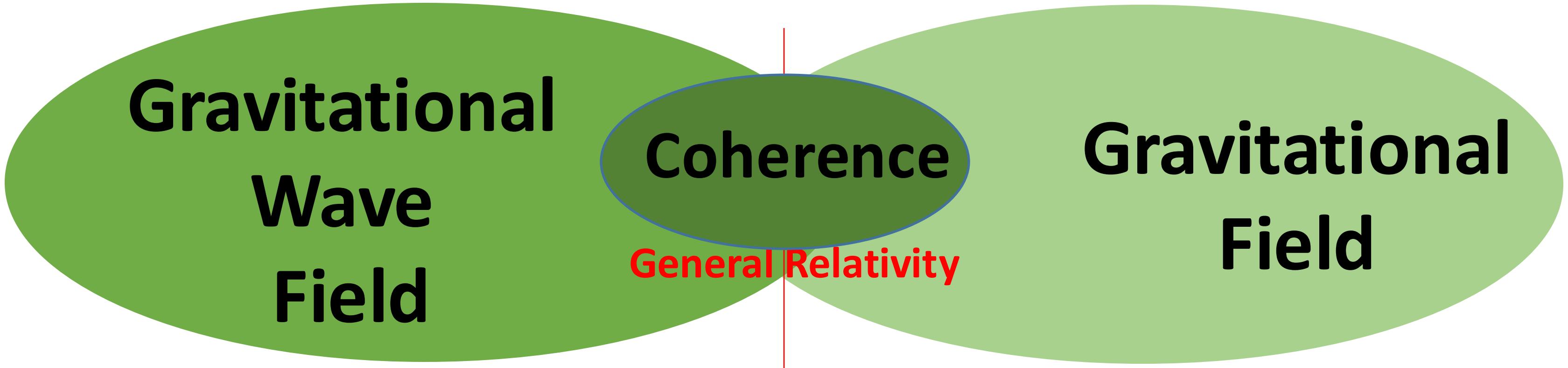
The Physics of Heliophysics

A Very Short Introduction

Tom Bogdan

All the heliosphere is a stage,
*And all **matter** and **classical fields** are merely players;*
They have their exits and entrances,
And each, in their time, plays several parts.

Classical Fields



$$R^{\alpha\beta} - \frac{1}{2} R g^{\alpha\beta} = \frac{8\pi G}{c^4}\left[T_m^{\alpha\beta} + T_{E+M}^{\alpha\beta} + T_{rad}^{\alpha\beta}\right]$$

statistical deterministic

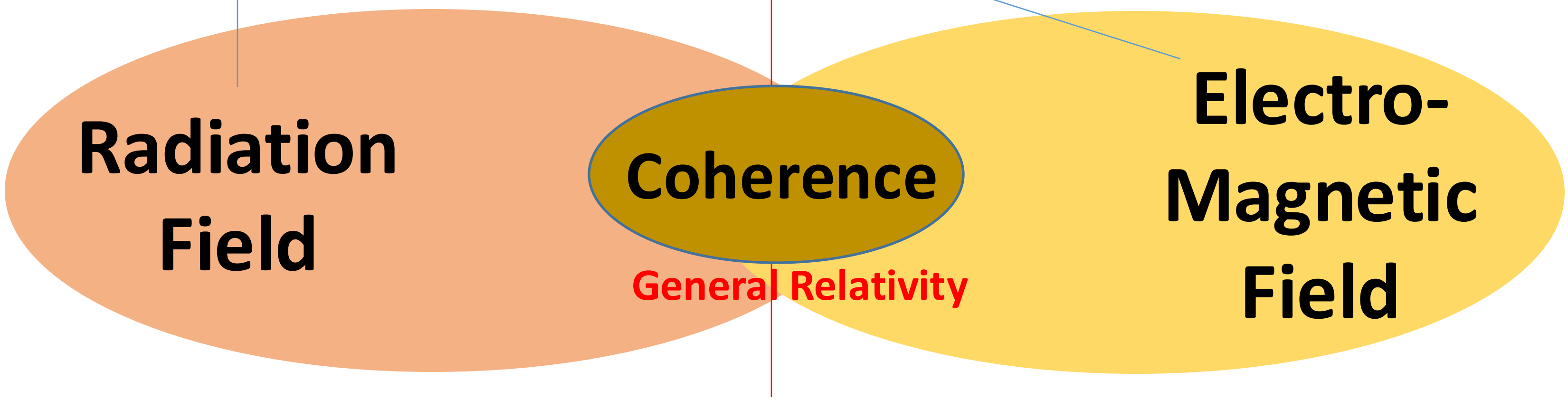
$$F^{\alpha\beta}_{\ ;\beta} = \mu_0 J^\alpha$$

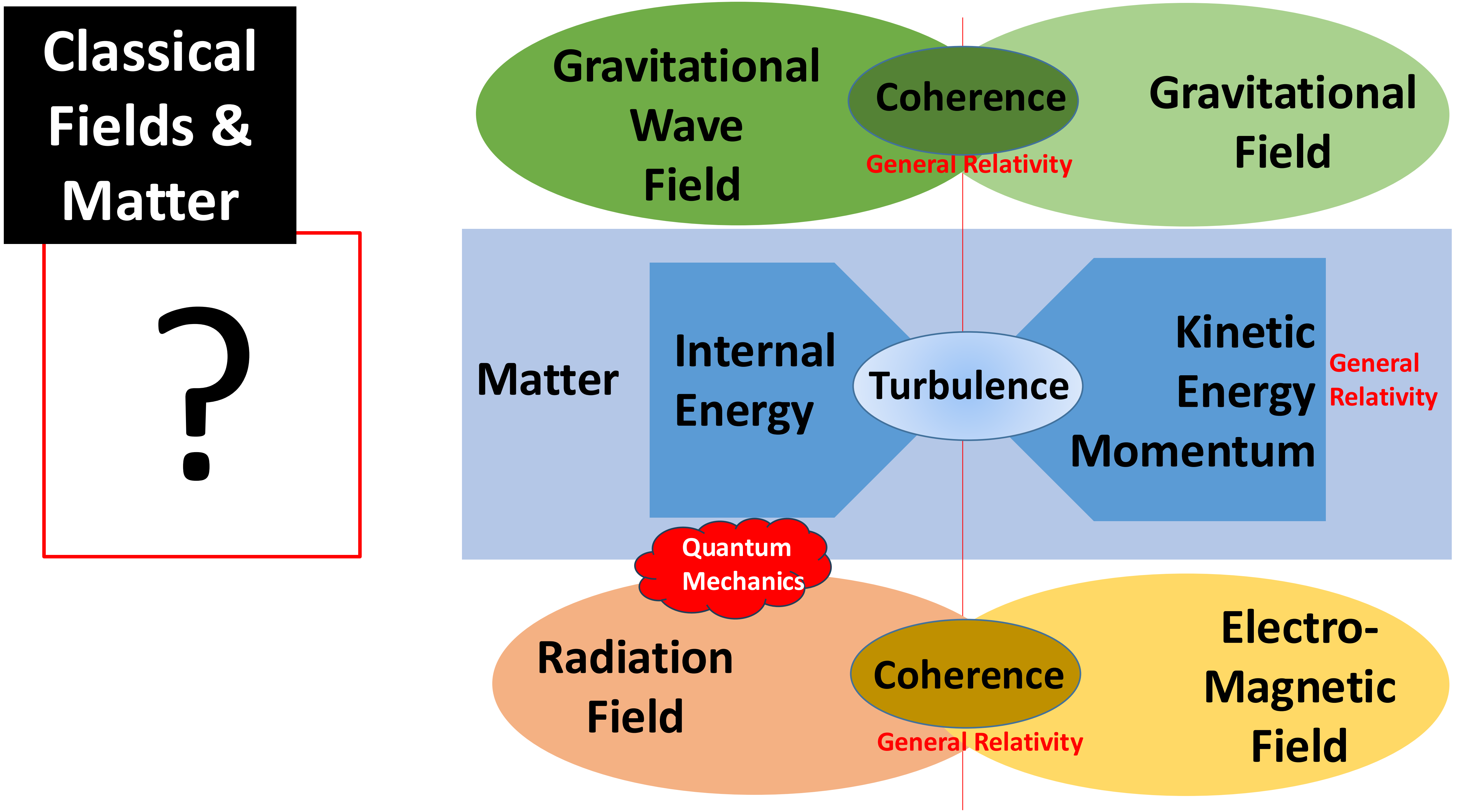
$$\nabla \cdot \boldsymbol{E} = \rho_e/\varepsilon_0 \qquad \nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \mu_0 \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}$$

in a flat Minkowski spacetime





Classical Fields & Matter
?
Gravitational Wave Field
Coherence
General Relativity
Gravitational Field
Matter
Internal Energy
Turbulence
Kinetic Energy Momentum
General Relativity
Quantum Mechanics
Radiation Field
Coherence
General Relativity
Electro-Magnetic Field

Matter

Scale	Equation	Theory
*Nano*scopic	$i\hbar\frac{\partial}{\partial t}\lvert\psi\rangle = \mathcal{H}\lvert\psi\rangle$	quantum mechanics
*Micro*scopic	$\dot{q}_i = \frac{\partial\mathcal{H}}{\partial p_i} \qquad \dot{p}_i = -\frac{\partial\mathcal{H}}{\partial q_i}$	dynamical systems/ classical mechanics
*Meso*scopic	$\frac{\partial\psi}{\partial t} + \frac{1}{m}\boldsymbol{p}\cdot\frac{\partial\psi}{\partial\boldsymbol{x}} + \boldsymbol{f}\cdot\frac{\partial\psi}{\partial\boldsymbol{p}} = \frac{\delta\psi}{\delta t}$ $\frac{1}{c}\cdot\frac{\partial I_\nu}{\partial t} + \mathbf{n}\cdot\boldsymbol{\nabla}I_\nu = \eta_\nu - \chi_\nu I_\nu$	plasma/kinetic theory radiative transfer
*Macro*scopic	$\frac{\partial}{\partial t}\rho + \boldsymbol{\nabla}\cdot\rho\boldsymbol{u} = 0$	fluid/continuum mechanics
*Mondo*scopic	$\frac{1}{2}\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} = 2KE + 3P + EM + R + W$	thermodynamics/ virial theory

Transport Coefficients Suprathermal Particles

Radiation as a Relativistic Fluid Two Fluid MHD

Increasing granularity

HelioPHYSICS

$$\nabla^2\Phi = \pm 4\pi G\rho$$

2 $\gamma = \frac{c}{\sqrt{c^2 - \|\boldsymbol{u}\|^2}} \approx 1$

*Macro*scopic

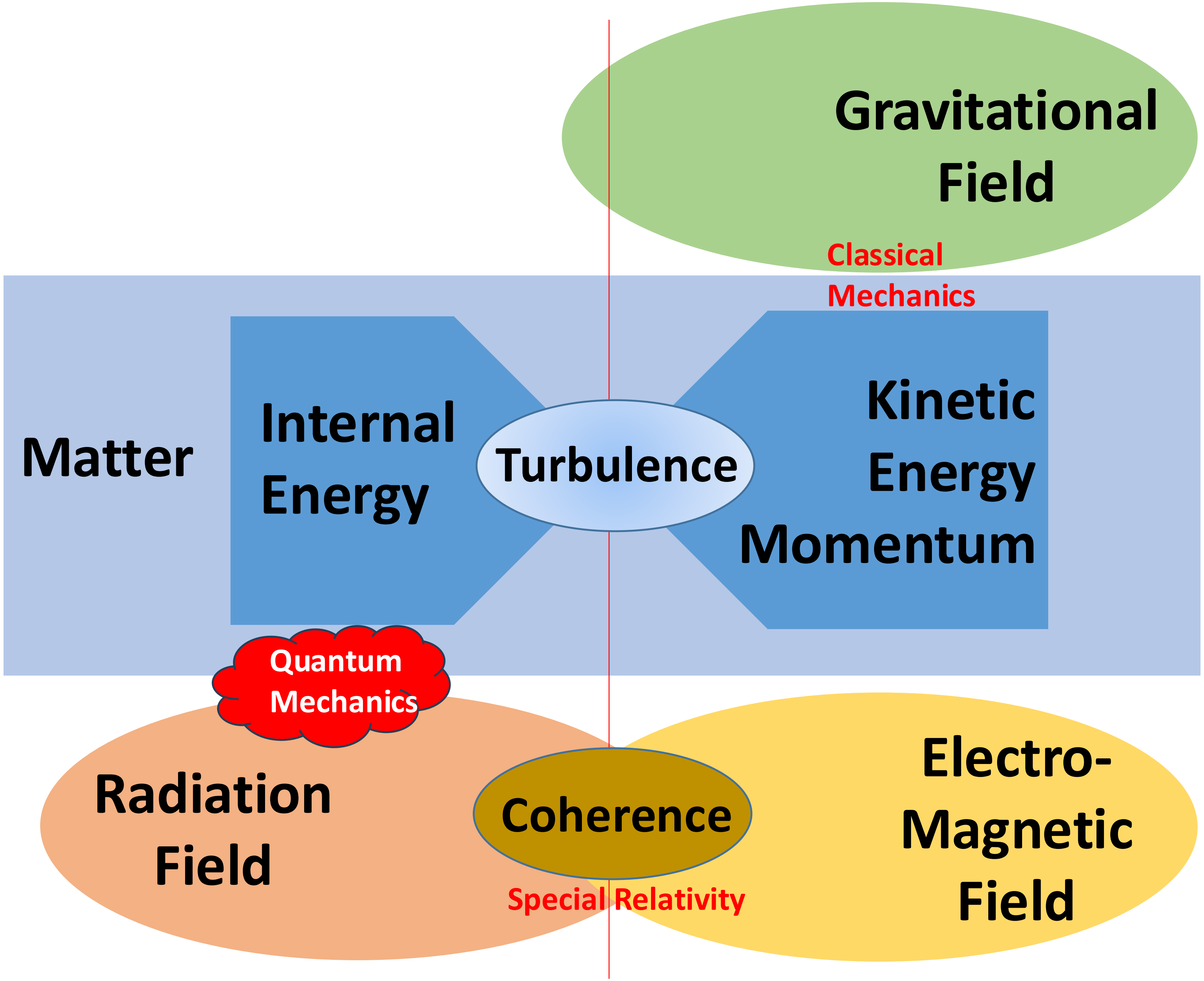
*Meso*scopic

$$\nabla \cdot \boldsymbol{E} = \rho_e/\varepsilon_0 \qquad \nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \mu_0\varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}$$

1 Flat Minkowski spacetime



Symmetry

$$t \rightarrow t' \pm t$$

Energy Conservation

Entropy Conservation

$$x \rightarrow x' + \mathbb{R}x$$

Momentum Conservation

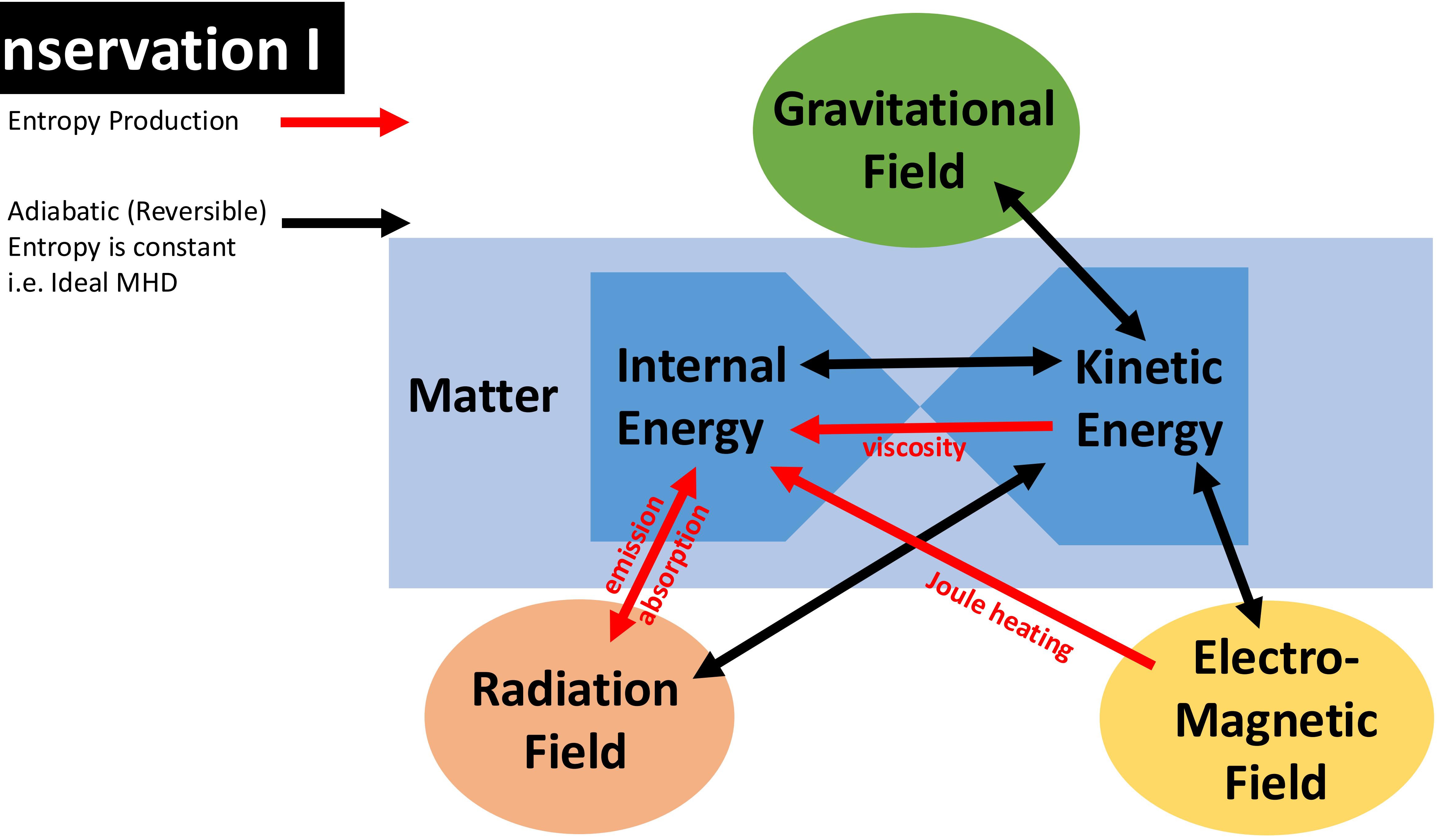
Parity Conservation (!)

Angular Momentum Conservation (!)

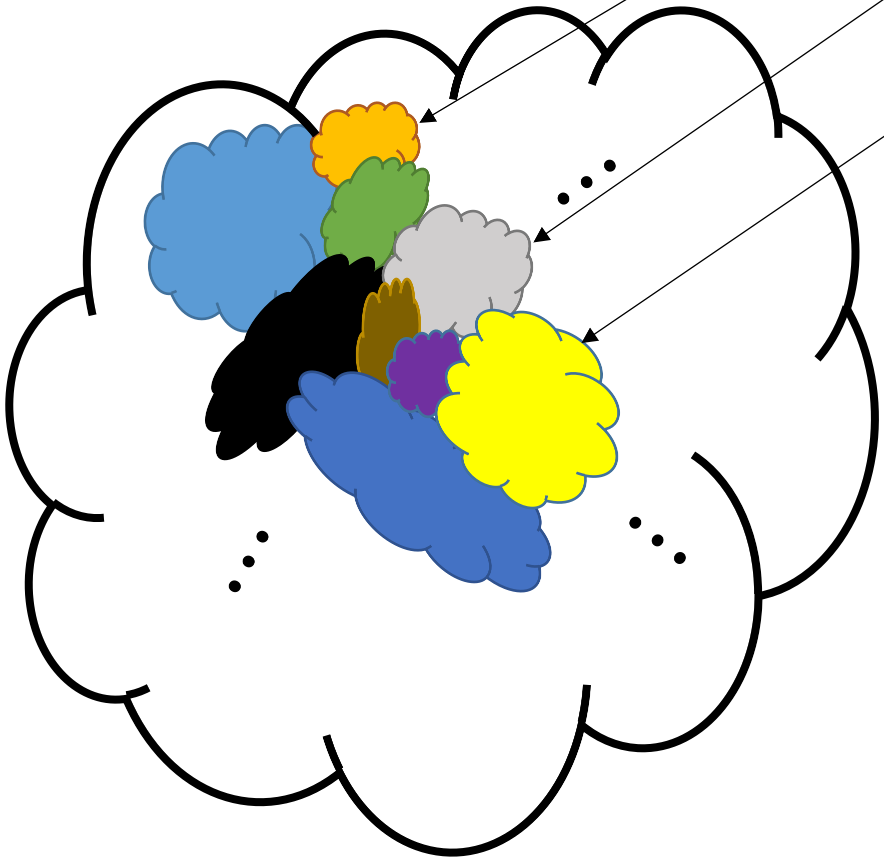
$$i \rightarrow j \qquad j \rightarrow i$$

Mass Conservation

Conservation I



Conservation II



V”, N”, **ε**”, {M”, E”, **P**”, **L**”}, ⋯, p”, T”, **μ**”	S”
V, N, **ε**, {M, E, **P**, **L**}, ⋯, p, T, **μ**	S
V’, N’, **ε**’, {M’, E’, **P**’, **L**’}, ⋯, p’, T’, **μ**’	S’

⋮

$\rho(\boldsymbol{x},t)$
$e(\boldsymbol{x},t)$
$\rho(\boldsymbol{x},t)\,\boldsymbol{u}(\boldsymbol{x},t)$
$p(\boldsymbol{x},t)$
$T(\boldsymbol{x},t)$
$\boldsymbol{\mu}(\boldsymbol{x},t)$
$S(\boldsymbol{x},t)$

Dynamic Equilibrium

$p = p' = p'' = \cdots$
P = **P**’ = **P**” = ⋯
L = **L**’ = **L**” = ⋯

$e(\boldsymbol{x},t)$, $T(\boldsymbol{x},t)$, $\boldsymbol{\mu}(\boldsymbol{x},t)$, $S(\boldsymbol{x},t)$

Thermal Equilibrium

T = T’ = T” = ⋯

$e(\boldsymbol{x},t)$, $\boldsymbol{\mu}(\boldsymbol{x},t)$, $S(\boldsymbol{x},t)$

Chemical Equilibrium

μ = **μ**’ = **μ**” = ⋯ S is maximal

Evaluate Interactions in the Comoving Frame

Comoving frame

Laboratory frame

$$\gamma = \frac{c}{\sqrt{c^2 - \|\boldsymbol{u}\|^2}} \approx 1$$

$$\boldsymbol{E}' = \gamma[\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}] - (\gamma - 1)(\boldsymbol{E} \cdot \boldsymbol{u})\boldsymbol{u} \,/\, \|\boldsymbol{u}\|^2$$

$$\boldsymbol{B}' = \gamma[\boldsymbol{B} - \boldsymbol{u} \times \boldsymbol{E}/c^2] - (\gamma - 1)(\boldsymbol{B} \cdot \boldsymbol{u})\boldsymbol{u} \,/\, \|\boldsymbol{u}\|^2$$

$\sigma \boldsymbol{E}'$ ⇐ $$\boldsymbol{J}' = \boldsymbol{J} - \gamma \boldsymbol{u}\rho_e + (\gamma - 1)(\boldsymbol{J} \cdot \boldsymbol{u})\boldsymbol{u} \,/\, \|\boldsymbol{u}\|^2$$

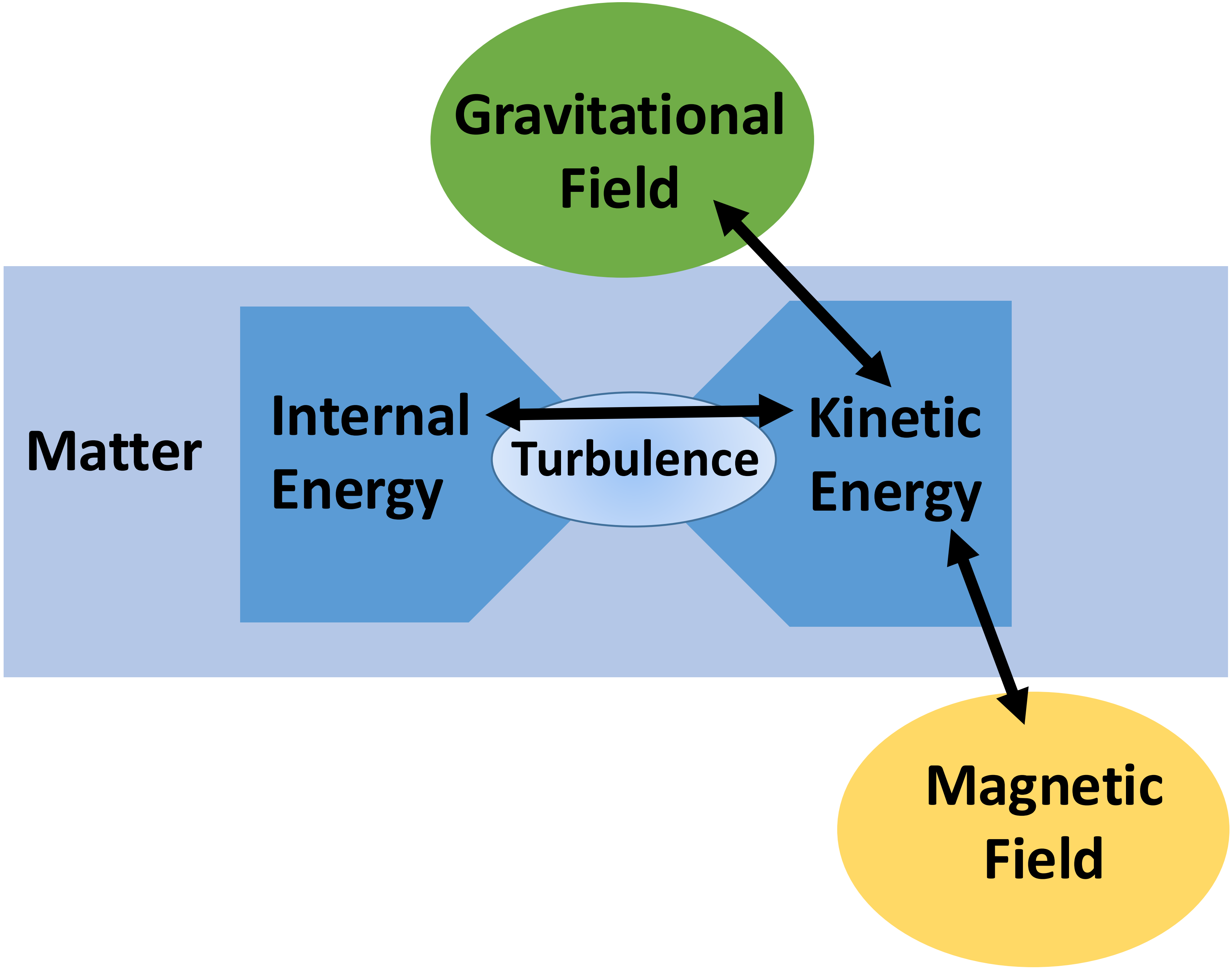
0 ⇐ $$\rho_e{}' = \gamma[\rho_e - \boldsymbol{u} \cdot \boldsymbol{J}/c^2]$$ ⇒ MHD

Ideal MHD

$$\nabla^2\Phi = \pm 4\pi G\rho$$

2 $\gamma = \frac{c}{\sqrt{c^2 - \|\boldsymbol{u}\|^2}} \approx 1$

*Macro*scopic



$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times (\boldsymbol{u} \times \boldsymbol{B}) = \frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J}$$

1 Flat Minkowski spacetime

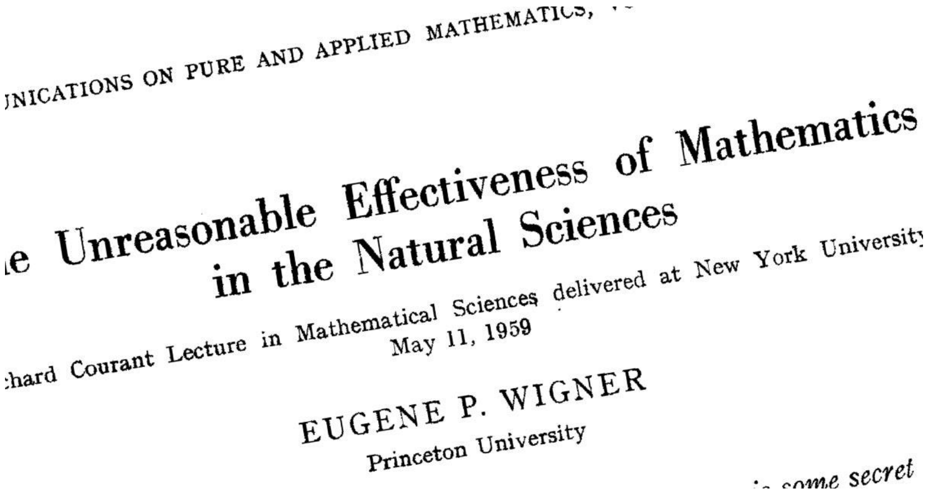
Unreasonable Effectiveness of the *Macro*scopic Description

"**If** *you conserve all the things that need to be conserved* **and** *you ensure that left to its own devices, entropy always increases,* **then** *things will often work out far better than one might have any right to expect. (…usually)*"

"**Always** *be certain that N is huge,* **and** *the physical system has both the time and ability to sample lots and lots of its available microstates consistent with a specified macrostate.*"

"**Always** *evaluate interactions between the matter and the classical fields in the* **comoving**, *i.e., rest-frame, of the material.*"

"**Solve** *your equations in any frame you like.*"


NICATIONS ON PURE AND APPLIED MATHEMATICS,

e Unreasonable Effectiveness of Mathematics in the Natural Sciences

hard Courant Lecture in Mathematical Sciences delivered at New York University May 11, 1959

EUGENE P. WIGNER
Princeton University

The Unreasonable Effectiveness of the Macroscopic Description

A Very Incomplete Introduction

We at some times are minions of our theories,
The fault, dear Brutus, is not in ourselves,
But in our stars, that we are underlings.

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