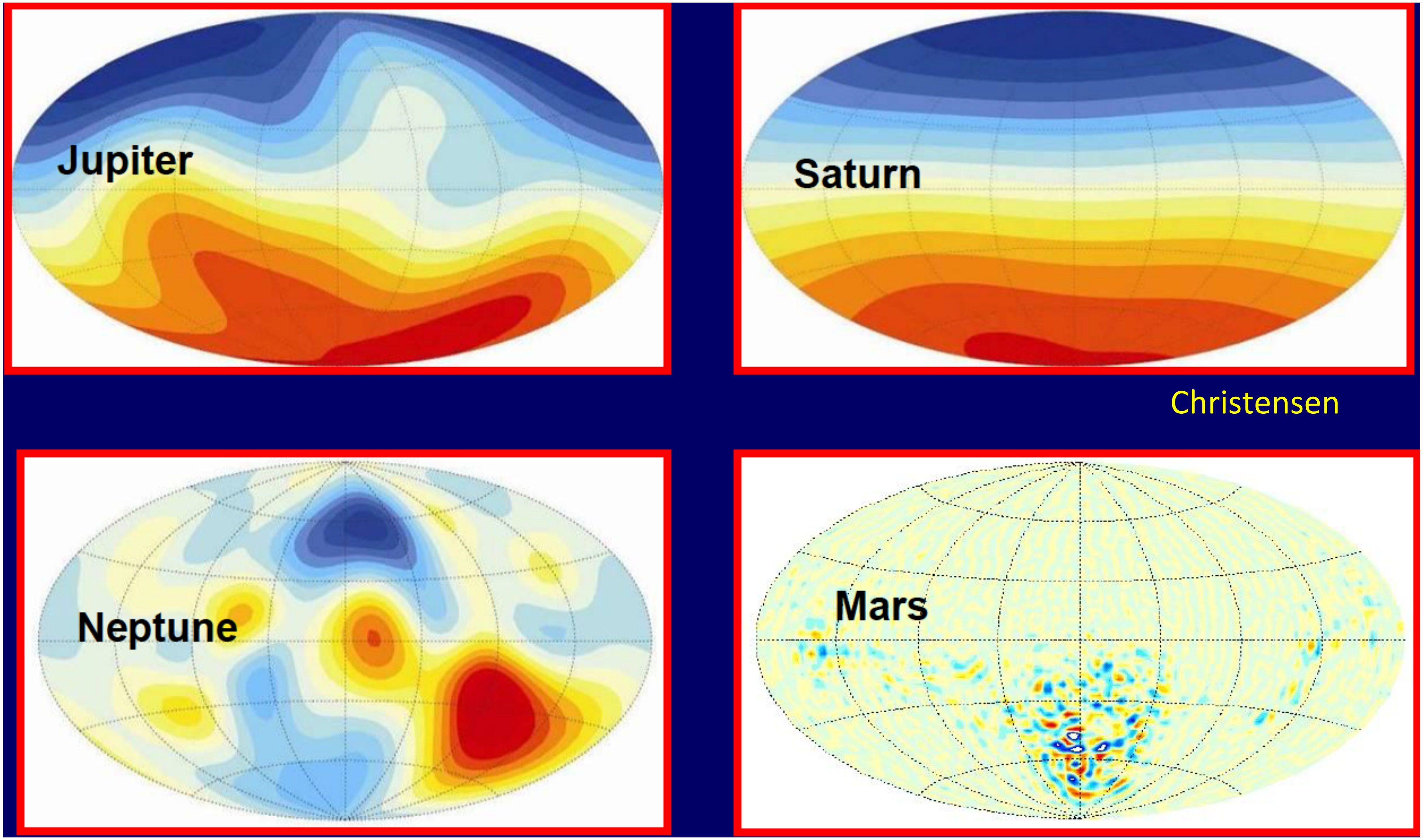
Christensen

Other planets

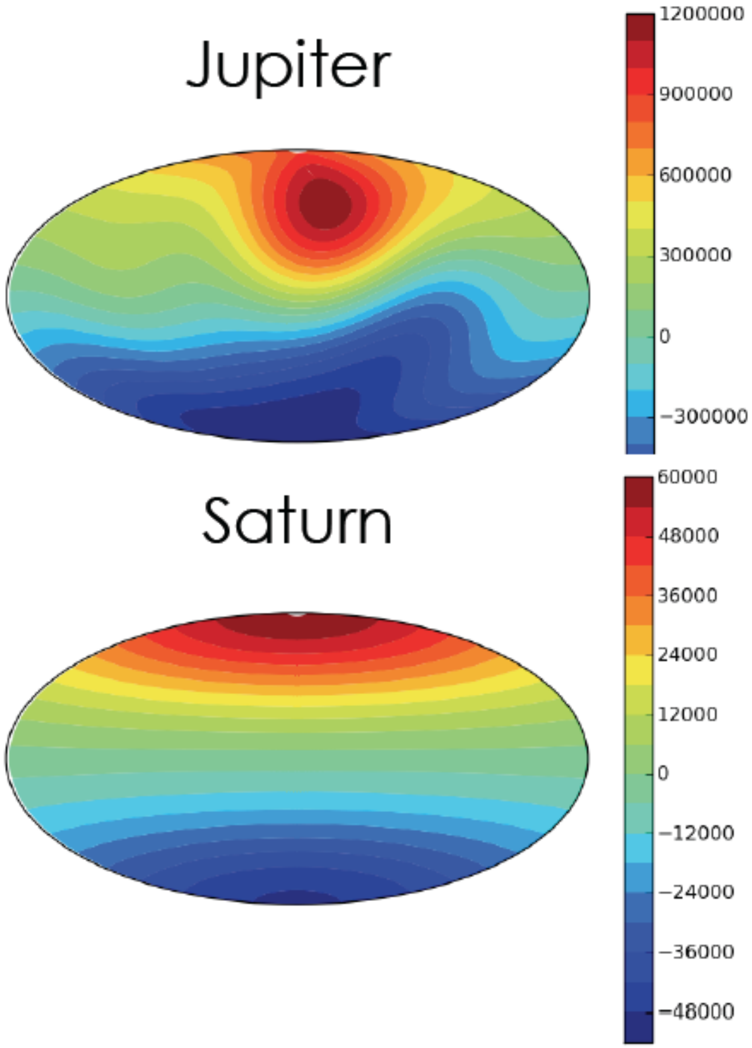
Christensen

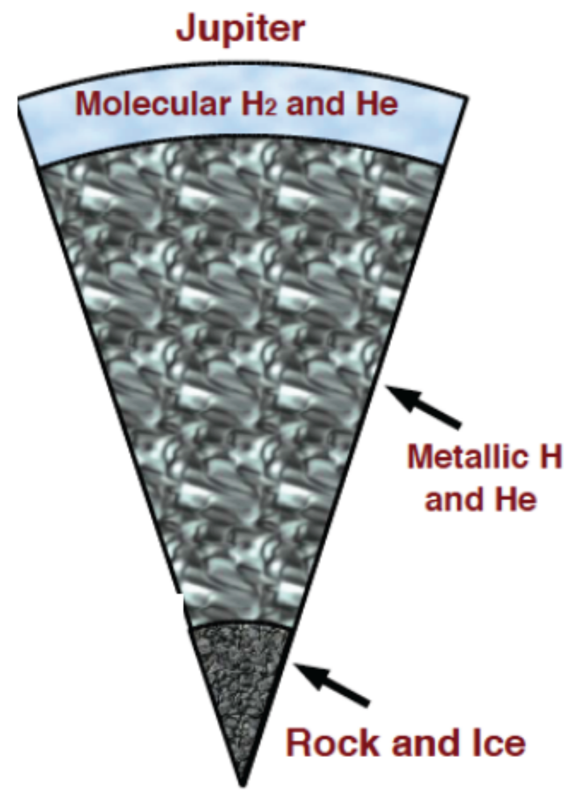
Planet	Dynamo	R_c/R_p	B_s [μT]	Dip. tilt	$\frac{\text{Quadr}}{\text{Dipole}}$
Mercury	Yes (?)	0.75	0.35	<5° ?	0.1-0.5
Venus	No	0.55			
Earth	Yes	0.55	44	10.4°	0.04
Moon	No	0.2 ?			
Mars	No, but in past	0.5			
Jupiter	Yes	0.84	640	9.4°	0.10
Saturn	Yes	0.6	31	0°	0.02
Uranus	Yes	0.75	48	59°	1.3
Neptune	Yes	0.75	47	45°	2.7
Ganymede	Yes	0.3 ?	1.0	< 5° ?	?

What that means



Gas Giants





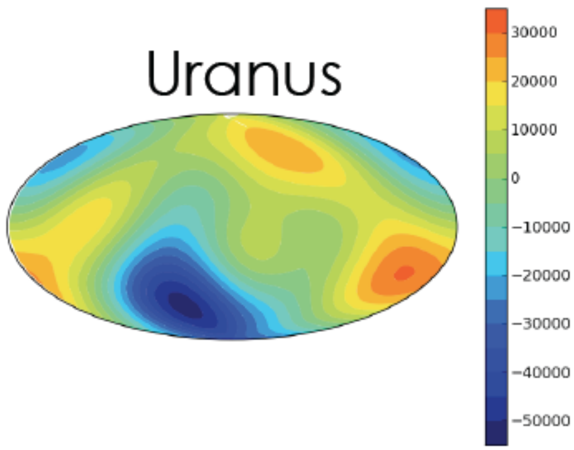
- large pressure range → radially variable properties can be important

Stanley

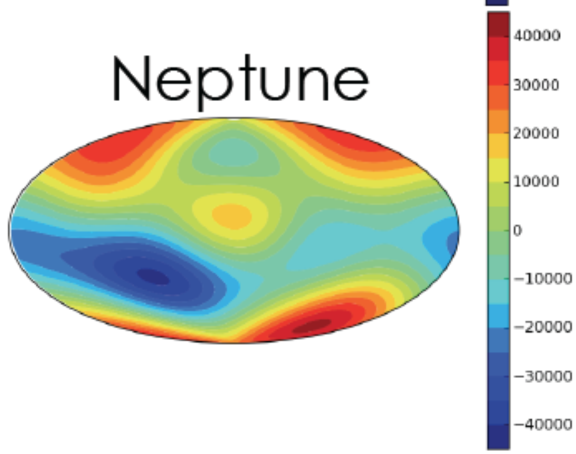
Ice Giants

Stanley

Uranus

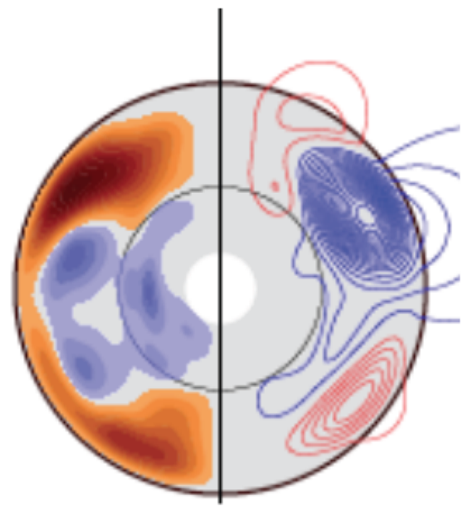


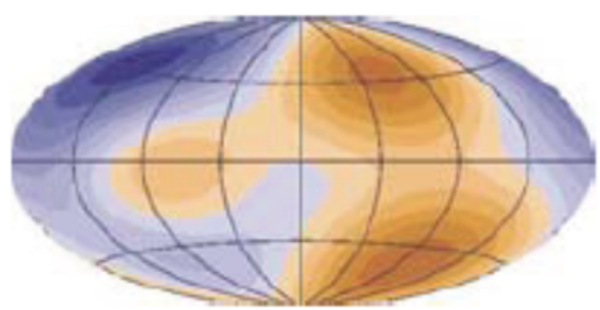
Neptune

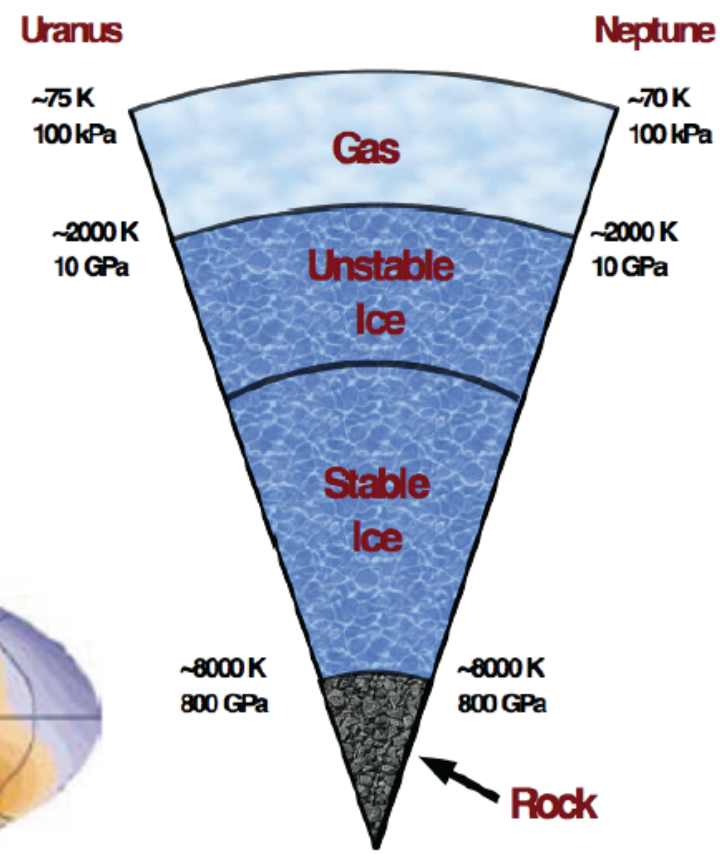


Stanley & Bloxham 2004, 2006

- work in a geometry suggested by low heat flow observations

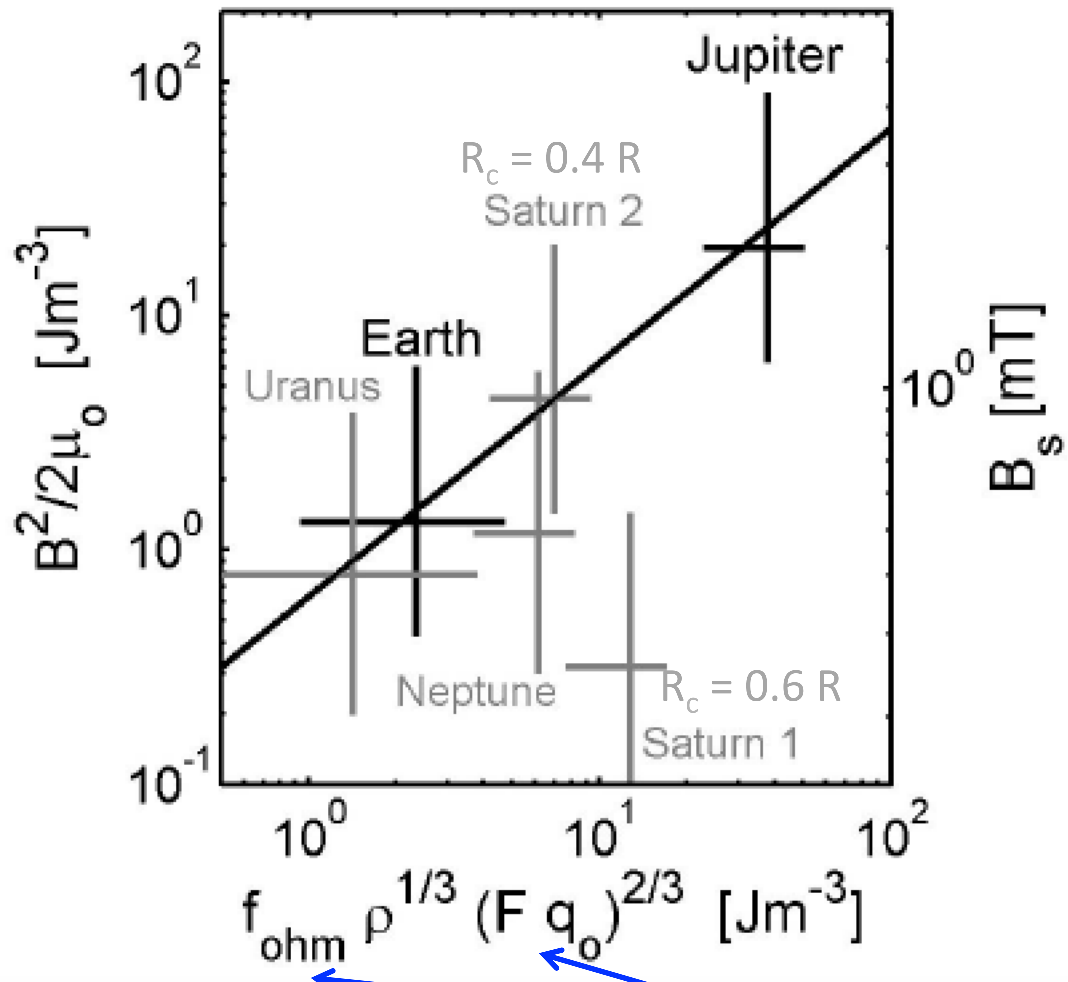






Level of saturation

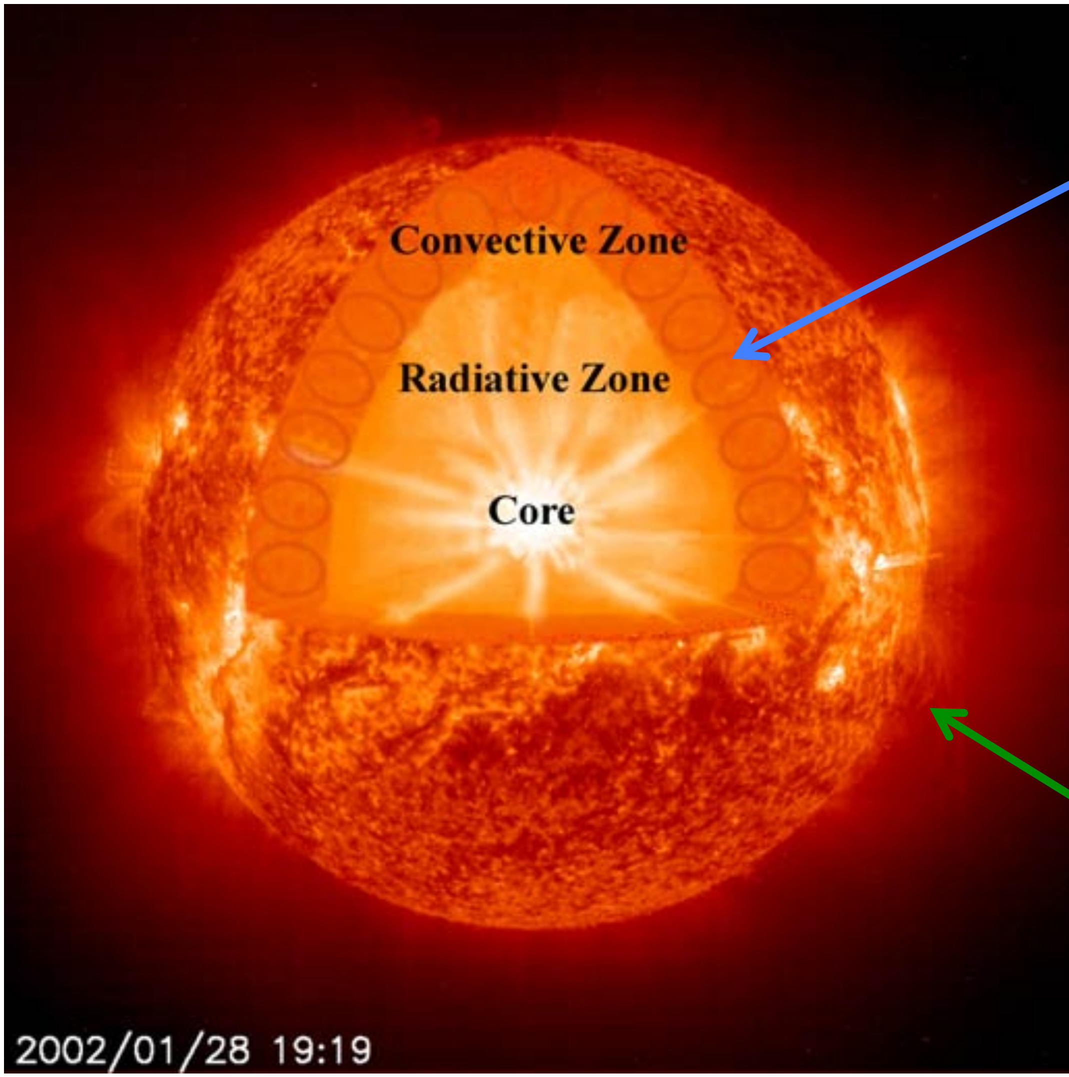
from Christensen



dimensionless factors

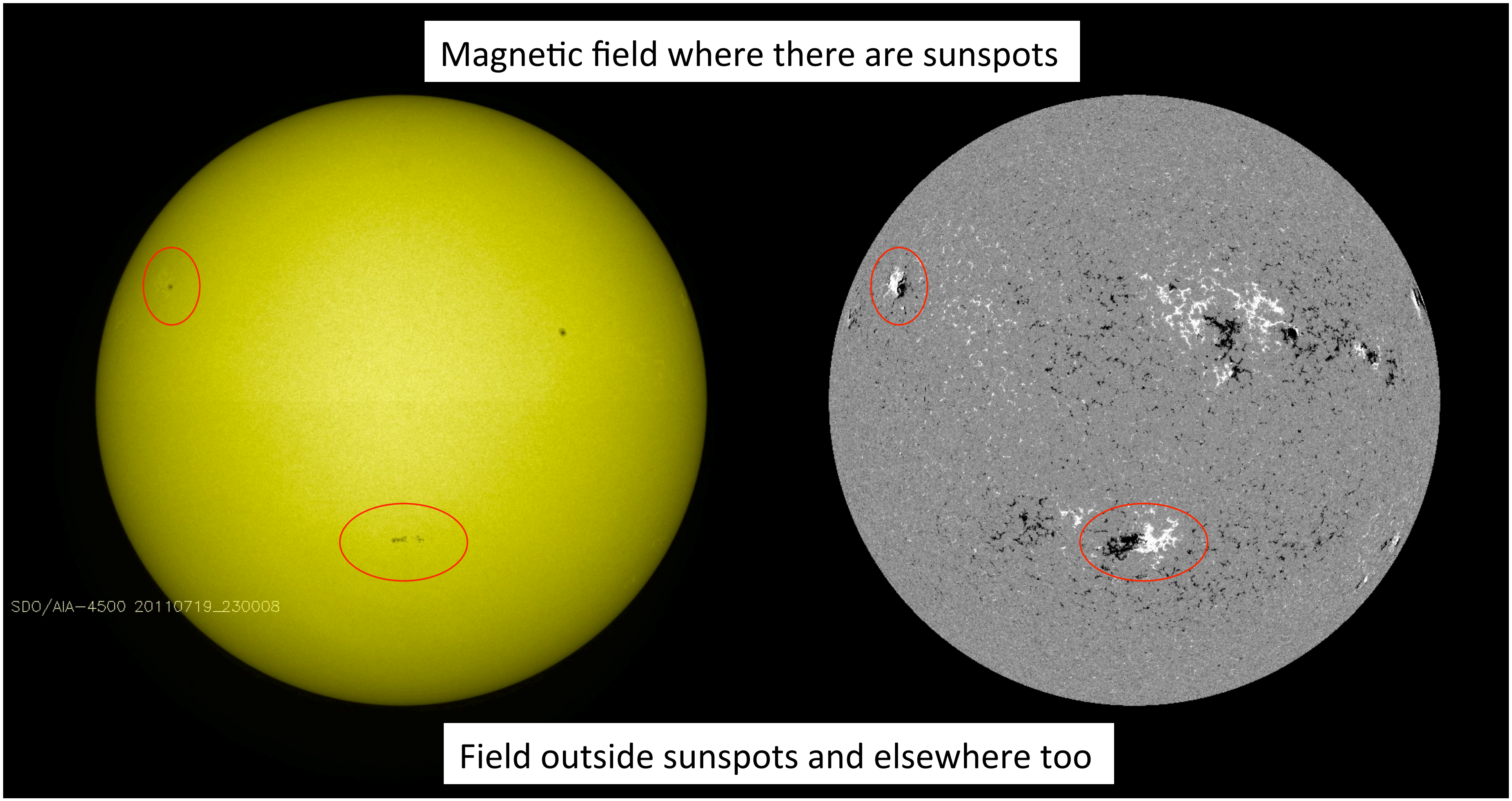
B saturates (exp growth ends) when driving power – thermal conduction q_0 – balances Ohmic dissipation

How it works in the Sun

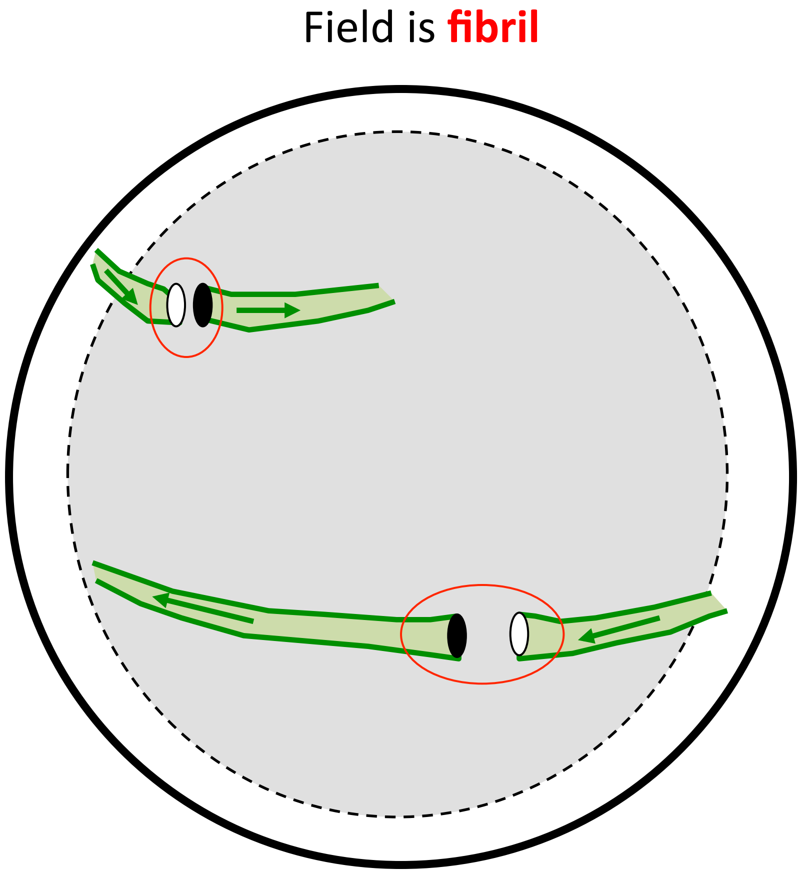


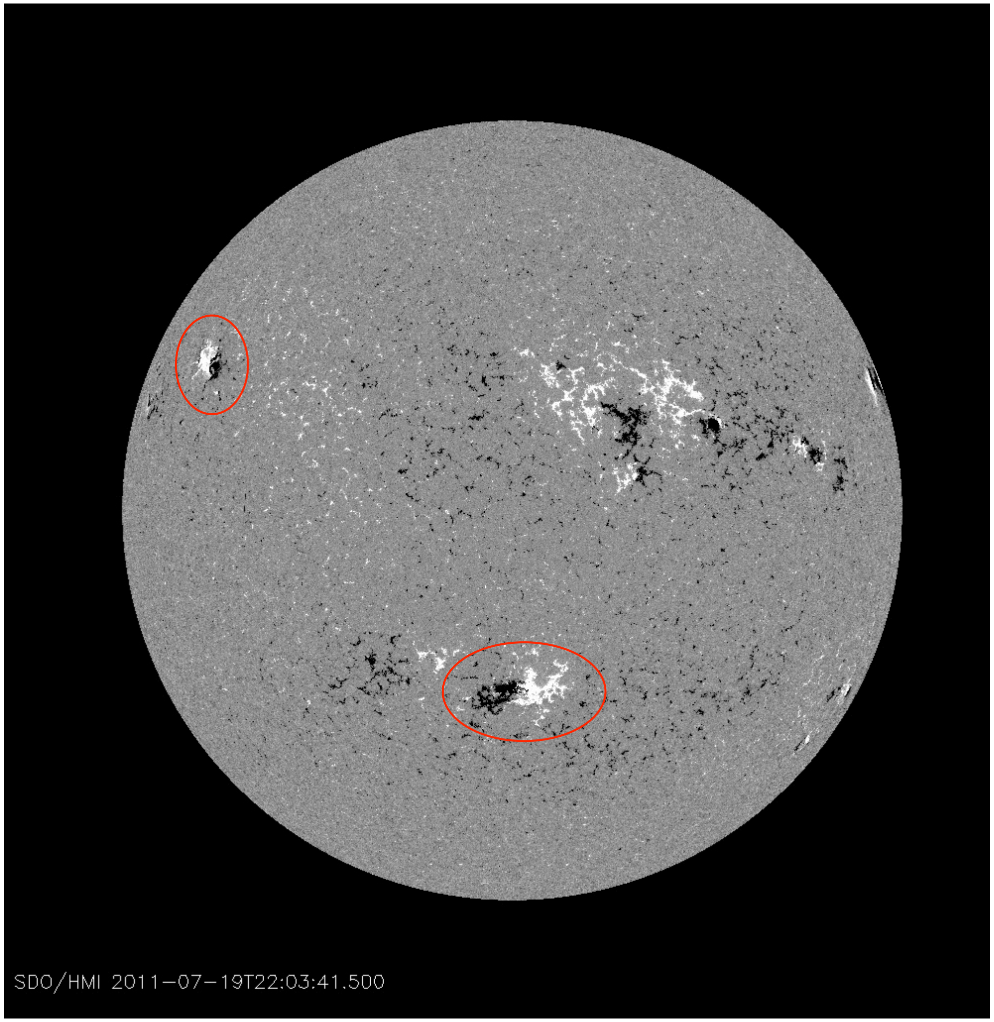
- Entire Star: H/He plasma
- Convection Zone (CZ)
  - Outer 200,000 km
  - Turbulence:
    $Re = 10^{10}$
  - Thermally driven
  - Good conductor
    $Rm = 10^{8}$
  - Rotation effective
    $Ro = 10^{-2}$
- Corona – conductive but tenuous:
  J smaller (~0?)

Evidence of the dynamo



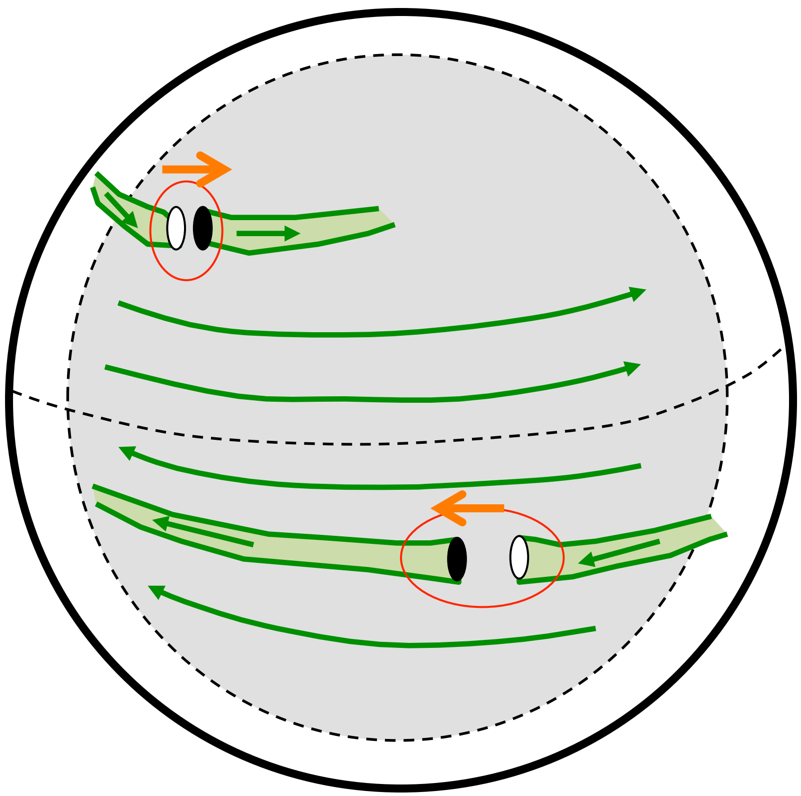
Evidence of the dynamo

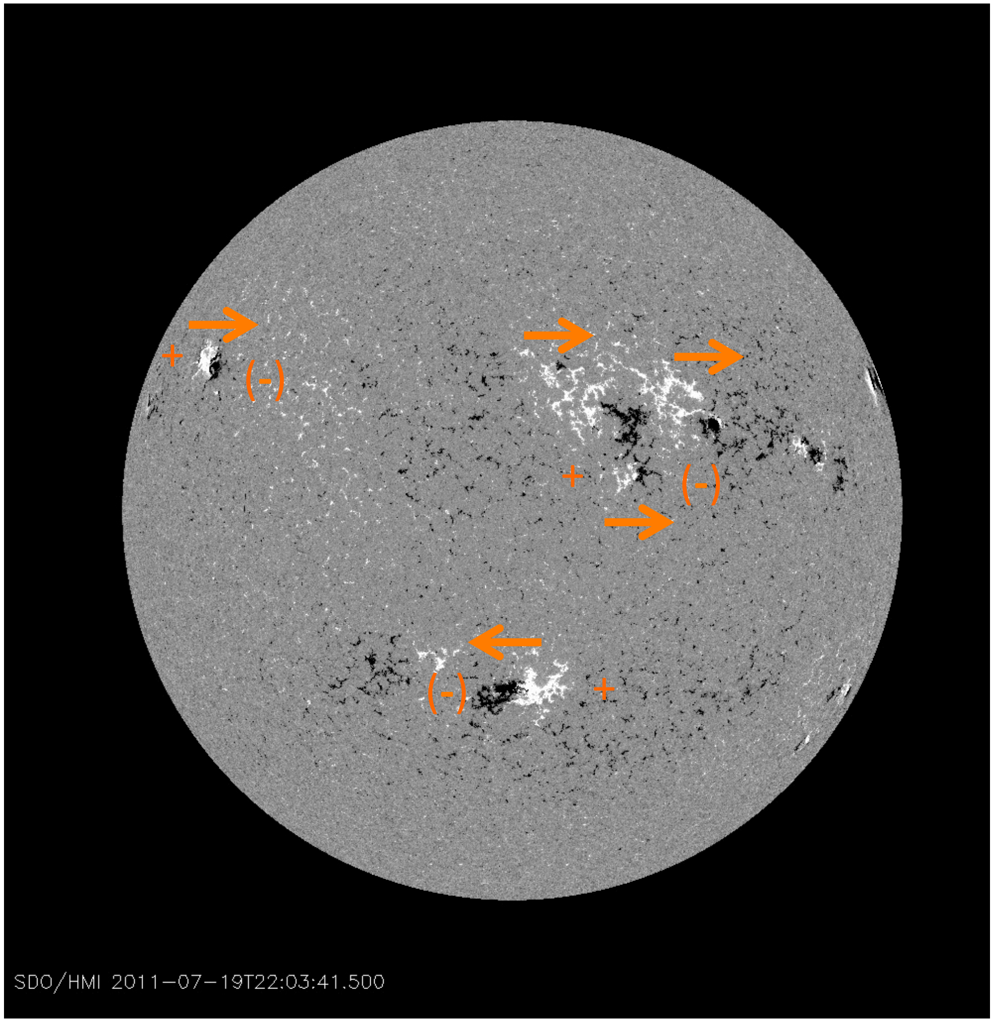


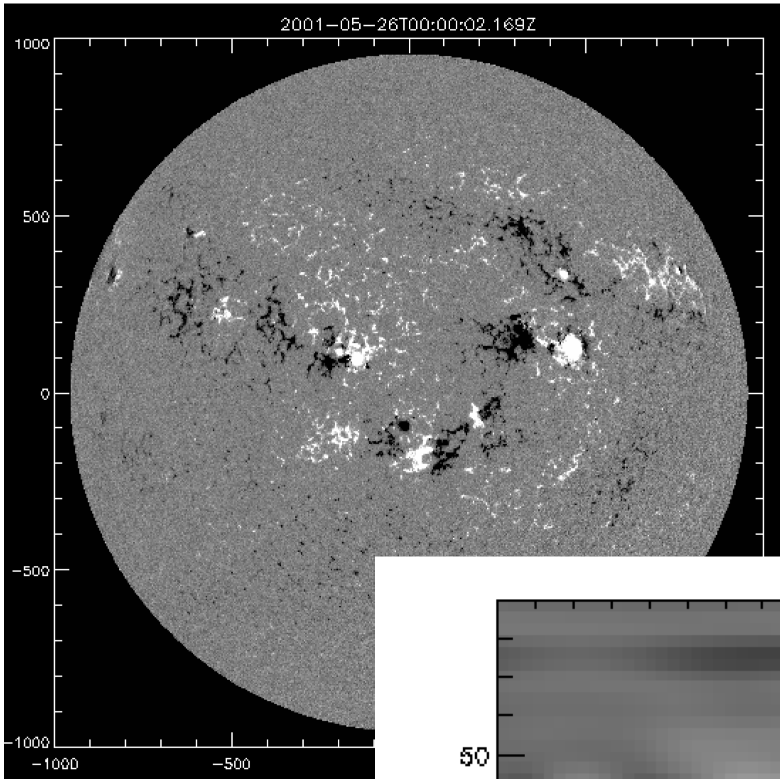


Evidence of the dynamo

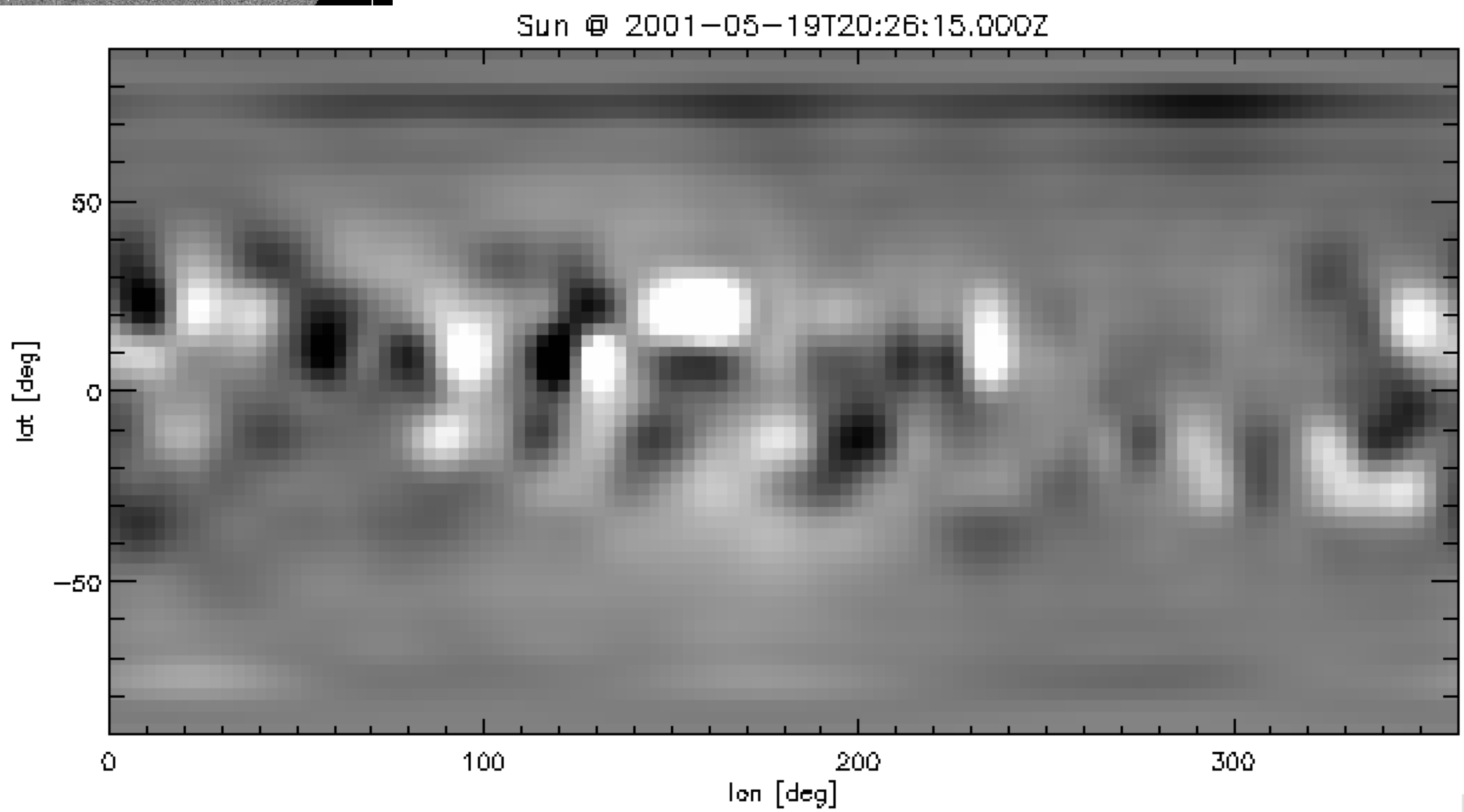
Field orientation: mostly toroidal

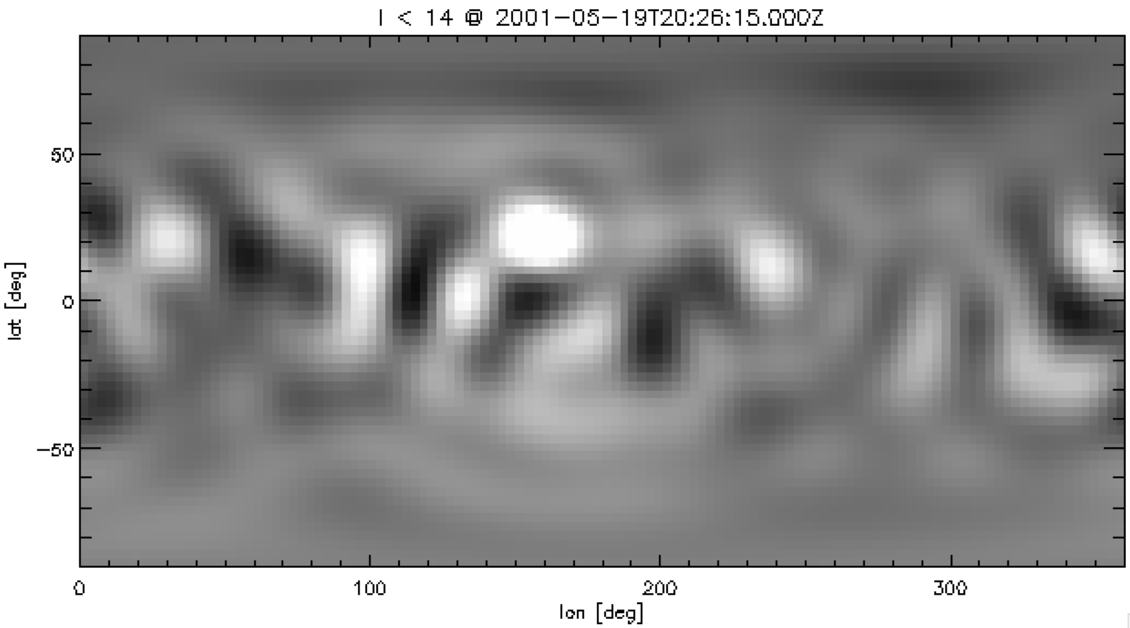


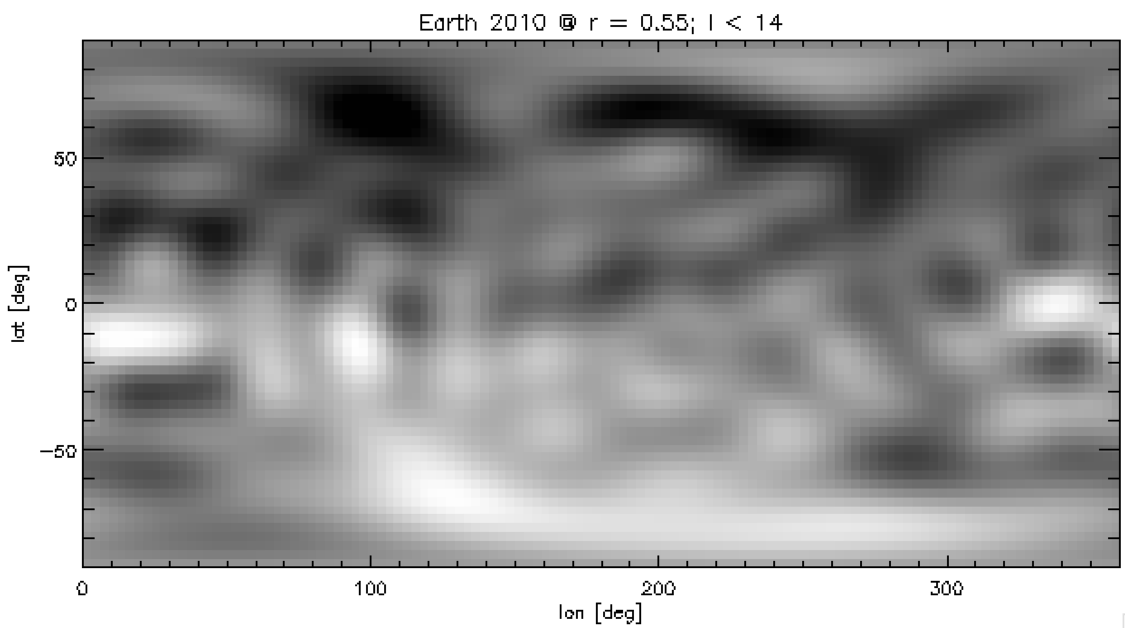




Synoptic plot: unwrapped view built up over time



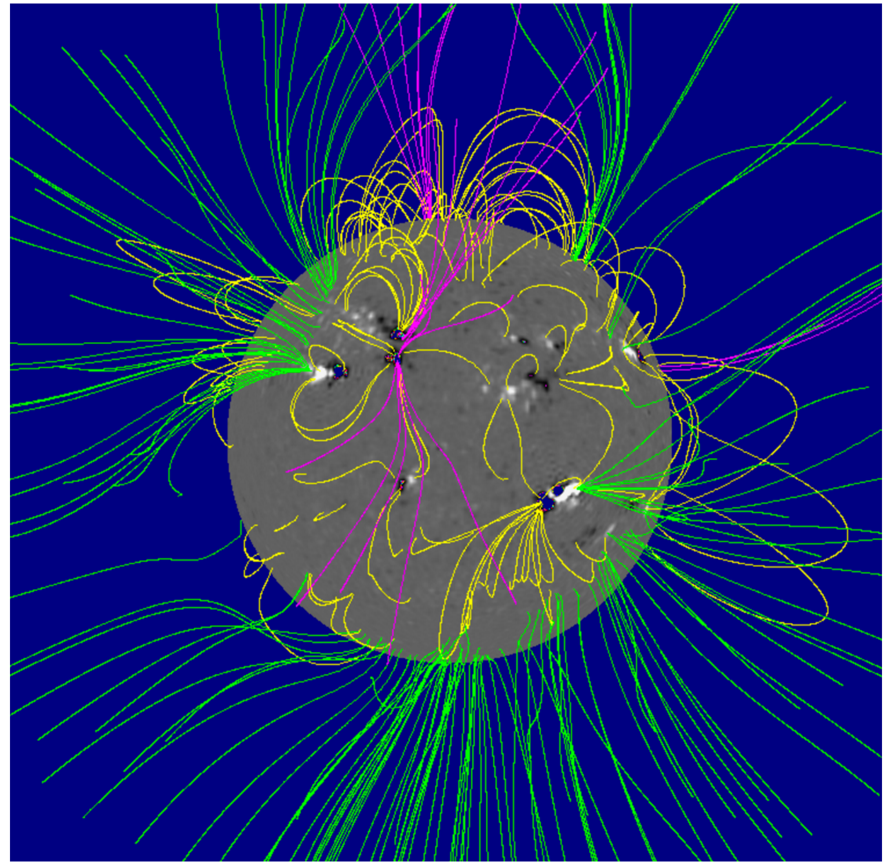




$Rm = 10^8$
$Ro = 10^{-2}$

Dynamo comparison: Sun vs. Earth

$Rm = 10^2$
$Ro = 10^{-6}$



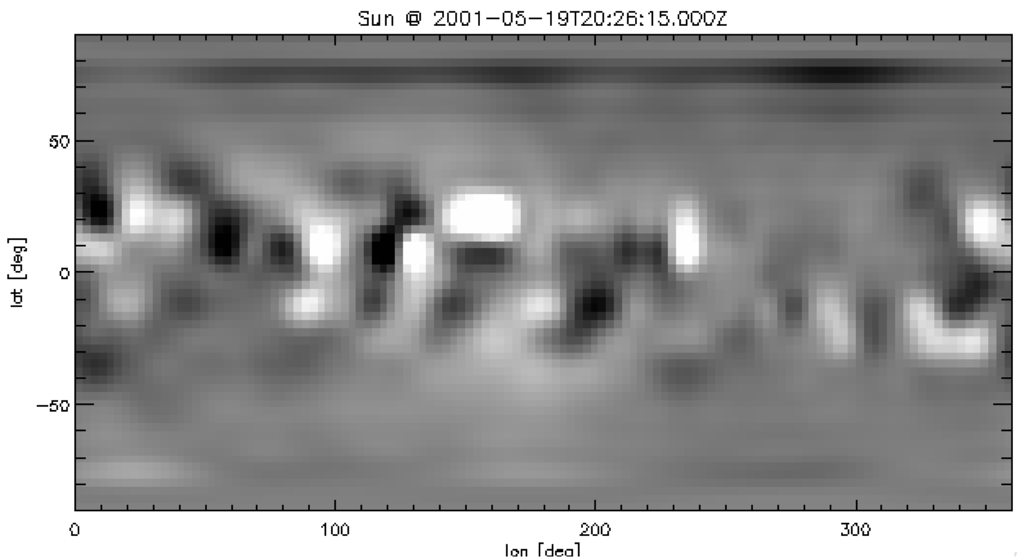
Assume corona has small (negligible) current:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} = 0$$

$$\mathbf{B} = -\nabla \chi$$

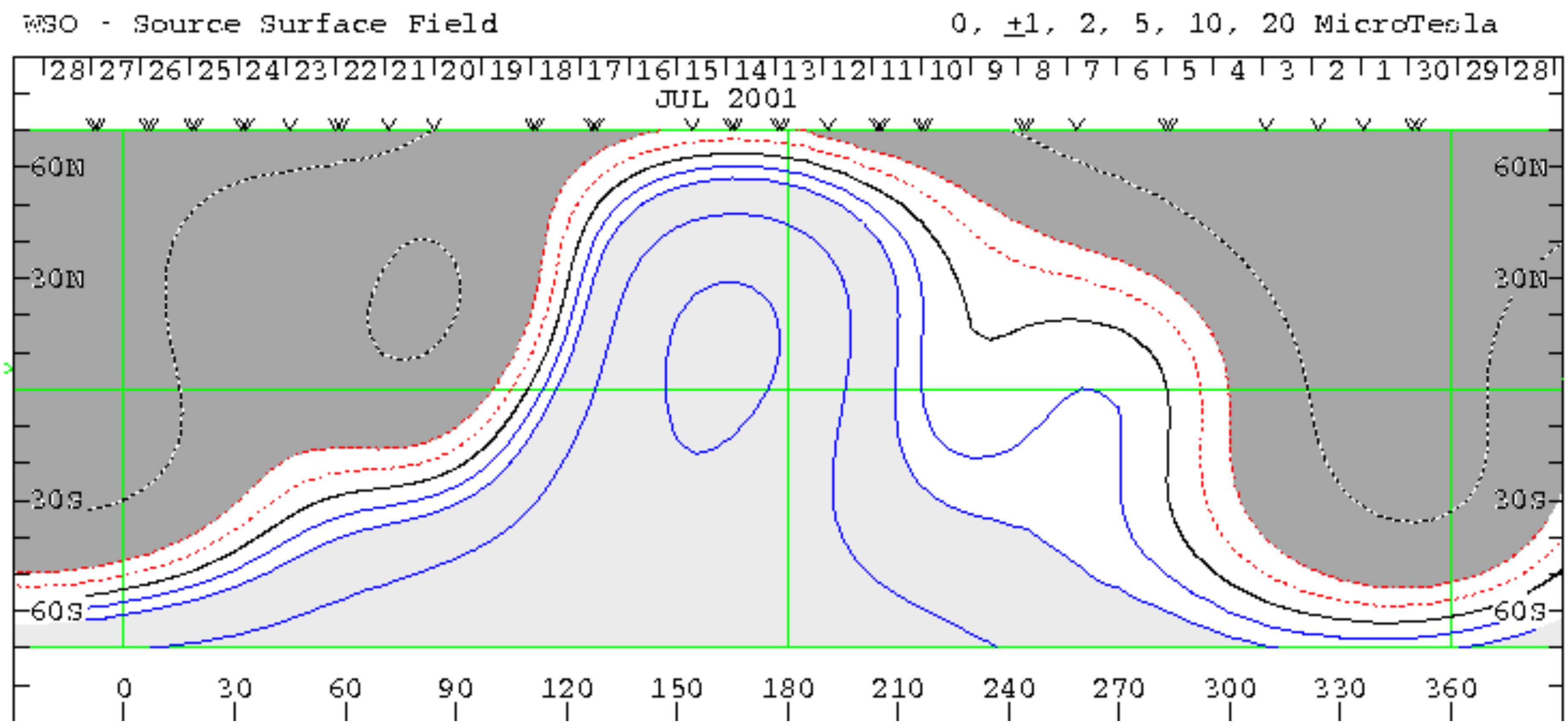
$$\nabla \cdot \mathbf{B} = -\nabla^2 \chi = 0$$

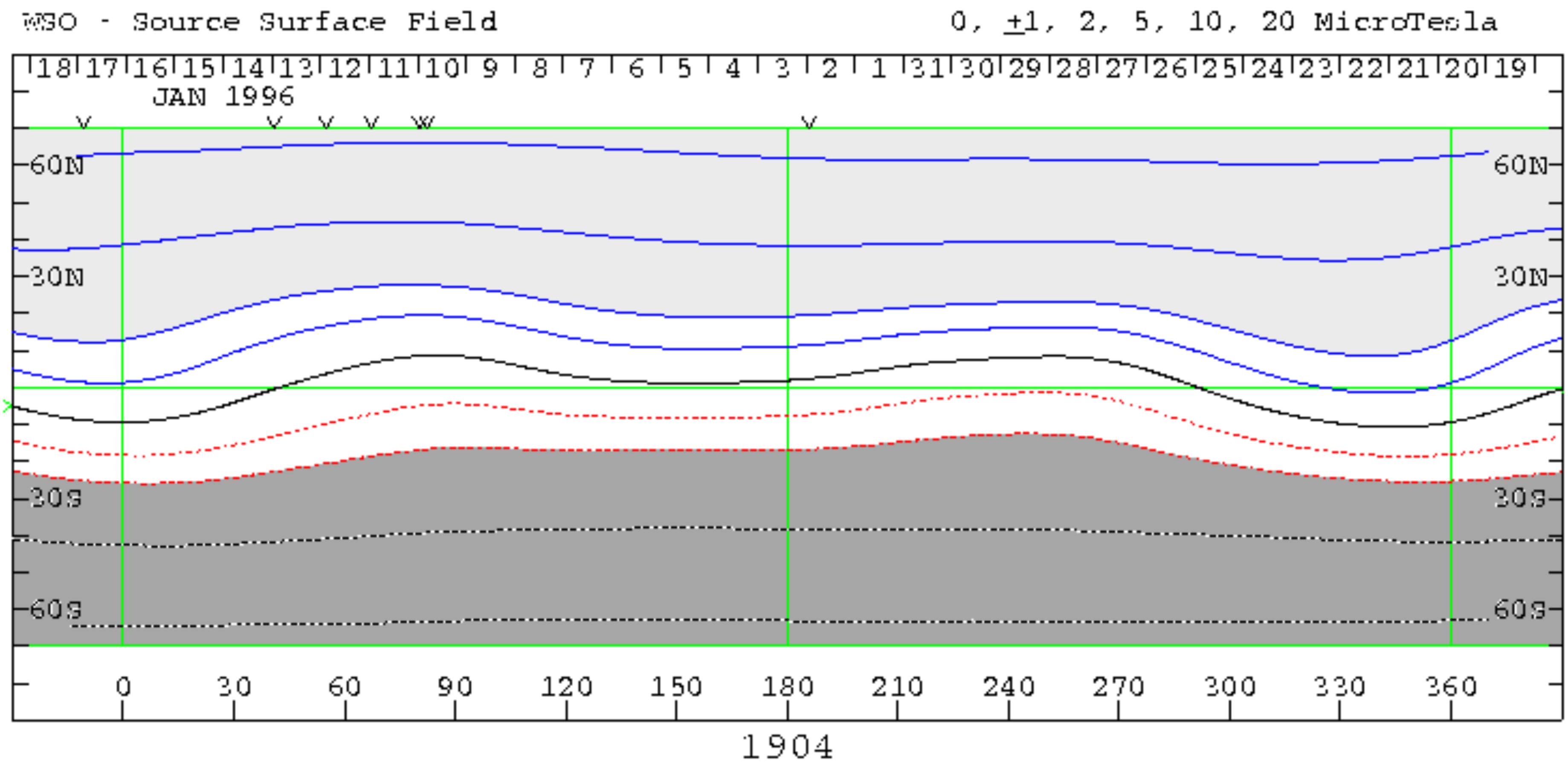
$$\chi(r,\theta,\phi) = \sum_{\ell,m} \tilde{g}_{\ell,m}\, Y_\ell^m(\theta,\phi) \left(\frac{R_\oplus}{r}\right)^{\ell+1}$$

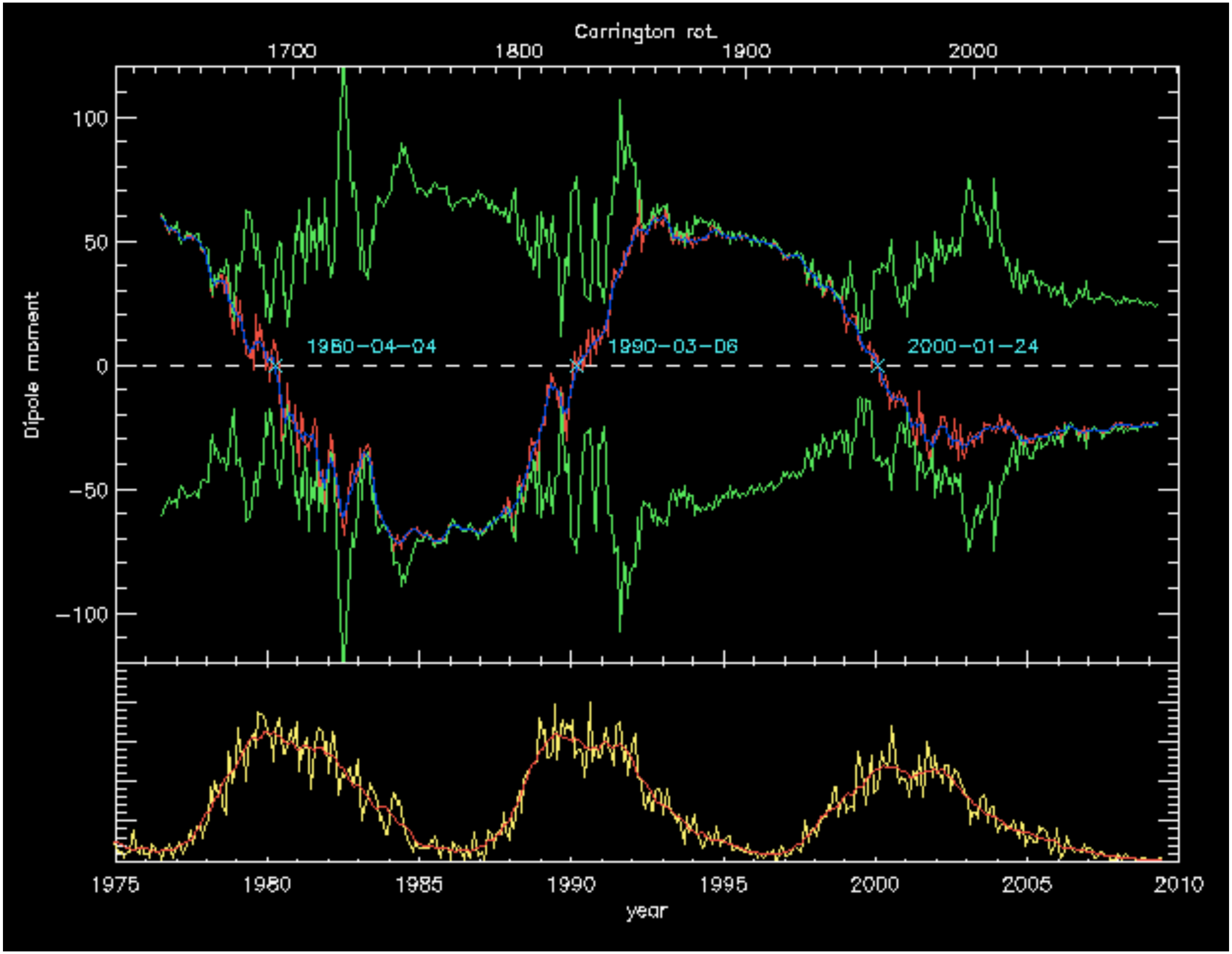


$r = R_\odot$

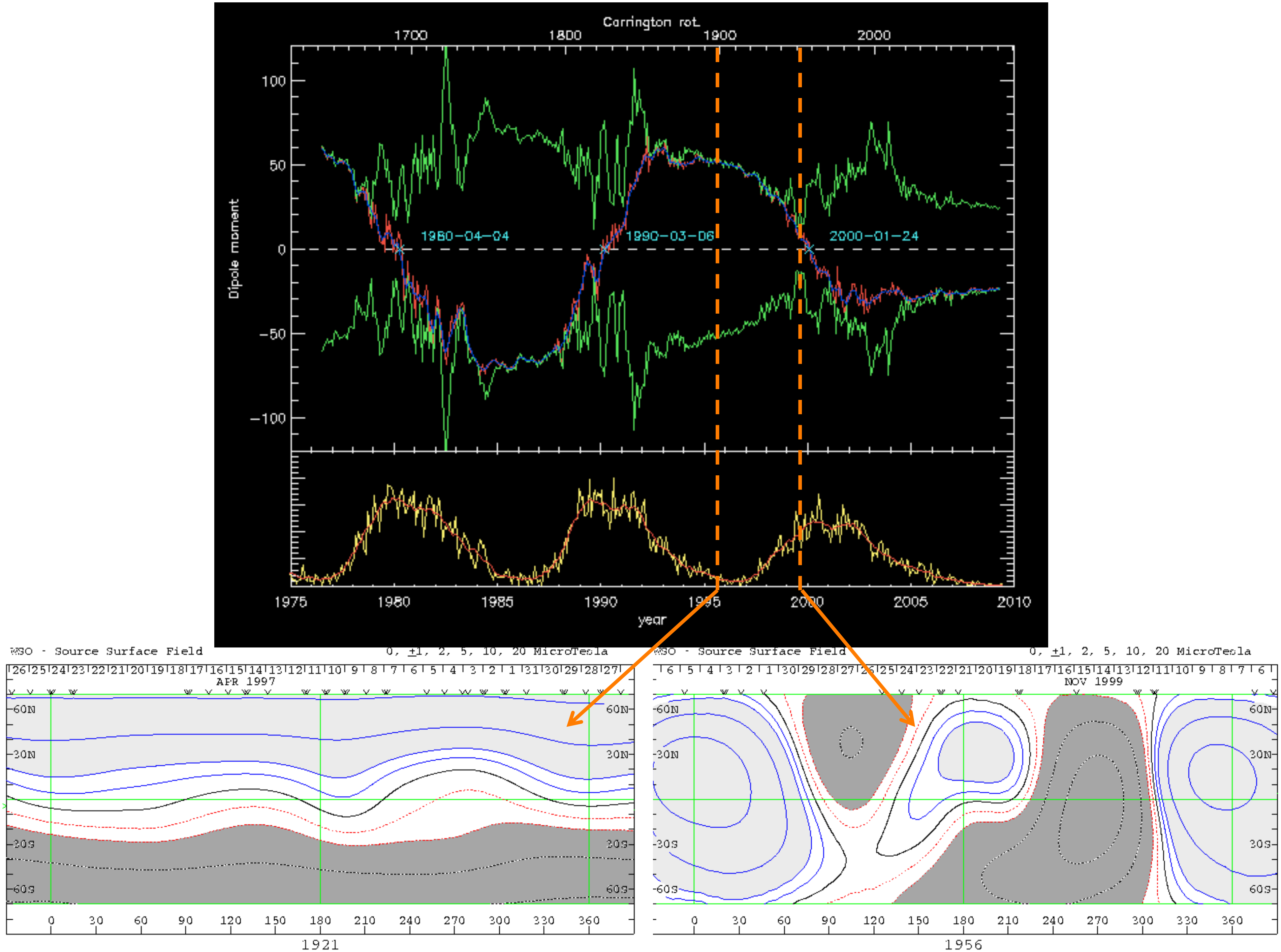
$r = 2.5\ R_\odot$




WSO - Source Surface Field
0, ±1, 2, 5, 10, 20 MicroTesla
18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 31 30 29 28 27 26 25 24 23 22 21 20 19
JAN 1996
60N
30N
30S
60S
0 30 60 90 120 150 180 210 240 270 300 330 360
1904

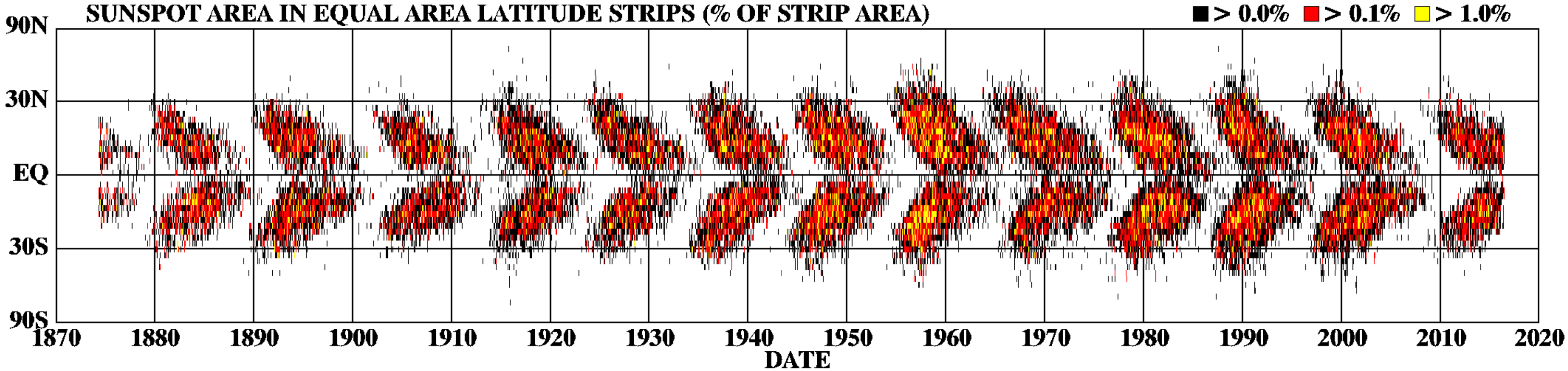


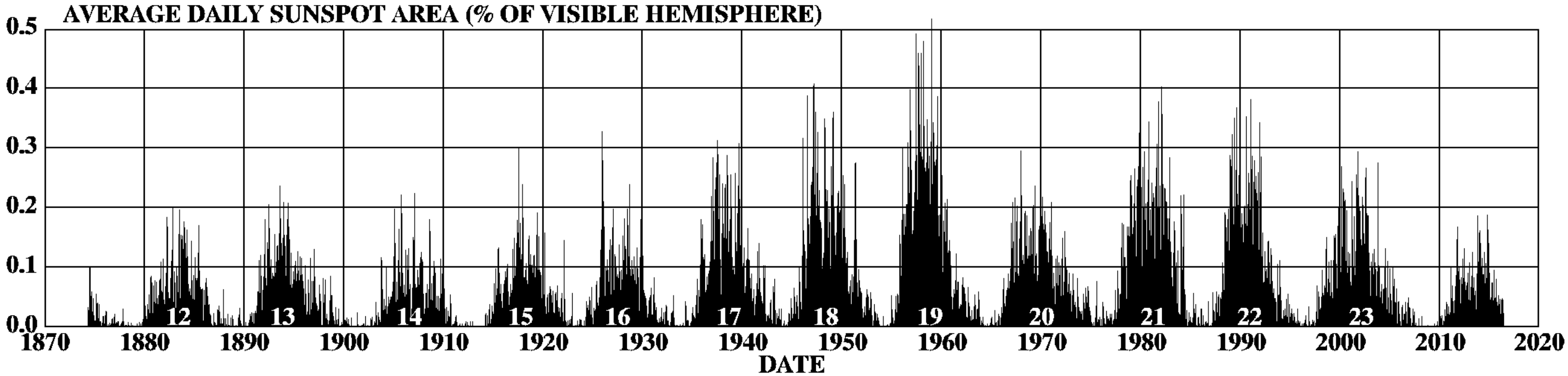
Carrington rot.
1700
1800
1900
2000
100
50
0
-50
-100
Dipole moment
1980-04-04
1990-03-06
2000-01-24
1975
1980
1985
1990
1995
2000
2005
2010
year



Carrington rot.
1700
1800
1900
2000
Dipole moment
100
50
0
−50
−100
1980-04-04
1990-03-06
2000-01-24
1975
1980
1985
1990
1995
2000
2005
2010
year
WSO - Source Surface Field
0, ±1, 2, 5, 10, 20 MicroTesla
APR 1997
1921
WSO - Source Surface Field
0, ±1, 2, 5, 10, 20 MicroTesla
NOV 1999
1956

DAILY SUNSPOT AREA AVERAGED OVER INDIVIDUAL SOLAR ROTATIONS

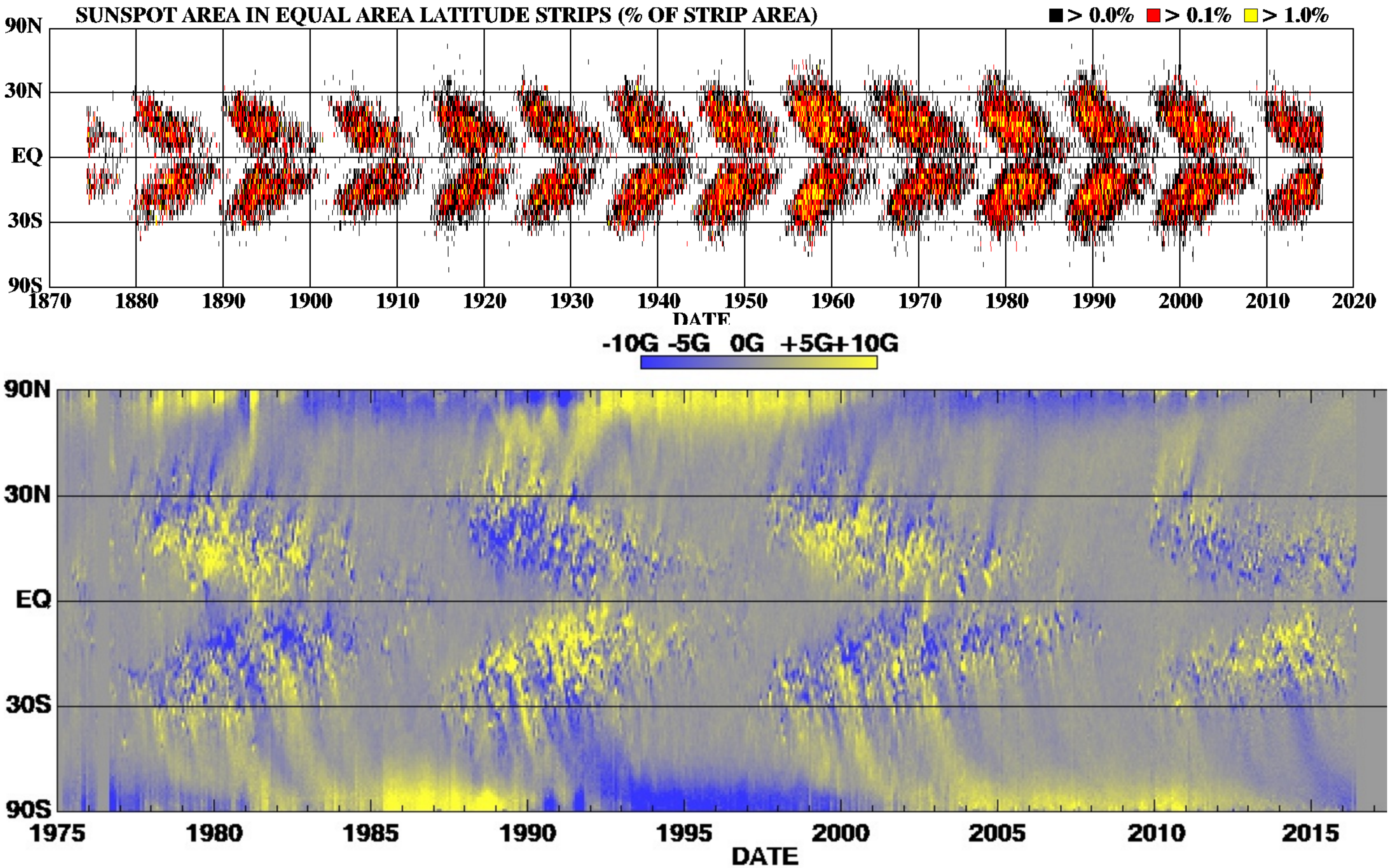




http://solarscience.msfc.nasa.gov/

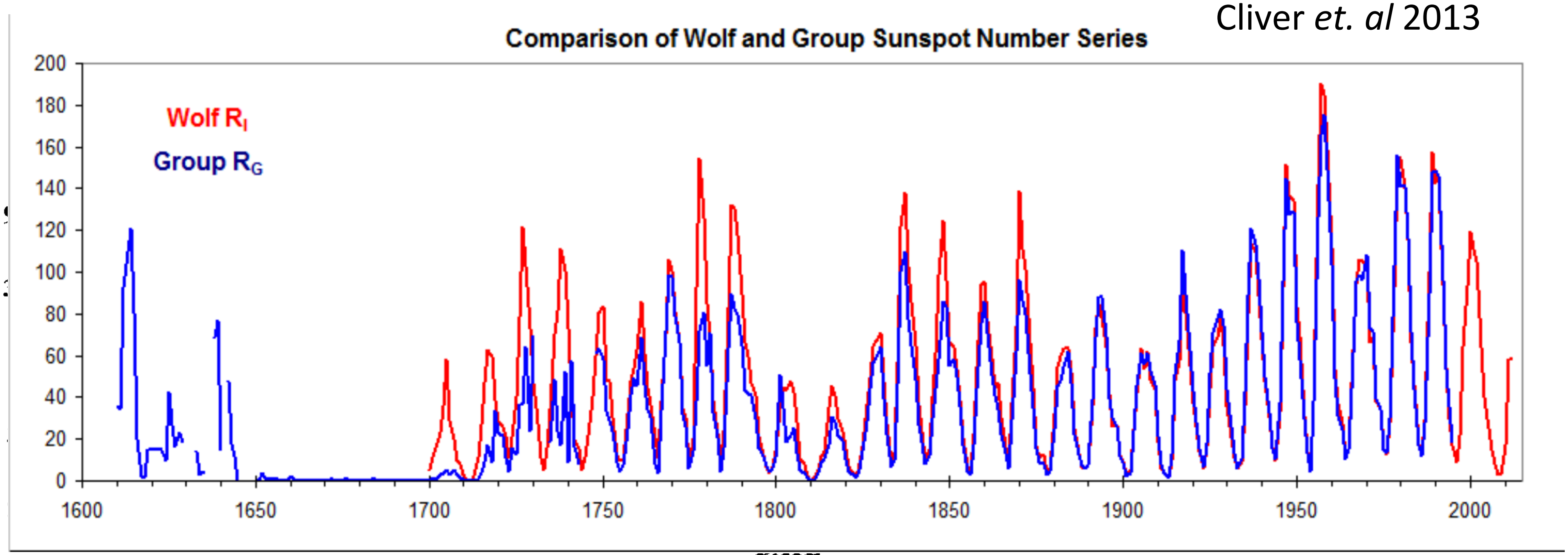
HATHAWAY NASA/ARC 2016/07

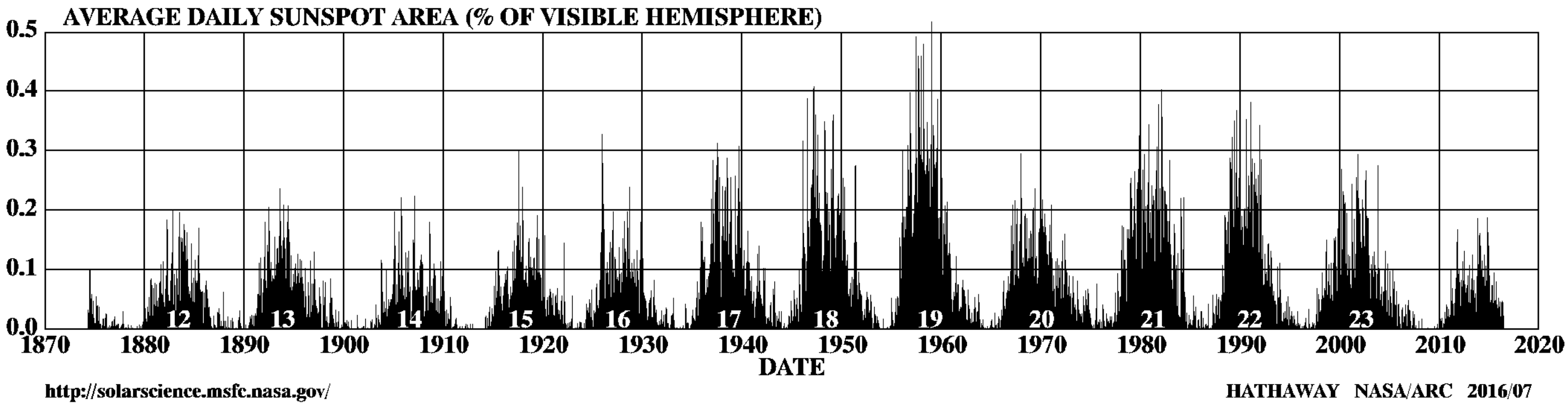
DAILY SUNSPOT AREA AVERAGED OVER INDIVIDUAL SOLAR ROTATIONS



Hathaway NASA ARC 2016/07

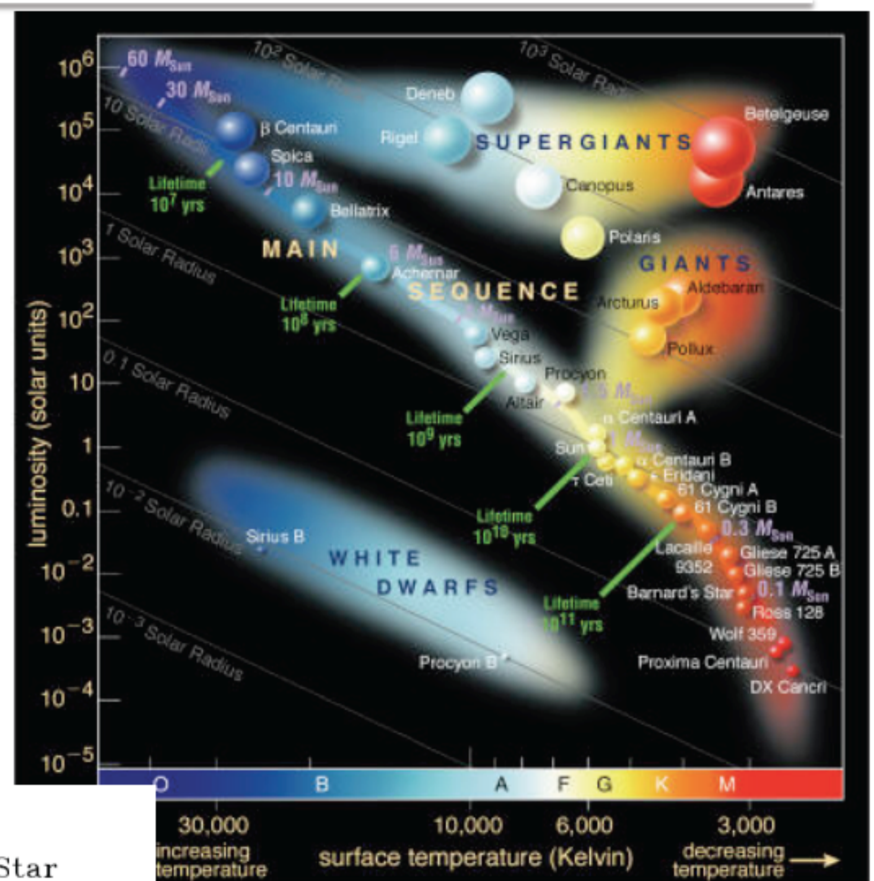
Cliver *et. al* 2013



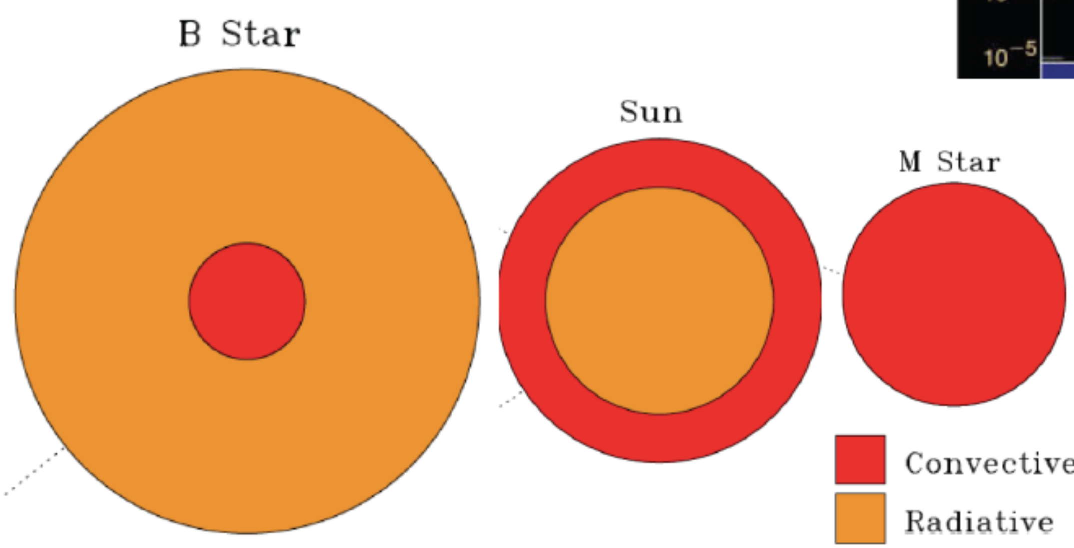


STELLAR MAGNETIC FIELDS

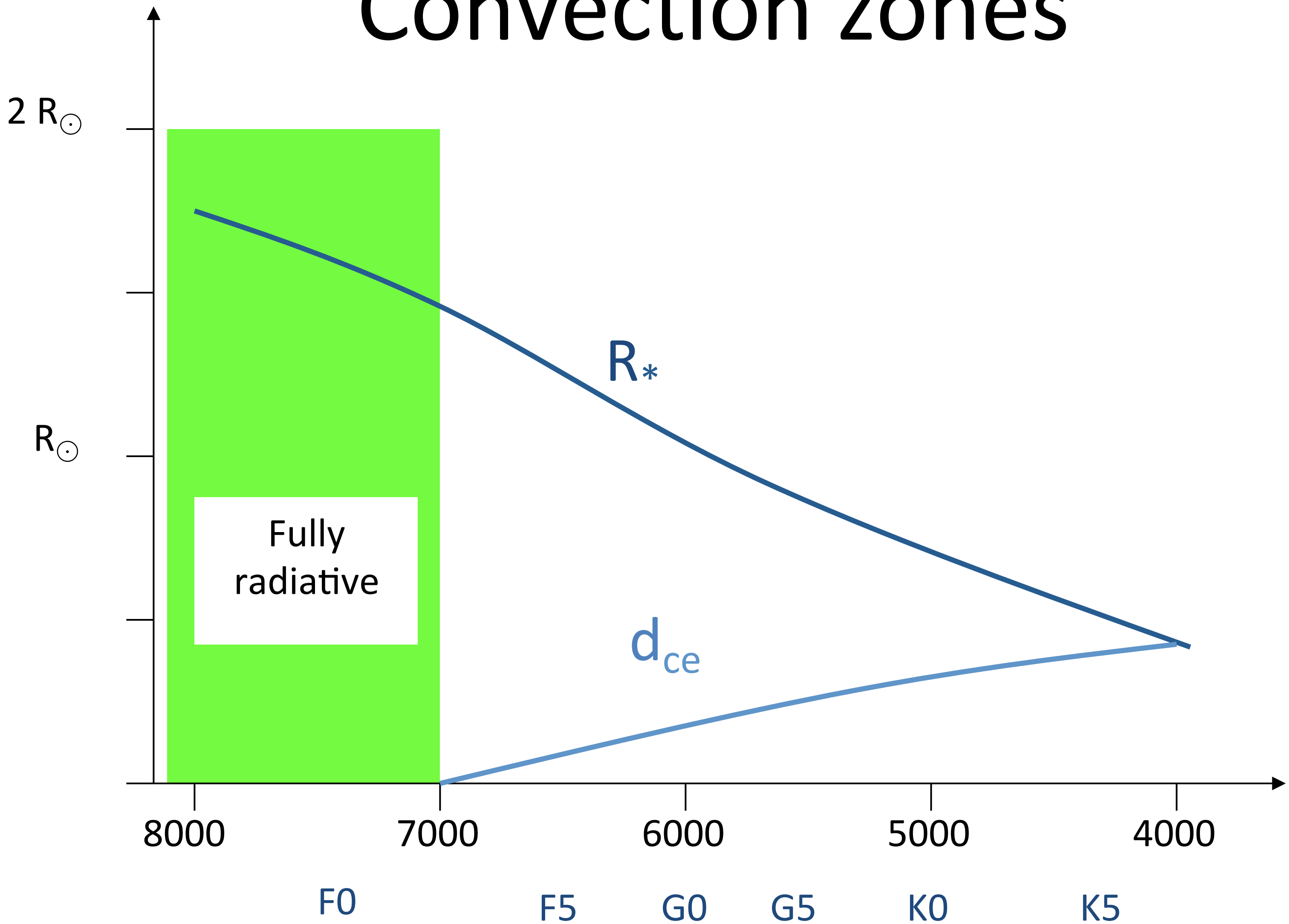
- correlations between stellar types and magnetic field properties, probably due to geometry of convection zones
- stars with outer convection zones (late-type stars) have observed magnetic fields whose strength tends to increase with their angular velocity
- Cyclic variations are known to exist only for spectral types between G0 and K7).



Stanley



Convection zones

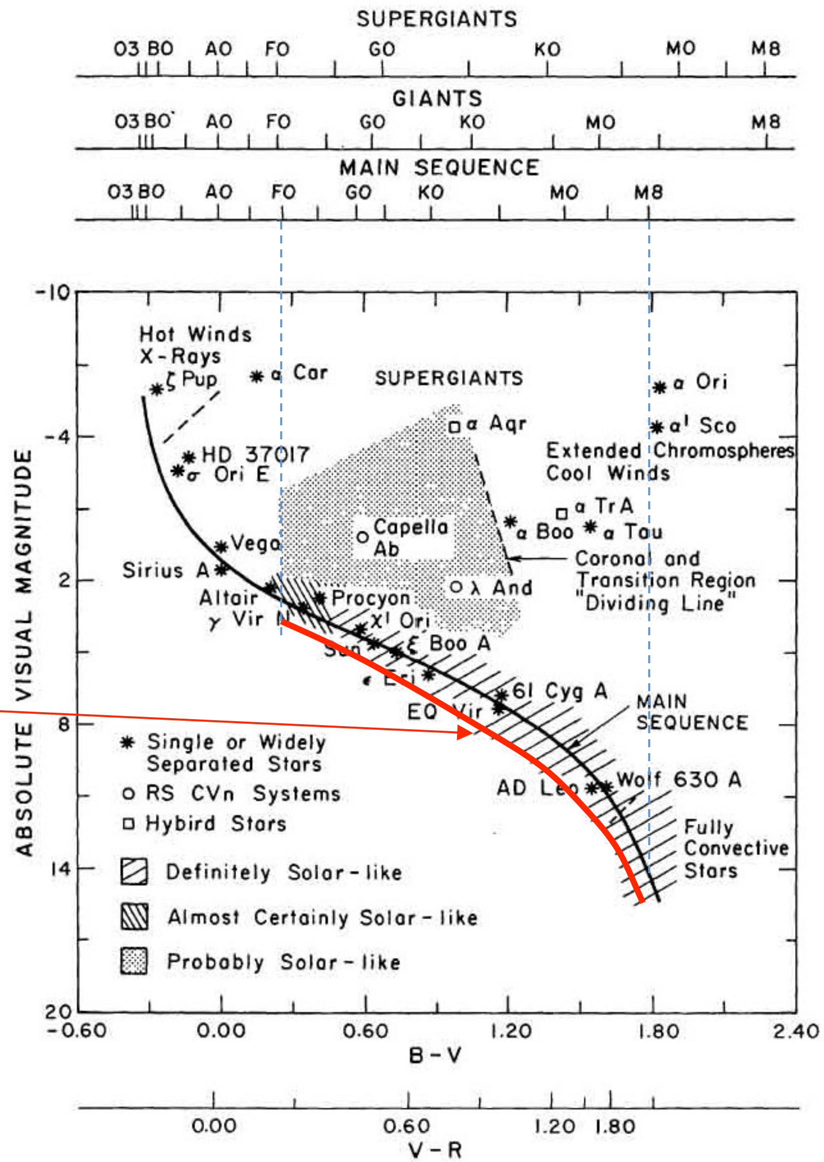


Other stars

Evidence of magnetic activity

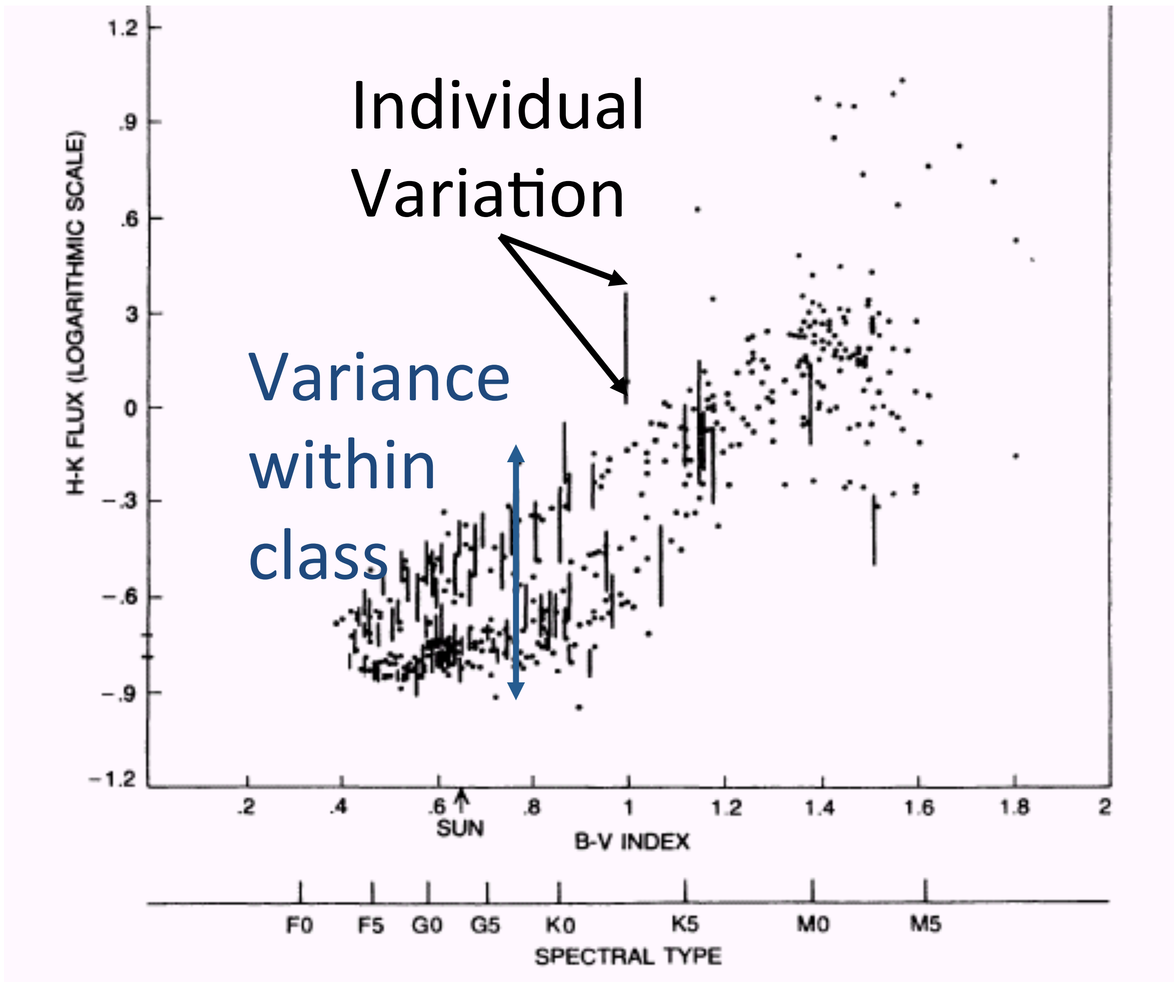
Activity on main sequence: types **F → M**

B-V > 0.4



(From Linsky 1985)

Explaining Activity Levels



The Dynamo Number

Parker's Dynamo #

$$N_D = \frac{\alpha_{\text{dyn}} \Omega' d^4}{\eta^2}$$

Dynamo is linear instability for $N_D > N_{crit}$

Dynamo α-effect: $\alpha_{\text{dyn}} \equiv \tau \langle \mathbf{v} \cdot (\nabla \times \mathbf{v}) \rangle \sim \Omega d$

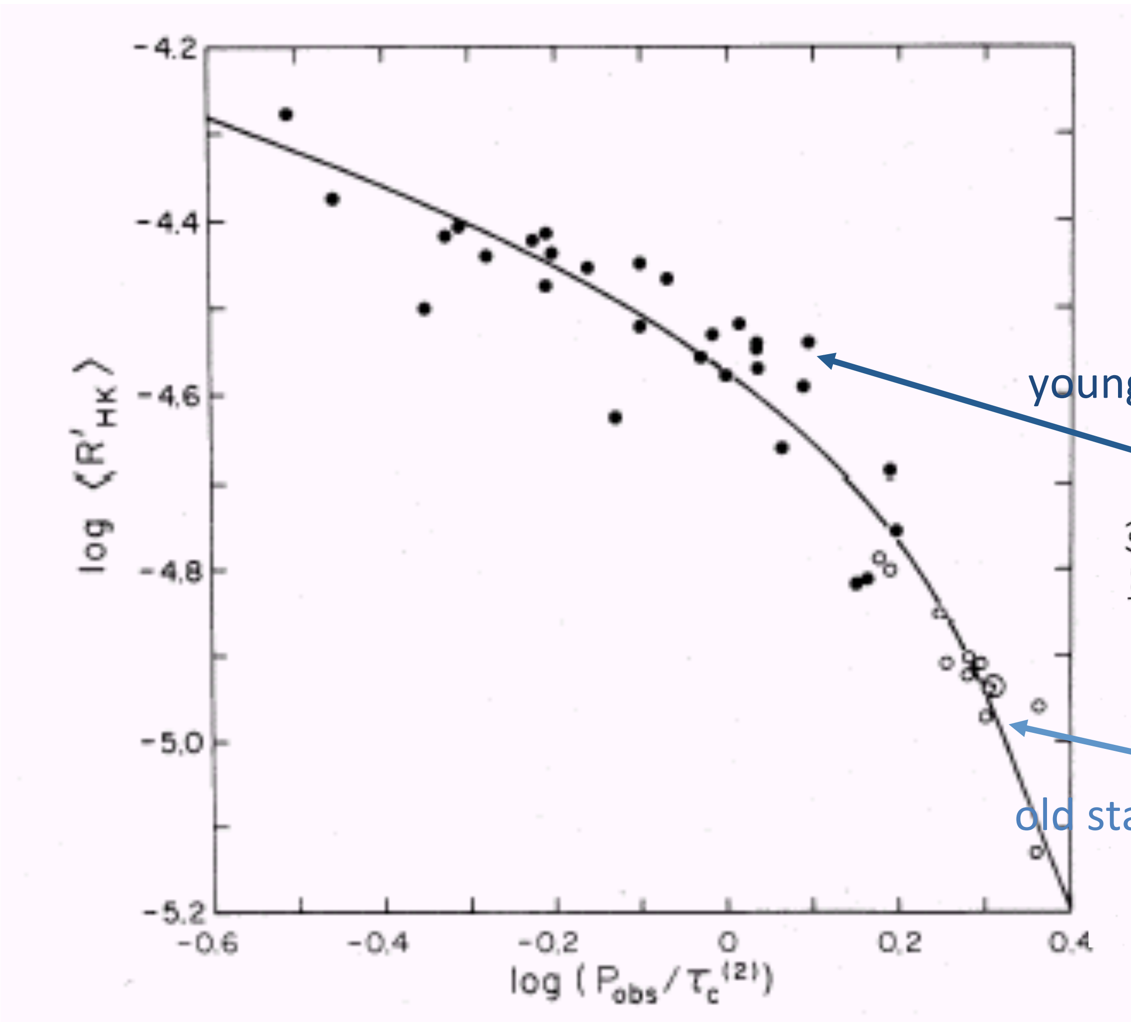
$$\eta = \eta_{\text{turb}} \sim \frac{d^2}{\tau_c} \qquad \Omega' = \frac{d\Omega}{dr} \sim \frac{\Omega}{d}$$

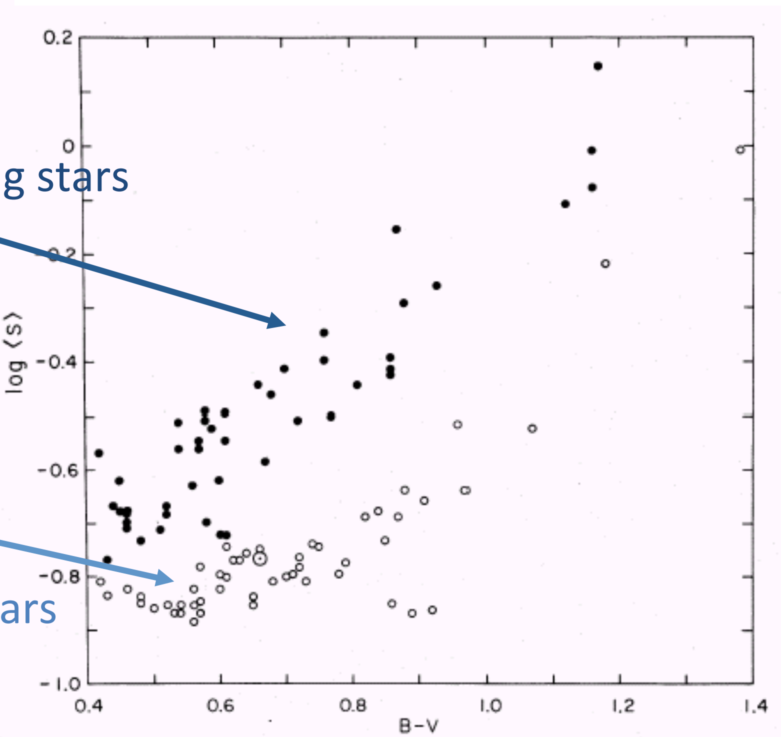
$$N_D \sim (\Omega \tau_c)^2 \sim (P_{\text{obs}} / \tau_c)^{-2} = Ro^{-2}$$

Activity vs. Rossby Number

41 Local stars

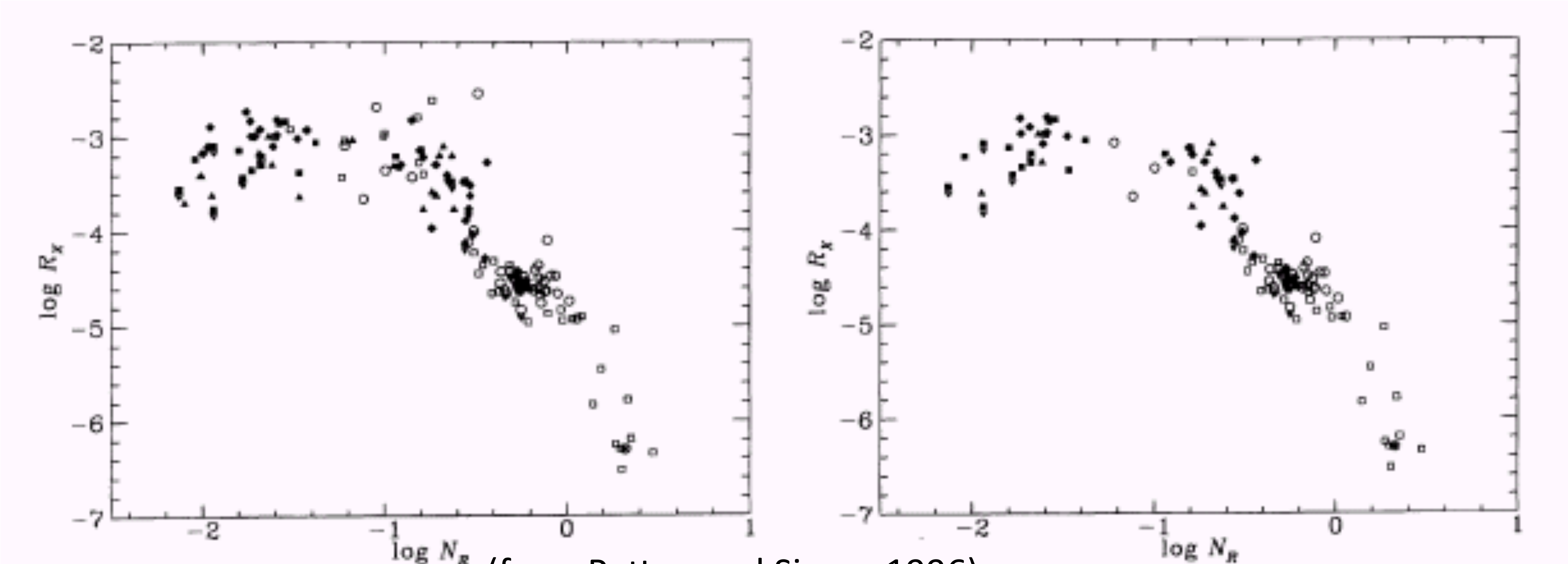
P_{obs} from S(t)





(from Noyes et al. 1984)

Activity vs. Rossby Number



(from Patten and Simon 1996)

- Stars in open cluster 2391 (30My old)
- R_X from ROSAT observations
- Rotation periods P_{obs} from optical photometry
- $N_R = Ro = P_{obs}/t_c$

Summary

- Magnetic fields – all from dynamos
  - Conducting fluid
  - Complex motions w/ enough *umph*
- Create complex fields
- Fields evolve in time – reverse occasionally
- Differences from different parameters:

  Rm, Re, Ro