

Heliophysics Summer School



Part 1

- **Motivation**: Why heliophysics?
- My view of plasma physics: kinetic, multi-fluid, and MHD approaches.
- Quick review of particle motion and adiabatic invariants.
- Shocks (basics) and why they are important (collisionless plasmas)
- Magnetic reconnection (basics) and why it is important (collisionless plasmas)

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Why Heliophysics?



I Am Motivated by the "Physics" Part

В



A motion of a charged particle is governed by a magnetic field, and vice-versa.



Early Mystery: Aurora and Particle Acceleration



Later Discovery: Three Particle Acceleration Processes





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Early Mystery: Cosmic Rays

The discovery of cosmic rays by balloons and cloud chambers was the beginning of particle physics.





Later Discoveries: Cosmic Ray Accleration

Cosmic rays are believed to be generated by collisionless shocks at supernova shells.

This work was based on heliophysics observations: the Earth's bow shock.





Early Mystery: Solar and Stellar Winds



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Heliophysics Discoveries: Solar and Stellar Winds



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On-Going Heliophysics Work: Solar and Stellar Winds



Early Mystery: Solar Flares



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Heliophysics Discovery: Magnetic Reconnection



On-Going Heliophysics Work: Magnetic Reconnection



On-Going Heliophysics Work: Magnetic Reconnection



Early Mystery: Collisionless Shocks



Same Phenomenon Viewed from the Inside





Heliophysics Discovery: The Electron Cyclotron Maser



Radio emissions:



On-Going Heliophysics Work: Turbulence



On-Going Heliophysics Work: Turbulence



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My View of a Plasma



Basic Approaches to Plasma Physics

Least

detailed but

highly useful.

Kinetic

(1) For each particle, calculate its motion given B and E using the Lorentz equation.

(2) At every particle location, calculate B and E from Maxwell's equations.

Most accurate method, but very unwieldy; basis of PIC simulations.

Multi-Fluid

(1) For each species (*s*, typically ions and electrons), calculate the density (n_s) , velocity (\boldsymbol{u}_s) , and temperature (T_s) using fluid equations for a given **B** and **E**.

(2) Calculate *B* and *E* from Maxwell's equations.

MHD

Treat the plasma as a single fluid. Calculate the density (n), velocity (u), and pressure (P) along with **B** and **E** using the MHD equations.

Kinetic Calculations

Kinetic

(1) For each particle, calculate its motion given \boldsymbol{B} and \boldsymbol{E} using the Lorentz equation.

(2) At every particle location, calculate *B* and *E* from Maxwell's equations.



$$m \frac{d\boldsymbol{v}}{dt} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$
$$\nabla \cdot \boldsymbol{E} = \frac{\rho_q}{\varepsilon_o}$$
$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$
$$1 \ \partial \boldsymbol{E}$$

 $\boldsymbol{\nabla} \times \boldsymbol{B} = \frac{1}{c^2} \frac{\partial \boldsymbol{L}}{\partial t} + \mu_o \boldsymbol{J}$

 $\nabla \cdot \boldsymbol{B} = 0$

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Distribution Functions



The distribution function as a function of v_x is the number of particles per unit velocity.

In free-space the units are number of particles per unit velocity per m³ (s/m⁴). In three dimensions, the units of *f* are s^{3}/m^{6} .

The most common type of distribution is a Maxwellian or Gaussian:

$$f = f_o e^{-\frac{1}{2}mv^2/k_B T}$$



Reducing Distribution Functions



Multi-Fluid Approach

Multi-Fluid

(1) For each species (*s*, typically ions and electrons), calculate the density (n_s), velocity (u_s), and temperature (T_s) using fluid equations for a given **B** and **E**.

(2) Calculate *B* and *E* from Maxwell's equations.

$$\nabla \cdot \boldsymbol{E} = \frac{\rho_q}{\varepsilon_o}$$
$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$
$$\nabla \times \boldsymbol{B} = \frac{1}{c^2} \frac{\partial \boldsymbol{E}}{\partial t} + \mu_o \boldsymbol{J}$$
$$\nabla \cdot \boldsymbol{B} = 0$$

$$\frac{\partial n}{dt} + \boldsymbol{\nabla} \cdot n\boldsymbol{u} = 0$$

$$m n \frac{\partial \boldsymbol{u}}{\partial t} + m n (\boldsymbol{u} \cdot \boldsymbol{\nabla}) \boldsymbol{u} = -\boldsymbol{\nabla} P + n q (\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}) + n m g \quad \text{(Collisionless)}$$

 $\nabla P = \gamma T \, \nabla n$

Ideal MHD

$$\begin{array}{ll} \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0 \end{array} \begin{array}{ll} \text{Continuity Equation:} & \text{Define:} \\ \rho \equiv m_i n_i + m_e n_e \\ \rho \equiv m_i n_i + m_e n_e \\ u \equiv \frac{m_i n_i u_i + m_e n_e u_e}{m_i n_i + m_e n_e} \\ u \equiv \frac{m_i n_i u_i + m_e n_e u_e}{m_i n_i + m_e n_e} \\ p \equiv p_i + p_e \\ J \equiv en(u_i - u_e) \\ \end{array}$$

$$\begin{array}{ll} \nabla \times B = \mu_o J \\ \nabla \times B = \mu_o J \end{array} \qquad \text{Ampere's Law} \qquad \qquad \text{Assumed:} \quad n_i \approx n_e \\ (1) \quad \text{"Principle of Quasi Neutrality"} \\ (2) \quad \text{"Frozen-In"} \end{array}$$

Basic Approaches to Plasma Physics

Kinetic

(1) For each particle, calculate its motion given B and E using the Lorentz equation.

(2) At every particle location, calculate *B* and *E* from Maxwell's equations.

To solve problems, one often must start with MHD to obtain a largescale solution, then use either multi-fluid or kinetic approach to understand the small-scale physics.

Multi-Fluid

(1) For each species (*s*, typically ions and electrons), calculate the density (n_s) , velocity (\boldsymbol{u}_s) , and temperature (T_s) using fluid equations for a given **B** and **E**.

(2) Calculate *B* and *E* from Maxwell's equations.

MHD

Treat the plasma as a single fluid. Calculate the density (n), velocity (u), and pressure (P) along with **B** and **E** using the MHD equations.

Collisional Versus Collisionless Plasmas

Collisional

(1) The interior of the Sun and planetary ionospheres examples of collisional plasmas; they have high densities

(2) Momentum and energy exchange between ions and electrons (and/or neutral particles) can be dominated by collisions.

(3) The force equation must include viscosity and collision terms related to momentum exchange.

(4) Collisions often lead to aGaussian distribution as per the central limit theorem.

Collisionless

(1) The solar corona, solar wind, Earth's magnetosphere, and many astrophysical plasmas can be treated as "collisionless".

(2) Momentum and energy exchange between ions and electrons is dominated by *B* and *E*.

(3) Due to low damping, collisionless plasmas are often turbulent.

(4) Collisionless plasmas often do not have Gaussian distributions and may have energetic tails.



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Quick Review of Particle Motion and Adiabatic Invariants

Examine the Lorentz force with a magnetic field (B). Let B be in the *z*-direction. Let's consider motion in the *x*-*y* plane.

$$\boldsymbol{F} = \boldsymbol{q}(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$

Newton:

Break the vector equation into components:



$$m\frac{d\boldsymbol{v}}{dt} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$

$$\frac{dv_x}{dt} = \frac{qE_x}{m} + \frac{qB_z}{m}v_y$$
$$\frac{dv_y}{dt} = \frac{qE_y}{m} - \frac{qB_z}{m}v_x$$

Particle Motion and Adiabatic Invariants

Let's start easy. Let $\boldsymbol{E} = 0$.



This equation is that of a harmonic oscillator.



$$v_x = v_o \cos(\Omega t + \vartheta)$$
$$v_y = -\frac{q}{|q|} v_o \sin(\Omega t + \vartheta)$$

 ϑ is an arbitrary angle (phase).

Particle Motion and Adiabatic Invariants

Let E_y be finite.



This equation is that of a harmonic oscillator **that moves at a constant speed**!



$$v_x = v_o \cos(\Omega t + \vartheta) + \frac{E_y}{B_z}$$

$$v_y = -\frac{q}{|q|} v_o \sin\left(\Omega t + \vartheta\right)$$

 $\boldsymbol{\vartheta}$ is an arbitrary angle (phase).



Single Particle Motion

General Equation

$$\boldsymbol{v}_E = \frac{\boldsymbol{E} \times \boldsymbol{B}}{\boldsymbol{B}^2}$$

$$\boldsymbol{v}_F = \frac{\boldsymbol{F} \times \boldsymbol{B}}{q B^2}$$

Other drifts:

$$oldsymbol{v}_{
abla B}=\pmrac{1}{2}rac{{oldsymbol{v}_{ot}}^2}{\Omega}rac{oldsymbol{B} imes
abla_{ot}oldsymbol{B}}{B^2}$$

$$\boldsymbol{v}_{\rm C} = \pm \frac{{v_{\parallel}}^2}{\Omega R_C} \frac{\boldsymbol{\widehat{R}}_C \times \boldsymbol{\widehat{b}}}{B^2}$$

Adiabatic Invariants

If one can identify a conserved quantity (e.g. energy, momentum, etc.), problem solving becomes much easier. A few things one should realize:

- Since *B* can only impart a force normal to the particles motion (*v* x *B*), energy is conserved. (*E*, however, can change a particle's energy).
- (2) We define:

$$W_{\perp} = \frac{1}{2} m v_{\perp}^{2} \qquad \qquad W_{\parallel} = \frac{1}{2} m v_{\parallel}^{2}$$

- (3) Therefore, $W_{\parallel} + W_{\perp} = W$, is constant.
- (4) Since a change in **B** can change v_{\parallel} , it must also change v_{\perp} in a way that energy is conserved.

$$m\frac{dv_{\parallel}}{dt} = -\mu\frac{dB}{dz} \qquad \qquad \mu \equiv \frac{W_{\perp}}{B}$$

(5) Conservation of energy:

$$\frac{dW}{dt} = 0 \quad \rightarrow \quad \frac{d}{dt}(W_{\parallel} + W_{\perp}) = 0 \quad \rightarrow \quad \frac{d}{dt}(W_{\parallel} + \mu B) = 0$$

Adiabatic Invariants

 $W_{\perp} = \frac{1}{2}mv_{\perp}^{2} \qquad W_{\parallel} = \frac{1}{2}mv_{\parallel}^{2} \qquad m\frac{dv_{\parallel}}{dt} = -\mu\frac{dB}{dz} \qquad \mu \equiv \frac{W_{\perp}}{B}$ So far: $\frac{dW}{dt} = 0 \quad \rightarrow \quad \frac{d}{dt}(W_{\parallel} + W_{\perp}) = 0 \quad \rightarrow \quad \frac{d}{dt}(W_{\parallel} + \mu B) = 0$ Chain rule and substitute $W_{\parallel} = \frac{1}{2} m v_{\parallel}^2$ $mv_{\parallel}\frac{dv_{\parallel}}{dt} + B\frac{d\mu}{dt} + \mu\frac{dB}{dt} = 0$ Use $m \frac{dv_{\parallel}}{dt} = -\mu \frac{dB}{dz}$ $-v_{\parallel}\mu\frac{dB}{dz} + B\frac{d\mu}{dt} + \mu\frac{dB}{dt} = 0$ Now use $v_{\parallel} = \frac{dz}{dt}$ $-\mu \frac{dB}{dt} \frac{dz}{dt} + B \frac{d\mu}{dt} + \mu \frac{dB}{dt} = 0$ $B\frac{d\mu}{dt}=0$

The first and third term cancel.

If *B* is not 0 then μ is conserved!

Adiabatic Invariant

We call μ an "adiabatic" invariant.

$$\mu \equiv \frac{W_{\perp}}{B}$$



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Ram Pressure/Shocks

Stellar winds travel 100's of AU then collide with the interstellar medium.

The collision causes as "shock", which appears as a curved structure.

The heliosphere's shock is toward α-Centauri.



Shocks are hypothesized to be at the root of cosmic ray acceleration.

Shocks lead to strong turbulence in plasmas.

Shocks convert ram energy into magnetic and thermal energy.



Magnetic Pressure

$$\rho \frac{\partial \boldsymbol{u}}{\partial t} + \rho(\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{u} = -\boldsymbol{\nabla}P - \boldsymbol{\nabla}\frac{B^2}{2\mu_o} + \frac{(\boldsymbol{B} \cdot \boldsymbol{\nabla})\boldsymbol{B}}{\mu_o} + \rho g \qquad \boldsymbol{J} \times \boldsymbol{B} = \frac{\boldsymbol{\nabla} \times \boldsymbol{B}}{\mu_o} \times \boldsymbol{B} = -\boldsymbol{\nabla}\frac{B^2}{2\mu_o} + \frac{(\boldsymbol{B} \cdot \boldsymbol{\nabla})\boldsymbol{B}}{\mu_o}$$

Assume steady state, u = 0, g = 0, and that **B** is straight.

$$0 = -\nabla P - \nabla \frac{B^2}{2\mu_o} \quad \rightarrow \quad \nabla \left(P + \frac{B^2}{2\mu_o} \right) = 0 \quad \rightarrow \quad P + \frac{B^2}{2\mu_o} = C$$



Ram Pressure / Shocks

Suppose a plasmas impinges on a object, which compresses the plasma as slows the flow. Steady state, no curvature in B, and not gravity.

Ram Pressure

 $\rho(\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u}$

Continuity Equation:

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{u}) = 0$$

Compressible & 1D (plasma shock)

∇ (pu) = 0 Constant

 $\rho(\boldsymbol{u} \boldsymbol{\nabla})\boldsymbol{u} = \boldsymbol{\nabla}(\rho u^2)$

$$\rho(\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u}=\boldsymbol{\nabla}(\rho u^2)$$

Incompressible (water pipe)

$$\nabla \cdot (\rho u) = 0$$

Constant
$$\rho (u \cdot \nabla) u = \frac{1}{2} \nabla (\rho u^2)$$

$$\rho(\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u}=\frac{1}{2}\boldsymbol{\nabla}(\rho u^2)$$

(We don't do water.)

$$\nabla(\rho u^2) = -\nabla P - \nabla \frac{B^2}{2\mu_o}$$

Suppose a plasmas impinges on a object, which compresses the plasma as slows the flow. Steady state, no curvature in B, and not gravity.

$$\rho u^2 + P + \frac{B^2}{2\mu_o} = C$$

Object or

barrier.

Ram Pressure $\rho(u \cdot \nabla)u$ $\rho($

Earth's Bow Shock



Ram Pressure / Shocks

$$\rho_2 u_2^2 + P_2 + \frac{B_2^2}{2\mu_o}$$



Rankine-Hugoniot Jump Conditions $\rho_1 u_1 = \rho_2 u_2$ $B_1u_1 = B_2u_2$ $\rho_1 u_1^2 + P_1 + B_1^2 / 2\mu_0 = \rho_2 u_2^2 + P_2 + B_2^2 / 2\mu_0$ $u_1(1/2 \rho_1 u_1^2 + \gamma/(\gamma - 1) P_1 + B_1^2/\mu_0) = u_2(1/2 \rho_2 u_2^2 + \gamma/(\gamma - 1) P_2 + B_2^2/\mu_0)$

Example: Shocks







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Anti-Parallel Magnetic Reconnection



Magnetic Reconnection



The Solar Flare Puzzle

The puzzle (1950's):

Solar flares erupt in minutes (~ 10^2 s).

Magnetic diffusion time across 10 Mm is on the order of 10^{16} s using simple magnetic diffusion.



Breaking the Frozen-In Condition

The first thought (1950's) was that collisions are the primary process that break the frozenin condition resulting in resistance and diffusion:

 $\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} = \eta \boldsymbol{J}$

Generalized Ohm's Law

Resistance results in magnetic diffusion:

$$\nabla \times (\boldsymbol{E} + \boldsymbol{\nu} \times \boldsymbol{B} = \eta \boldsymbol{J})$$
Use
$$\nabla \times \boldsymbol{B} = \boldsymbol{\mu}_{o} \boldsymbol{J}$$
If $\boldsymbol{\nu} = 0$:
$$\frac{\partial \boldsymbol{B}}{\partial t} = \frac{\eta}{\mu_{o}} \nabla^{2} \boldsymbol{B}$$

$$t_{D} \approx \frac{\mu_{o}}{\eta} L^{2} \boldsymbol{B}$$



Magnetic Reconnection at Earth

Dungy (1961) realized that magnetic reconnection could "open" the Earth's magnetosphere allowing solar wind and its energy to enter. He correctly inferred that magnetic reconnection is part of the energy transfer from the solar wind that power aurorae.





Dungy's model was not well accepted. At the time (1960's), controversy raged. The plasma resistance is near zero, so magnetic reconnection could not occur.

$\boldsymbol{E} + \boldsymbol{\nu} \times \boldsymbol{B} = \eta \boldsymbol{J} = \boldsymbol{0}$

However, as research went forward (1970's) the first satellite observations showed evidence supporting the Dungy model.

Now we have a bigger mystery! Collisionless magnetic reconnection?_{ISS, 2024. Page 57}

The Tokamak Problem

First reported in 1974, magnetic fields in tokamaks would suddenly disrupt resulting in a loss of confinement. These events were named "sawtooth crashes" due to the nonlinear shape of the signals.

Ultimately, the root cause of sawtooth crashes has been determined to be magnetic reconnection.

Once again, the magnetic reconnection rate is much higher than expected.





Magntorotational Instability – Accretion Disks

The magnetorotational instability in accretion disks (*Velikov*, 1959; *Chandrasekhar*, 1960) is thought to be primarily driven by the Rayleigh-Taylor instability.

Recent work now includes magnetic reconnection and turbulence to describe the nonlinear evolution.

"If you don't understand it, invoke magnetic reconnection or turbulence!"





MRI simulation near a Kerr black hole.

MRI in an accretion disk.



γ-Ray Bursts

In high-σ (magnetic field energy to kinetic energy) outflows, magnetic reconnection can be more efficient than the internal shock (*Thompson* 1994; *Spruit et al.* 2001; *Lyutikov* & *Blandford* 2002; 2003).





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Pulsar Nebulae

Extreme particle acceleration (beyond the radiation reaction limit) in pulsar nebulae has been inferred from synchortron radiation. It has been postulated that particles can be accelerated well above the classical radiation reaction limit (160 MeV), by a relativistic Speiser mechanism inside of a magnetic reconnection layer (*Uzdensky, Cerutti, & Begelman,* 2011).

Simulations of particle acceleration beyond the classical synchrotron burnoff limit in magnetic reconnection: an explanation of the crab flares (*Cerutti et al.*, 2013).



Magnetic Reconnection in the Solar Wind

Collisionless magnetic reconnection has also been observed in the solar wind (Gosling et al., 2005; 2007)





Magnetic Reconnection, a Universal Process

Magnetic reconnection has been observed/postulated in vastly differing plasma environments, from tokamaks to neutron stars to pulsar nebulae, with scale sizes ranging from cm to Mm, with plasma densities scaling over 12 orders of magnitude, and magnetic field strengths from nT to >kT. It is associated with turbulence and particle acceleration. Yet, the physics of the diffusion region was not understood!

Major questions:

- How does collisionless magnetic reconnection occur?
- What physical process breaks the frozen in condition in fast magnetic reconnection?
- What supports the reconnection electric field?
- How is the magnetic field energy dissipated?



Magnetic Reconnection Research

20-30 years ago, it was recognized that fast reconnection and collisionless reconnection required examination of the full Generalized Ohm's Law.

Several types of MHD simulations are used to balance E + vxB:

- Ideal MHD: 0
- Resistive MHD: η*J*.
- Hall MHD: *J*x*B*/*en*

Adding electron pressure or inertia required one to abandon MHD.





Hall Reconnection

Ions, which have much larger skin depth (c/ω_{pi}) and gyroradii, decouple from **B** (via Hall term) resulting in two diffusion regions named the "ion diffusion region" and the "electron diffusion region".

Magnetic Reconnection Hall MHD was a large step forward. However, electrons do not decouple via the Hall term. **Ion Diffusion Region** And since: Dístance d_i **Electron Diffusion Region** $\boldsymbol{J} \times \boldsymbol{B} \cdot \boldsymbol{J} = 0$ Electrons Ions Hall MHD cannot account for magnetic energy conversion. -2 0 2 4 -4 Distance d

PIC Simulations

A number of advances came from PIC (Particle-In-Cell; kinetic) simulations:

Primarily in 2D.

- (1) Energy dissipation into electrons was quantified.
- (2) Island formation (tearing mode) was verified.
- (3) Particle acceleration methods were postulated.
- (4) Asymmetric magnetic reconnection (reconnection between differing plasmas) was studied.
- (5) In 3D, turbulence emerges.
- (6) Magnetic reconnection with a finite guide field differs for anti-parallel magnetic reconnection.
- (7) Electron jets can emerge from magnetic reconnection.
- (8) Much more...



Laboratory Experiments

Laboratory experiments were able to verify or disprove a number of the theories of magnetic reconnection.







Space Measurements of the EDR



Space Measurements of Magnetic Reconnection



What MMS observed in the electron diffusion region of anti-parallel magnetic reconnection:

(a) The Hall term breaks ion coupling (predicted).

(b) Electrons de-couple *kinetically* forming "crescent distributions". The reconnection electric field is supported by an *off-diagonal term in the electron pressure gradient* (predicted by some, e. g. *Hesse et al.*, 2014, but a major controversy prior to MMS). (c) Electron heating: $T_{\parallel} > T_{\perp}$ (partly predicted by some).

(d) Strong, localized energy conversion $J \cdot E > 0$ (predicted, except MMS observes much higher values and much more structure.)

The Frozen-In Condition