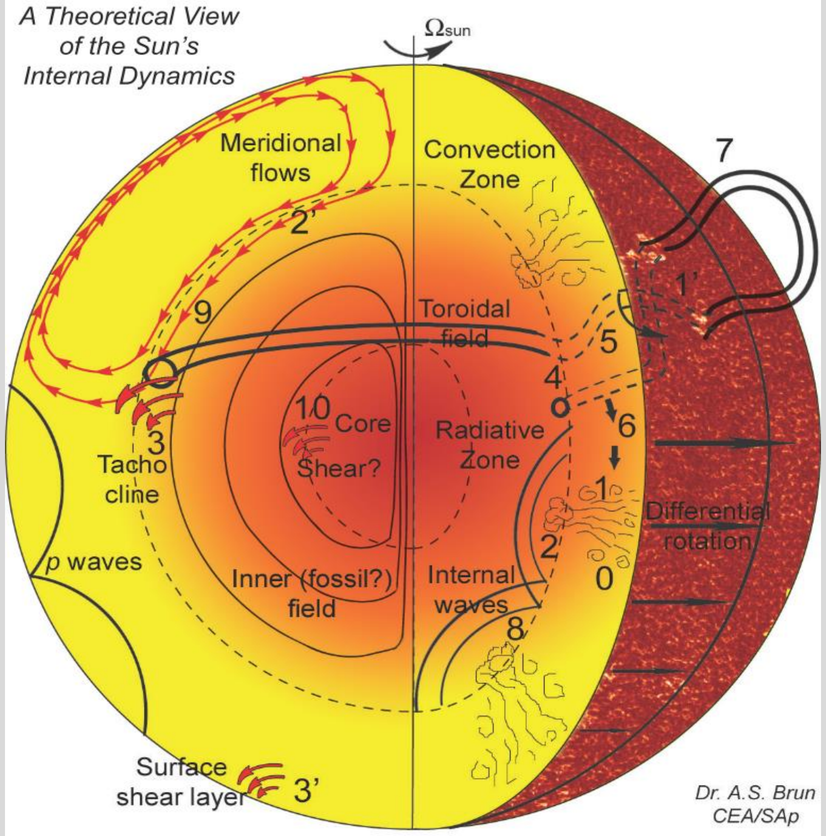
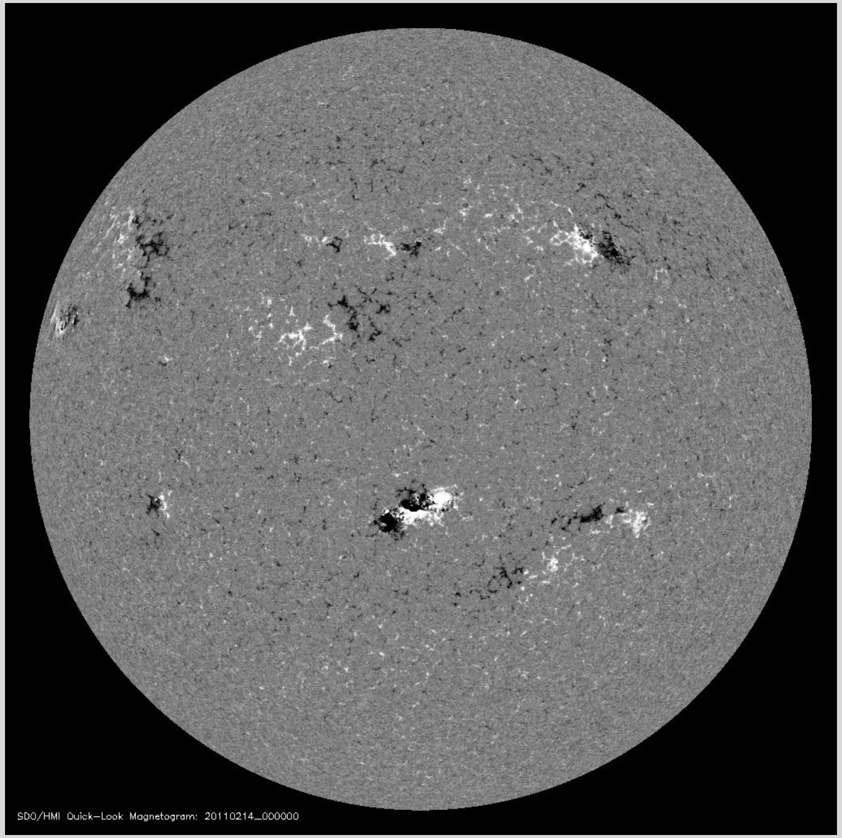
From the Sun to stars and planets: Applications of dynamo theory

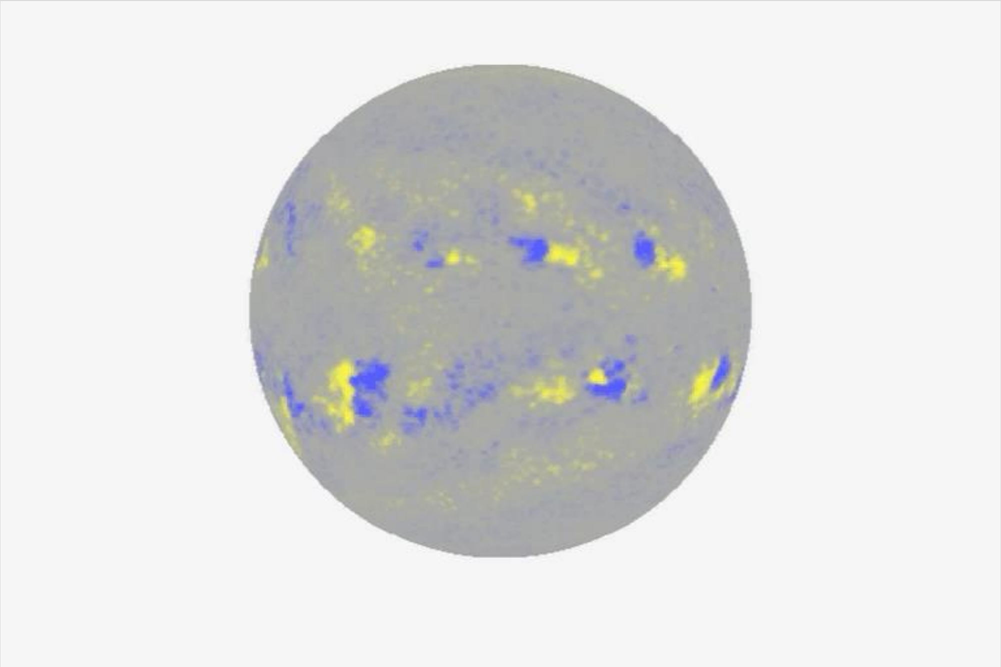


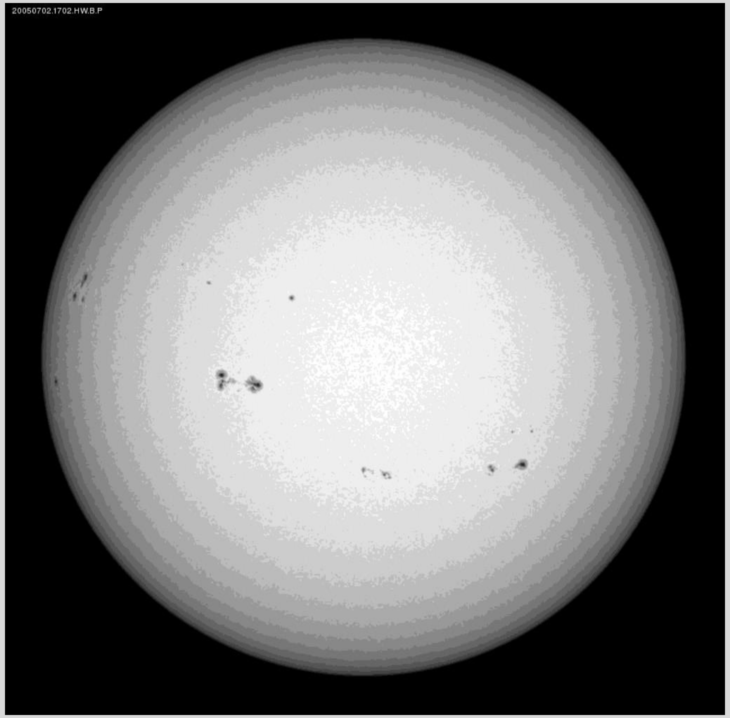


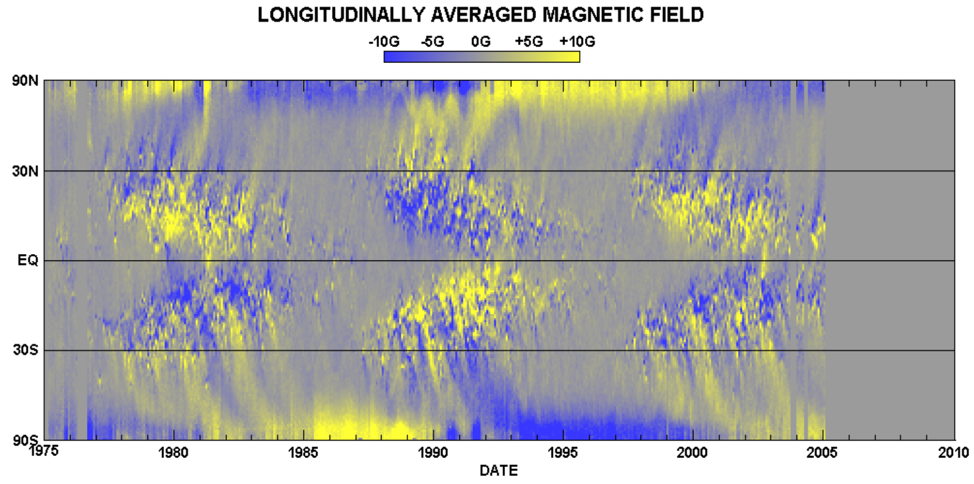
Matthias Rempel

HAO/NSF NCAR

Solar magnetic field

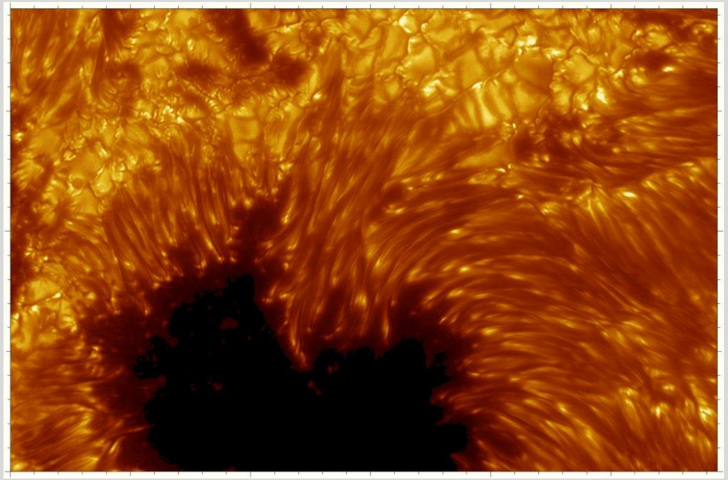






NASA/NSSTC/Hathaway 2005/02

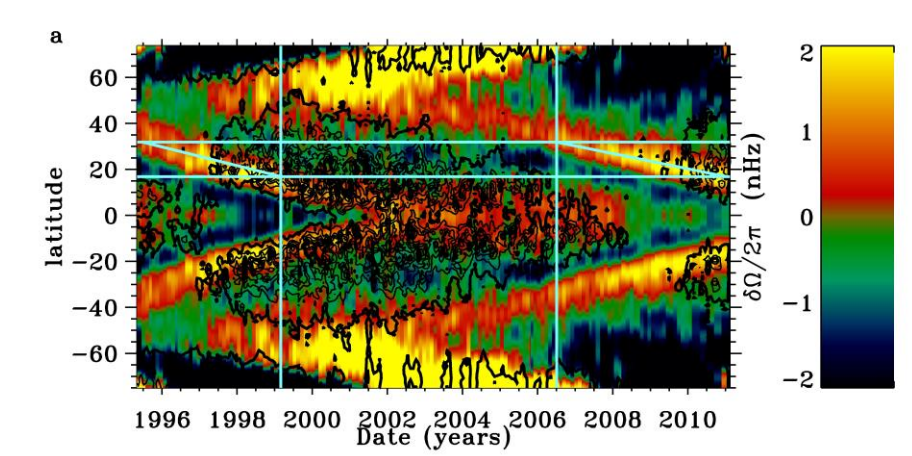
D. Hathaway

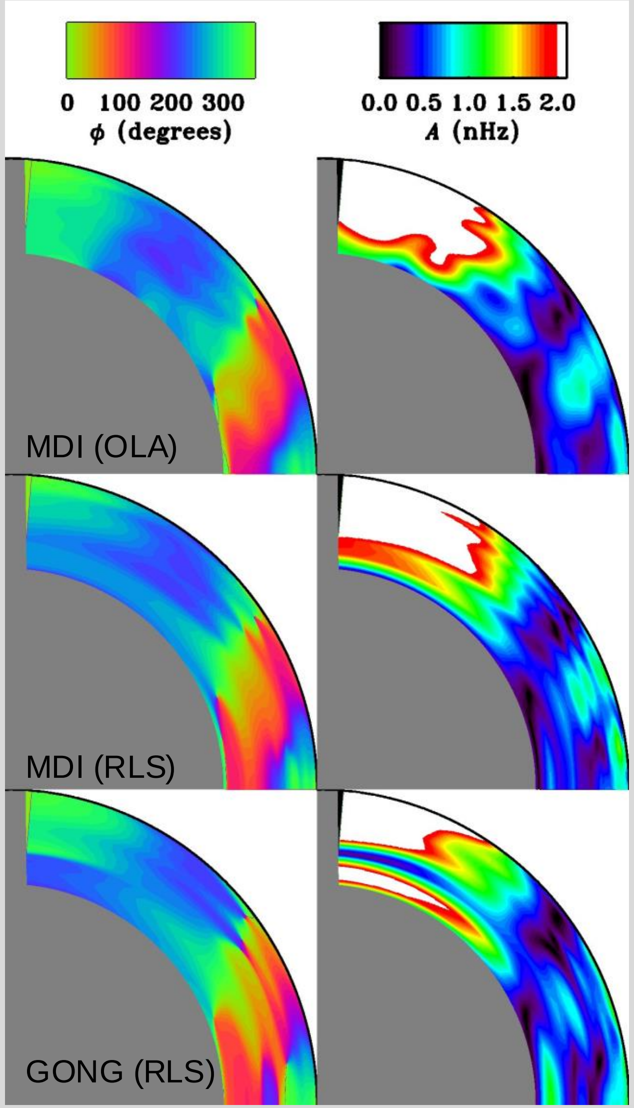


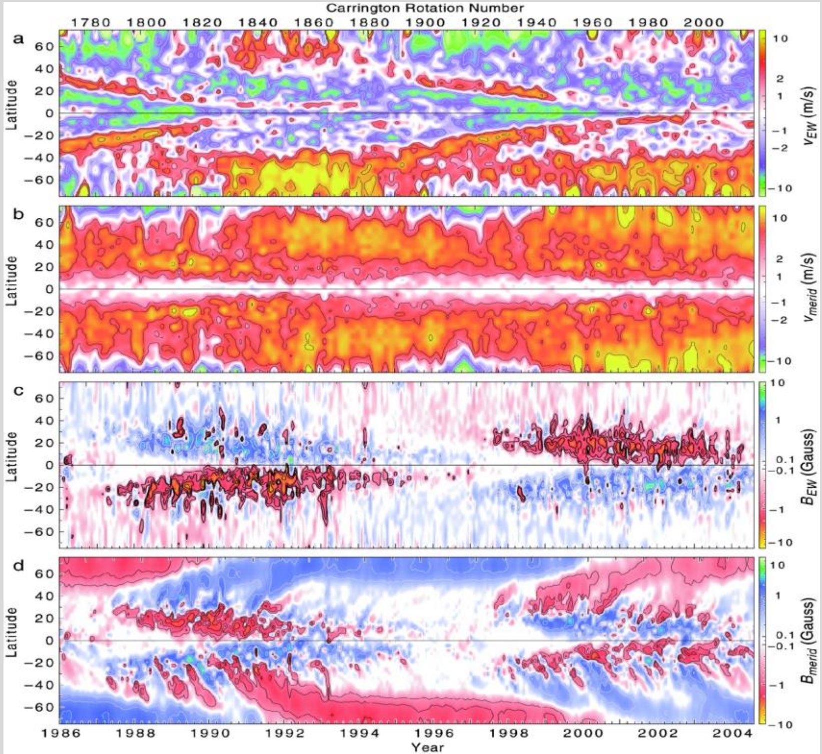
SST, La Palma

Large scale flow variations

Global Helioseismology (R. Howe)





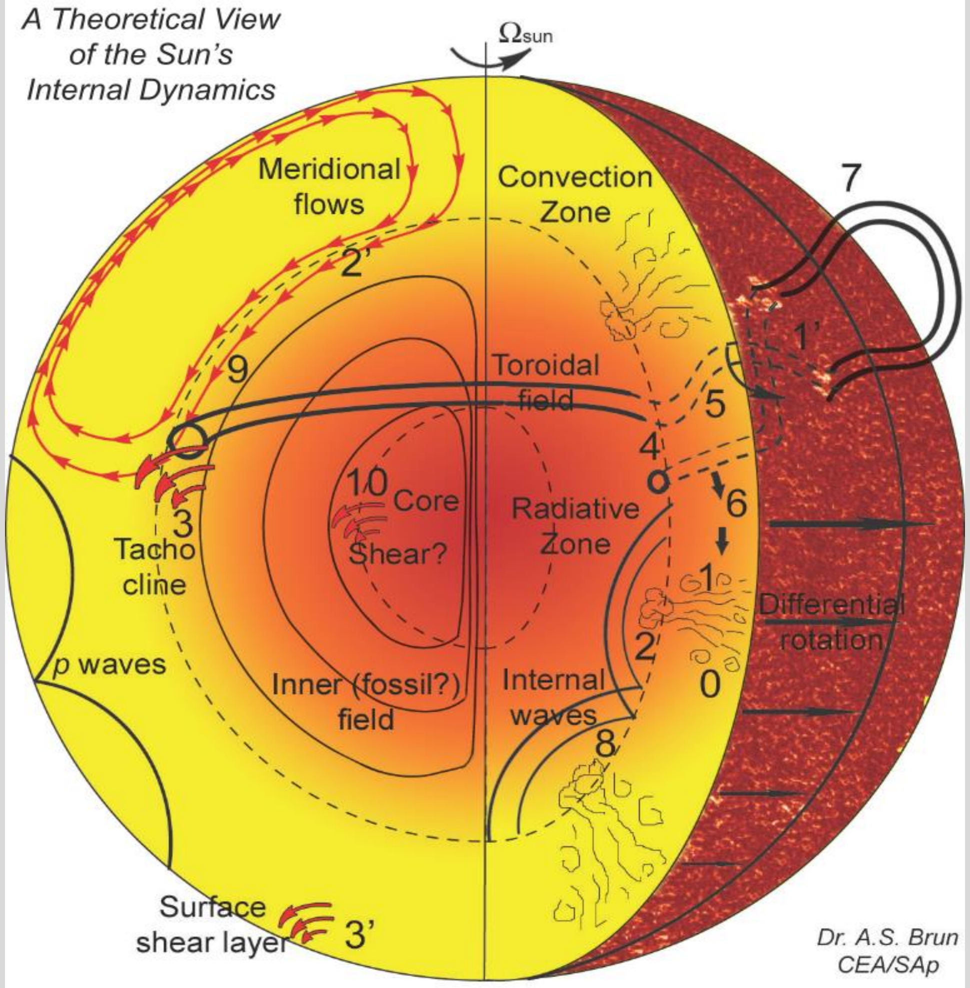


Surface Doppler (R. Ulrich)

Solar dynamo models – what is the goal?

- What is a solar dynamo model supposed to do?
  1) Show a “solar-like” activity pattern in terms of:
     - Cyclic behavior with equator-ward propagation of activity
     - Surface flux evolution consistent with observations
     - Large scale flow variations consistent with observations
  2) Show a “solar-like” amplitude variation from cycle to cycle
  3) Allow prediction of future activity
- Most models struggle already with point 1)
  - Focus this lecture on 1)
  - 2) and 3) can provide additional constraints on dynamo models

The basic dynamo ingredients



- ➢ Large-scale flows
  - – Differential rotation
  - – Meridional flow
  - – Mean and (cyclic) variation
- ➢ Turbulent induction
  - – Transport
    - Advective
    - Diffusive
  - – α-Effects
    - Key terms that enable dynamo action
- ➢ Flux emergence
  - – Links dynamo to photospheric field observations
  - – Might play role in dynamo process itself
    - Babcock-Leighton mechanism

Numerical modeling approaches

- Meanfield models
  - Solve equations for mean flows, mean magnetic field only
  - Inexpensive, but need good model for correlations of small scale quantities (e.g. turbulent angular momentum transport), see extensive work by Rüdiger & Kitchatinov)
  - Can address the full problem, but not from first principles (models have many degrees of freedom and tunable parameters)
- 3D numerical simulations
  - Solve the full set of equations (including small- and large-scale flows, magnetic field) from first principles
  - Very expensive:
    - Low resolution runs for long periods >10 years
    - High resolution for short periods
  - Good understanding of ingredients of solar dynamo, no complete model yet
- Advances in computing infrastructure shift balance toward 3D simulations, but we need both!

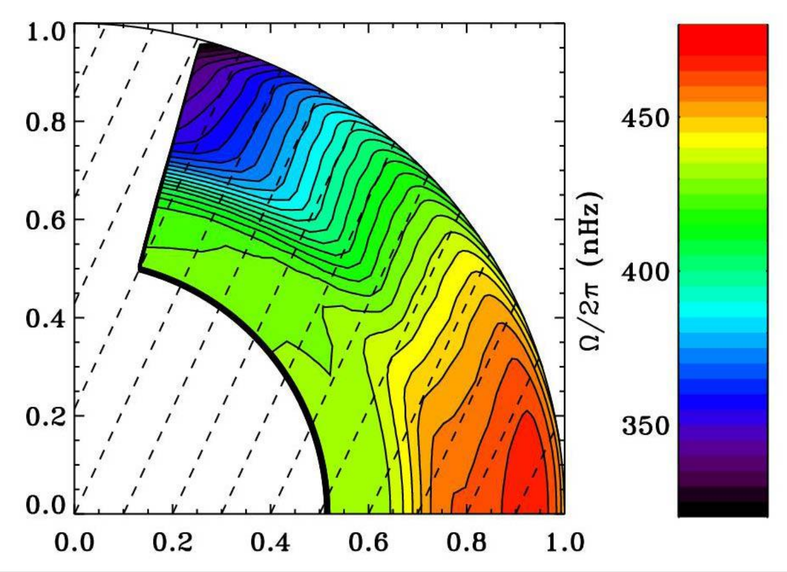
Mean field models

- Mean field models consider only average quantities
  - Sunspots are a key feature of the solar cycle, but they are averaged away
- Mean field models make strong assumptions that are not well justified from first principles
- Too many degrees of freedom require “educated guesses”

$$(\overline{v' \times B'})_i = a_{ik}\overline{B}_k + b_{ijk}\frac{\partial \overline{B}_j}{\partial x_k}$$

  - Contains 36!!! (mostly unknown) functions of r and ϑ, in most models only 2 are considered and even that allows for a lot of freedom
  - Computing mean field coefficients from 3D simulations (Schrinner et al. 2007, Ghizaru et al. 2011, Warnecke et al. 2018) shows that in general almost all of them are important!
- Mean field models allow us to study certain scenarios or they allow to analyze a complicated 3D simulation, but one has to be very lucky to find the “correct” model for the solar cycle without additional knowledge
- Non-linear feedback difficult to implement

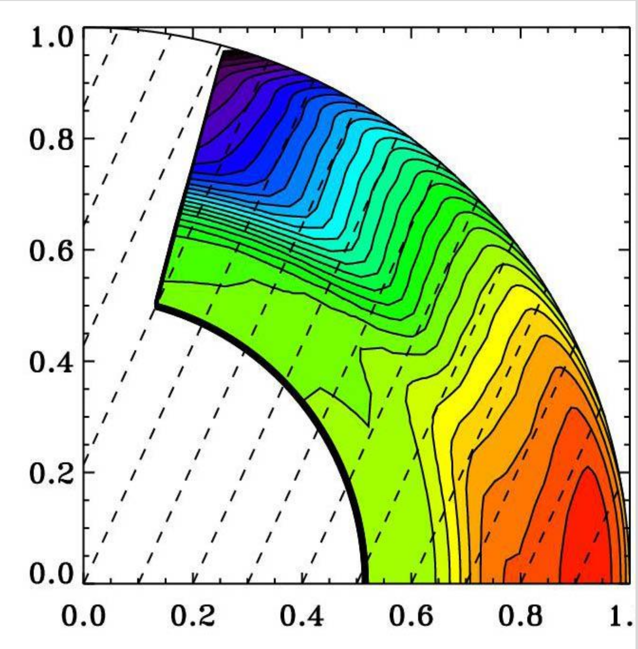
Solar dynamo models

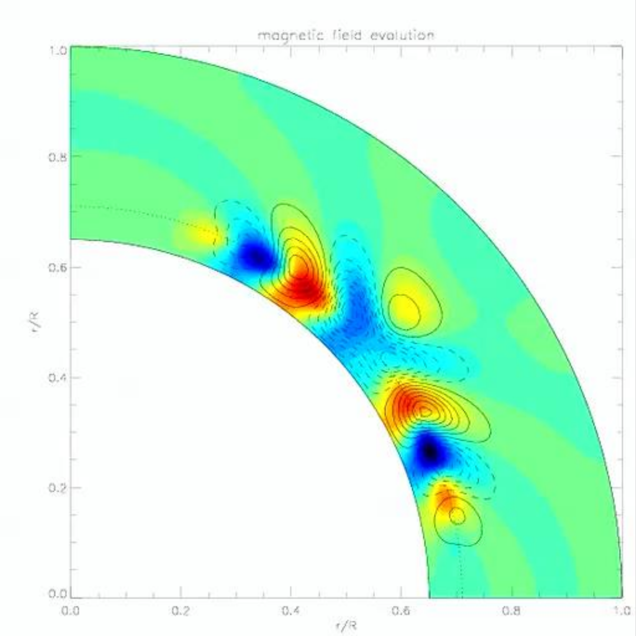


- Mean field models
  - Convection zone dynamos
  - Tachocline/interface dynamos
  - Near surface shear layer dynamos
  - Flux transport dynamos
- Main uncertainties
  - Location of dynamo
  - Poloidal field regeneration (B_r, B_ϑ from B_φ: α-effect)
  - Turbulent transport (magnetic pumping, turbulent diffusion vs. magnetic buoyancy)
  - Role of meridional flow (propagation of activity belt)

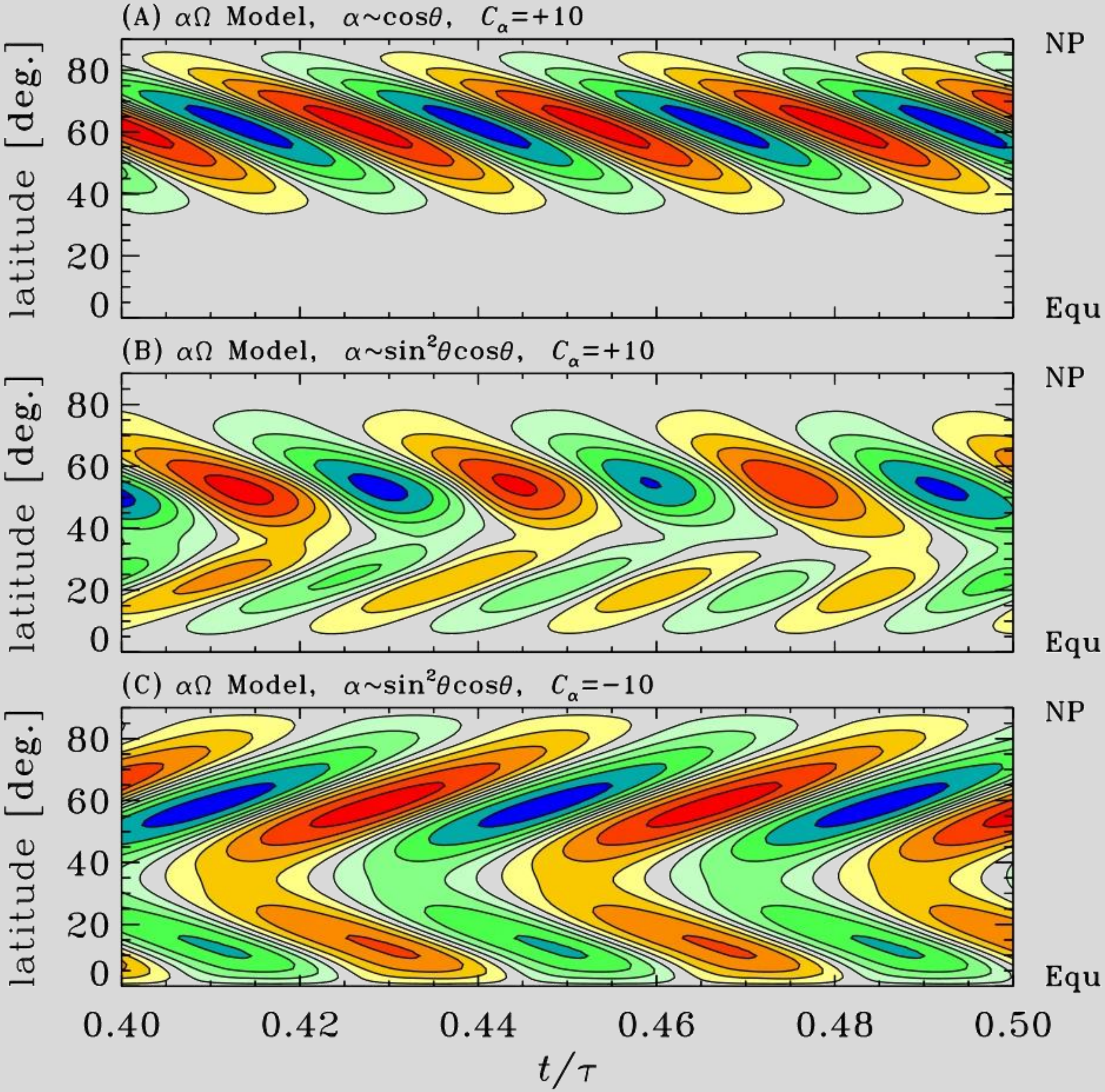
Mean field dynamos

- Thin layer dynamos
  - Overshoot/tachocline dynamos
    - Radial shear, αΩ-type dynamos, latitudinal propagating dynamo wave
    - Negative α in northern hemisphere for equatorward propagation
  - Surface shear layer?
  - Main problem:
    - Typically very short latitudinal wave length (several overlapping cycles)
- Distributed dynamos
  - Interface dynamos
    - Ω-effect in tachocline, α-effect in CZ, introduced to avoid problems with strong α-quenching
    - Solutions very sensitive to details





Challenge of solar-like butterfly diagrams

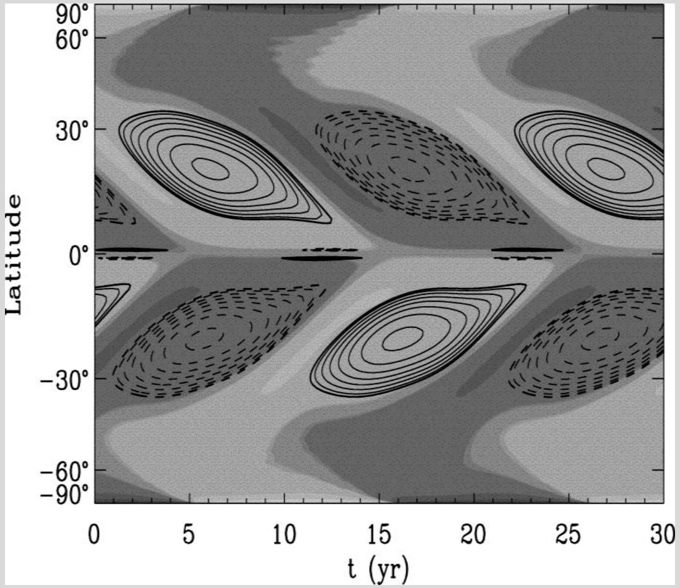


- Too strong polar branch unless alpha is restricted closed to equator
- Wrong propagation direction of low latitude branch (with positive alpha)
- Need negative alpha
  - Can be justified at base of CZ since flow helicity changes sign
- Too much cycle overlap
  - Too short wavelength of “dynamo wave”

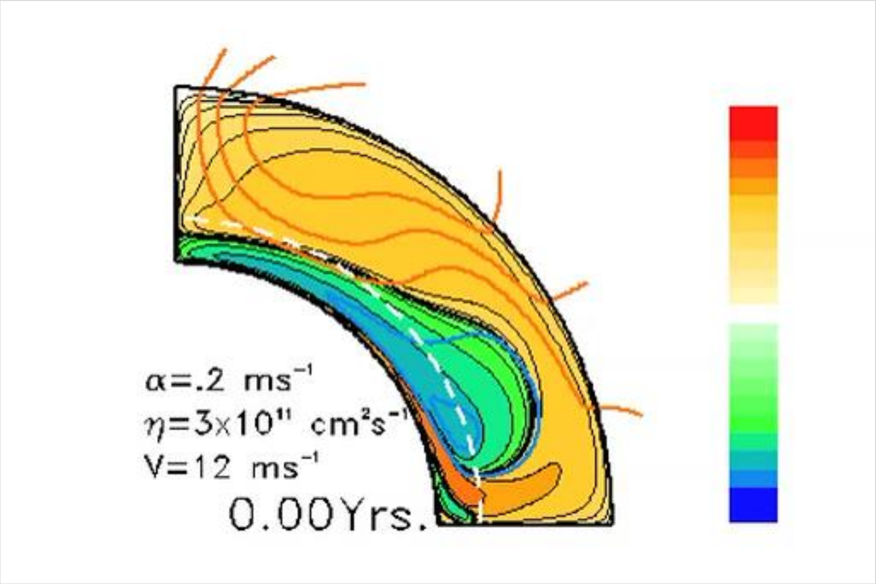
Mean field dynamos

➢ Distributed dynamos

- Flux transport dynamo
  - Advective transport of field by meridional flow
  - Propagation of AR belt advection effect
  - Cycle length linked to overturning time scale of meridional flow
- Central assumption:
  - Proper meridional flow profile (mostly single flow cell poleward at top, equatorward near bottom of CZ)
  - Weak turbulent transport processes
  - Babcock-Leighton α-effect
- Overall:
  - Most successful in reproducing solar like behavior



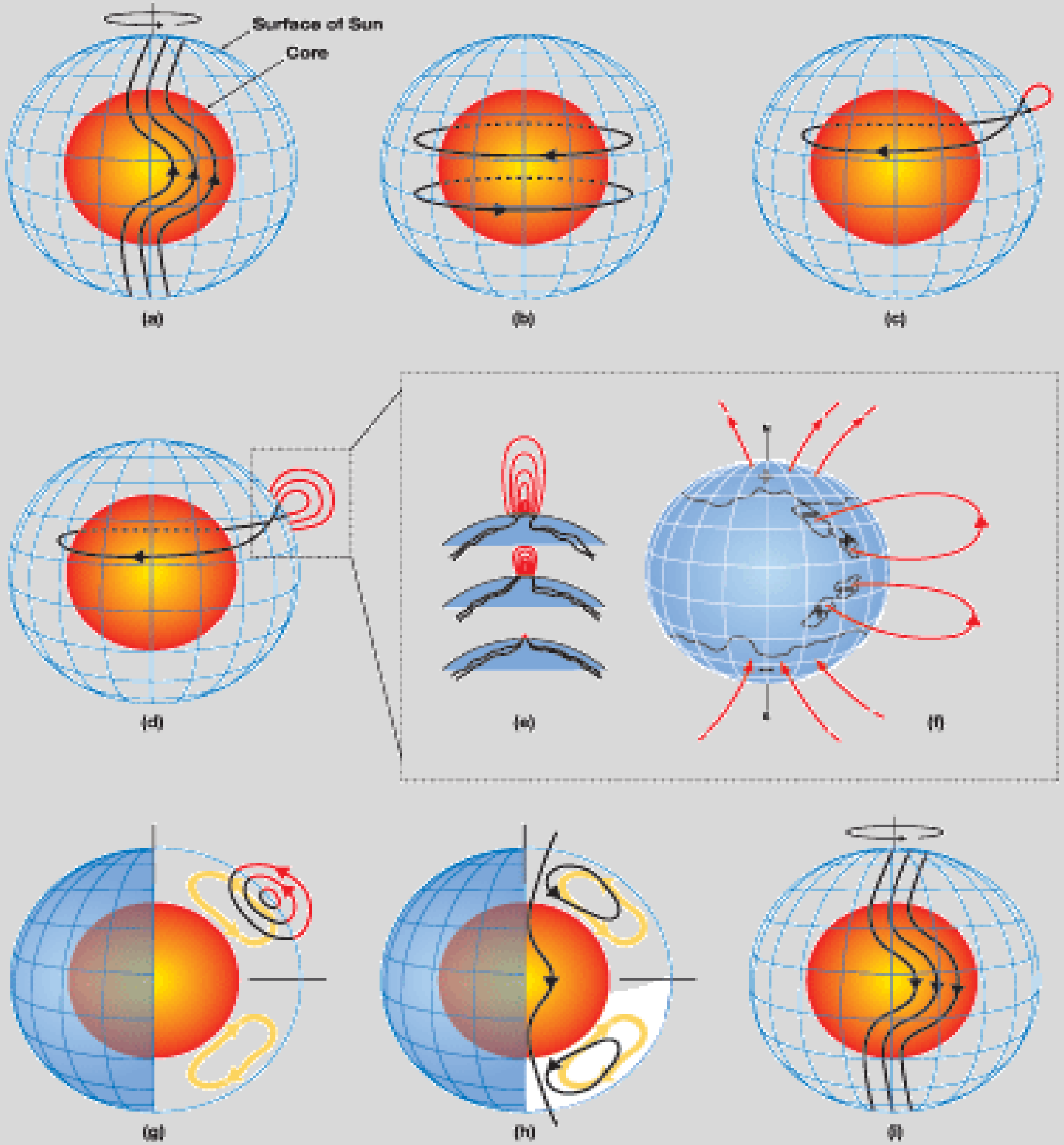
Dikpati et al. 2004

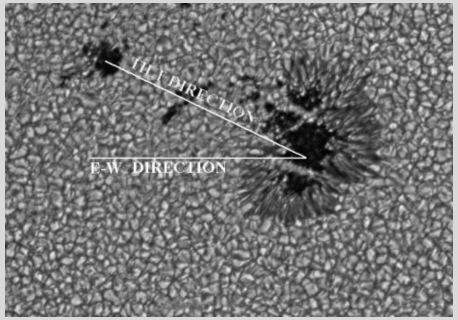


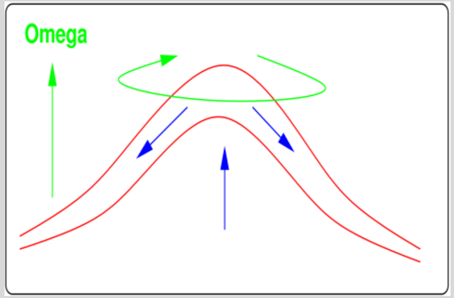
Schematic of a Babcock-Leighton flux transport model

(Durney,Choudhuri,Schüssler,Dikpati,Nandi,Charbonneau,Gilman,Rempel,Hotta)

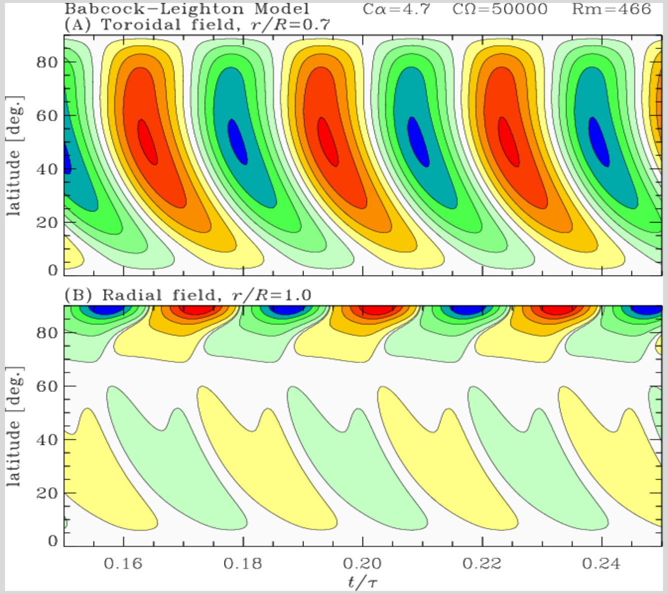
- Differential rotation
  - Toroidal field production
  - Stored at base of CZ
  - Rising flux tubes
- Babcock-Leighton α effect
  - Tilt angle of AR
  - Leading spots have higher probability to reconnect across equator
- Transport of magnetic field by meridional flow



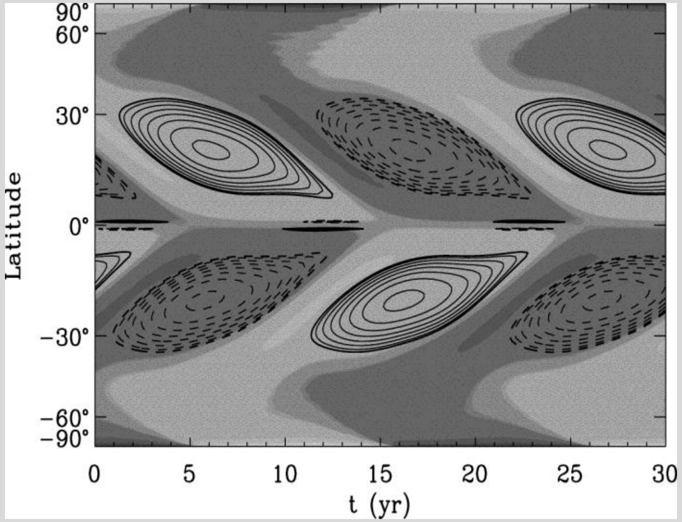




Solution properties flux transport dynamos



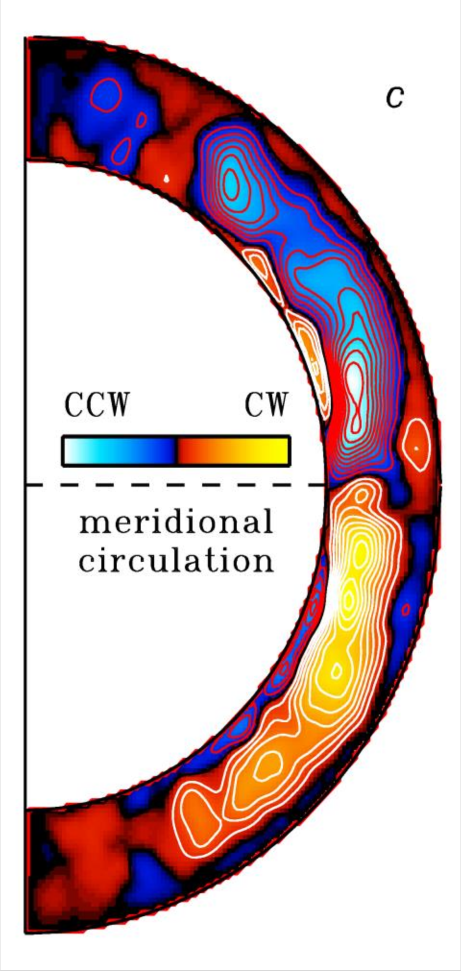
Dikpati & Charbonneau (1999)



Dikpati et al. (2004)

- Good agreement with basic cycle properties
  - – Equatorward propagation
  - – Weak cycle overlap
  - – Correct phase relation between poloidal and toroidal field
- Less good agreement
  - – Poleward extension of butterfly diagram?
  - – Polar surface field typically too strong
  - – Symmetry of solution (quadrupole preferred)
- More complicated ingredients can improve agreement
  - – Strong variation of magnetic diffusivity in CZ
  - – Strong turbulent pumping in surface regions
  - – Additional α-effect at base of CZ
- Expense: Strong sensitivity to many not well known ingredients

Meridional flow structure, assumptions flux transport dynamo

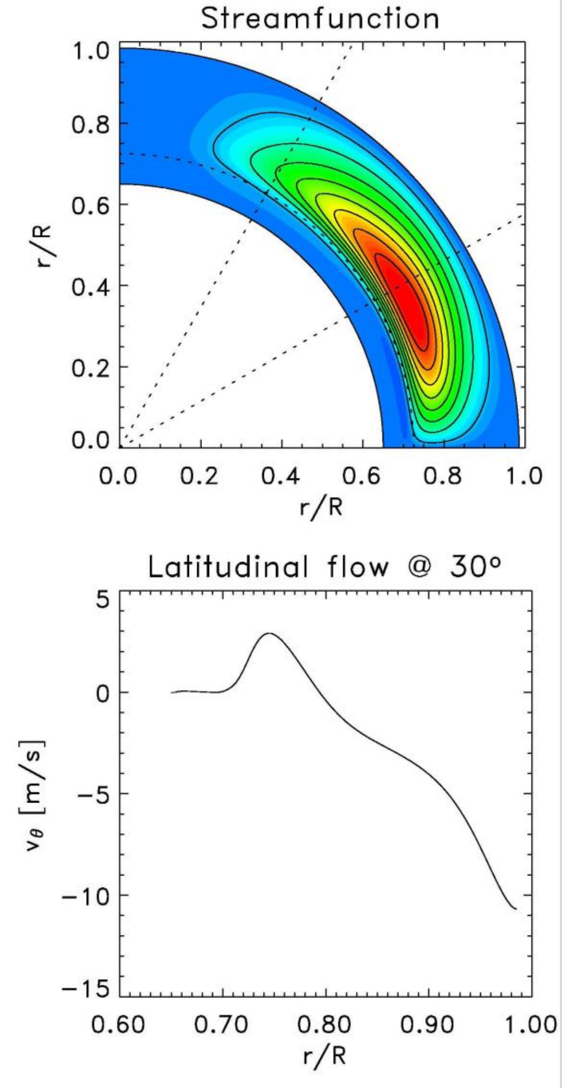


3D simulation
Miesch et al. (2008)

- Observations
  - Poleward near surface (surface Doppler and local helioseismology agree well)
  - Structure of flows in convection zone still heavily debated
- Theory
  - Mean field models: single flow cell, related to inward transport of angular momentum
  - 3D: Typical multi-cell for simulations that have a solar-like differential rotation
- Advection dominated regime difficult to realize:

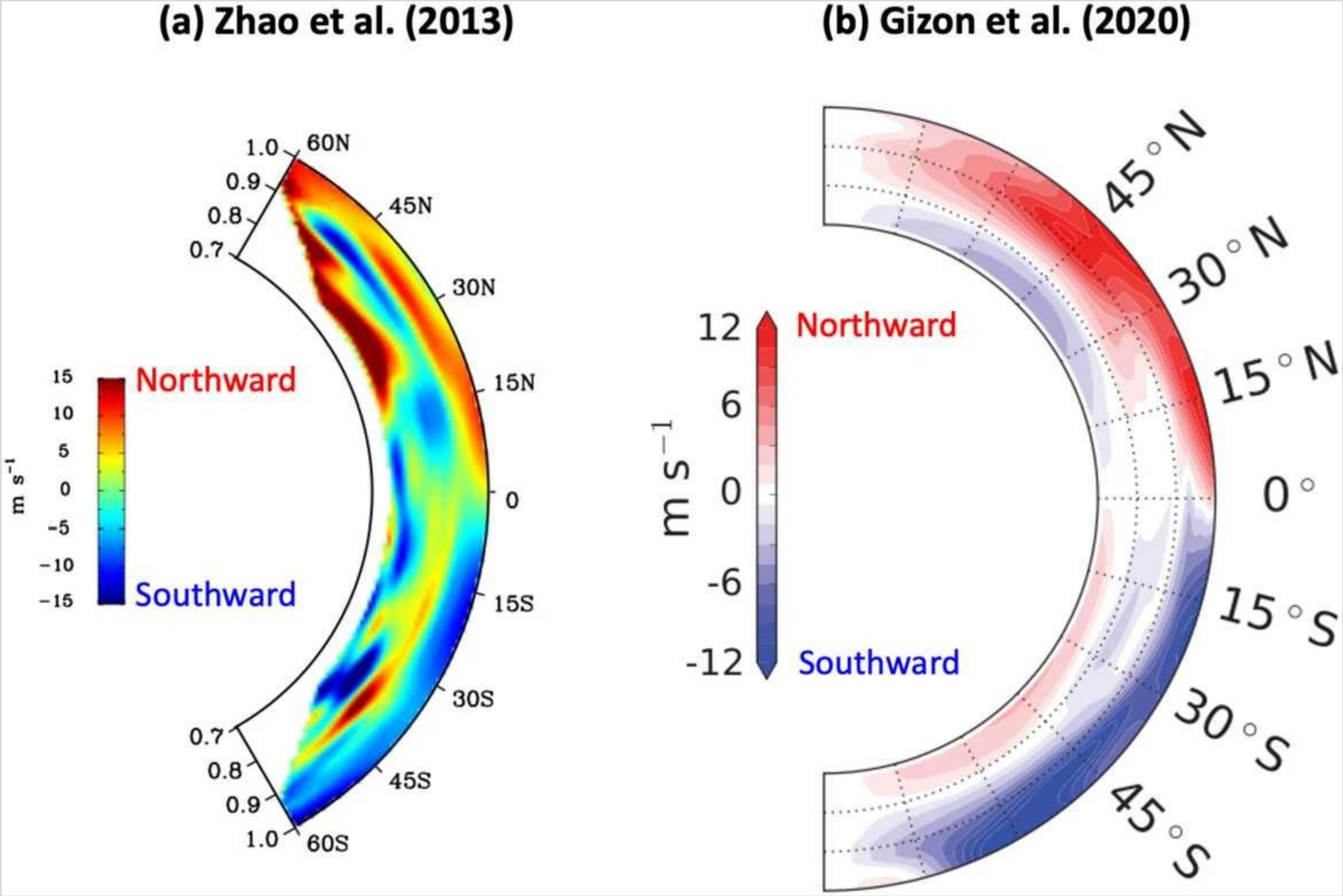
$$\eta_{turb} \propto H_p V_{rms}$$

$$V_{merid} \propto V_{rms}^2 / V_{rot}$$



Mean field model
Rempel (2005)

Flows inferred through helioseismology



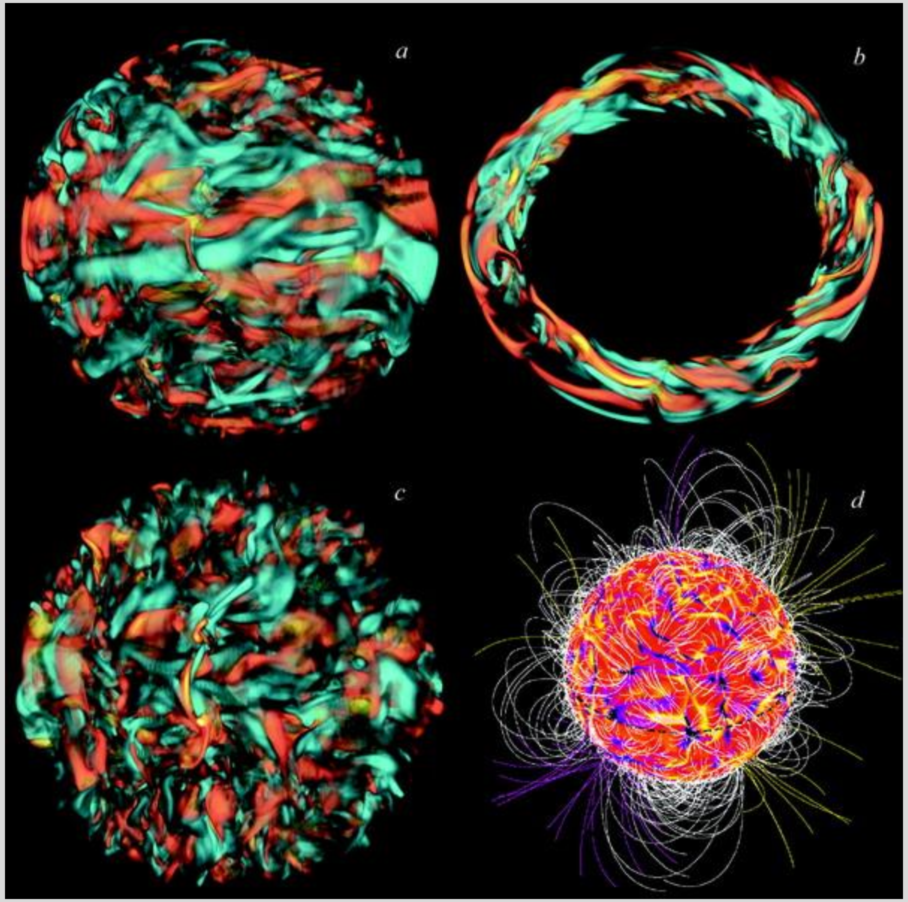
- ➢ Different methods lead to different results
- ➢ Different data-sets can lead to different results
  - – HMI vs. GONG
- ➢ More single celled in inversions that enforce conservation of mass

3D simulations

- Solve the full set of equations (including small and large scale flows, magnetic field) from first principles
  - No shortcuts, have to solve for the full problem including differential rotation and meridional flow
  - Non-linear effects automatically included
- Intrinsic limitations
  - Boundary conditions (radial direction)
    - Tachocline at base of CZ
    - Top boundary typically 20 Mm beneath photosphere
  - Cannot capture solar Re and Rm, how to treat small scales
    - DNS: resolve dissipation range with artificially increased diffusivities
    - (I)LES: do only the minimum required to maintain numerical stability
- Very expensive
  - Low resolution runs for long periods >10 years
  - High resolution for short periods
- Good understanding of ingredients of solar dynamo, no complete model yet

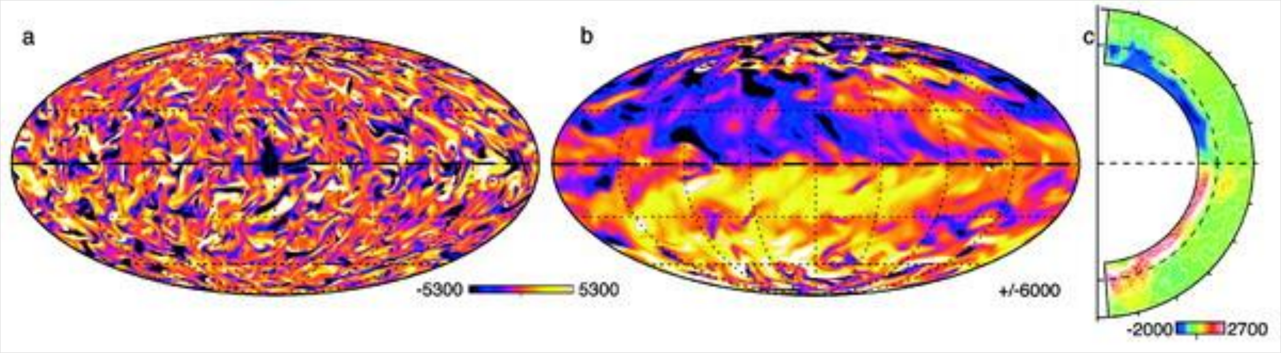
3D dynamo simulations

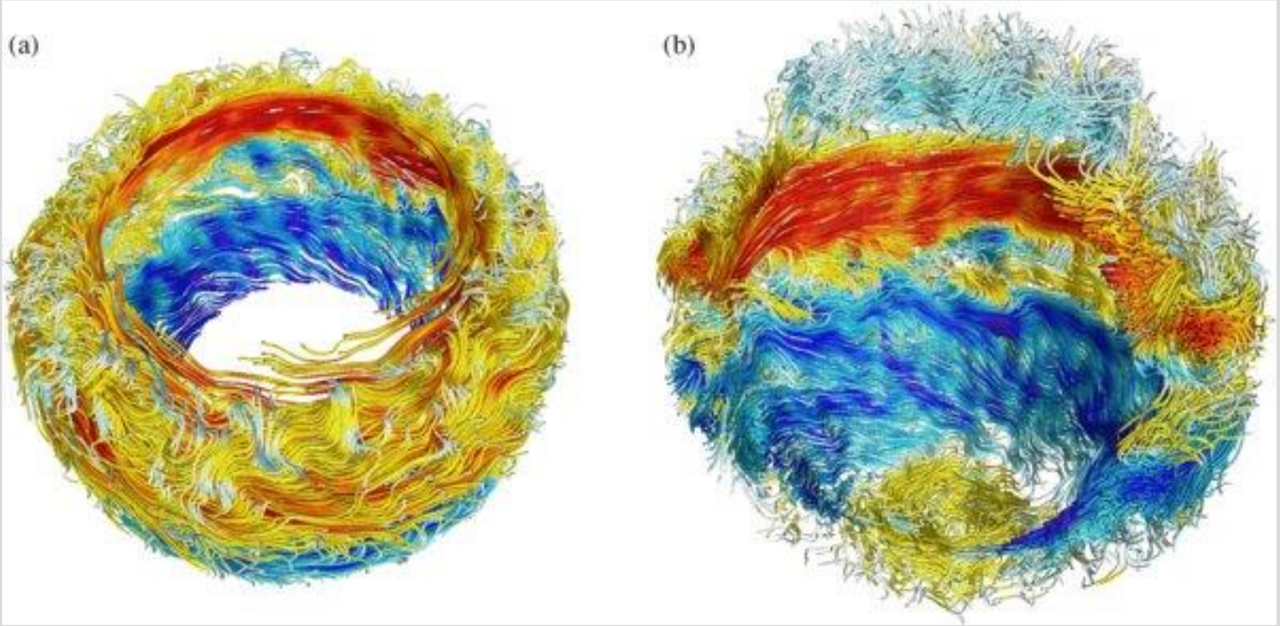
- 1981 Gilman & Miller
  - First 3D convective dynamos in a spherical shell (Boussinesq)
- 1983 Gilman
  - Dynamo simulations with reduced diffusivities
    - large scale field and periodic field reversal
    - poleward propagation
- 1985+ Glatzmaier …
  - Mostly 3D geodynamo models
- 2004 Brun, Miesch, Toomre
  - Turbulent dynamo (anelastic)
    - 800 G peak toroidal field
    - Mean field 2% of energy
    - No cyclic behavior



3D dynamo simulations

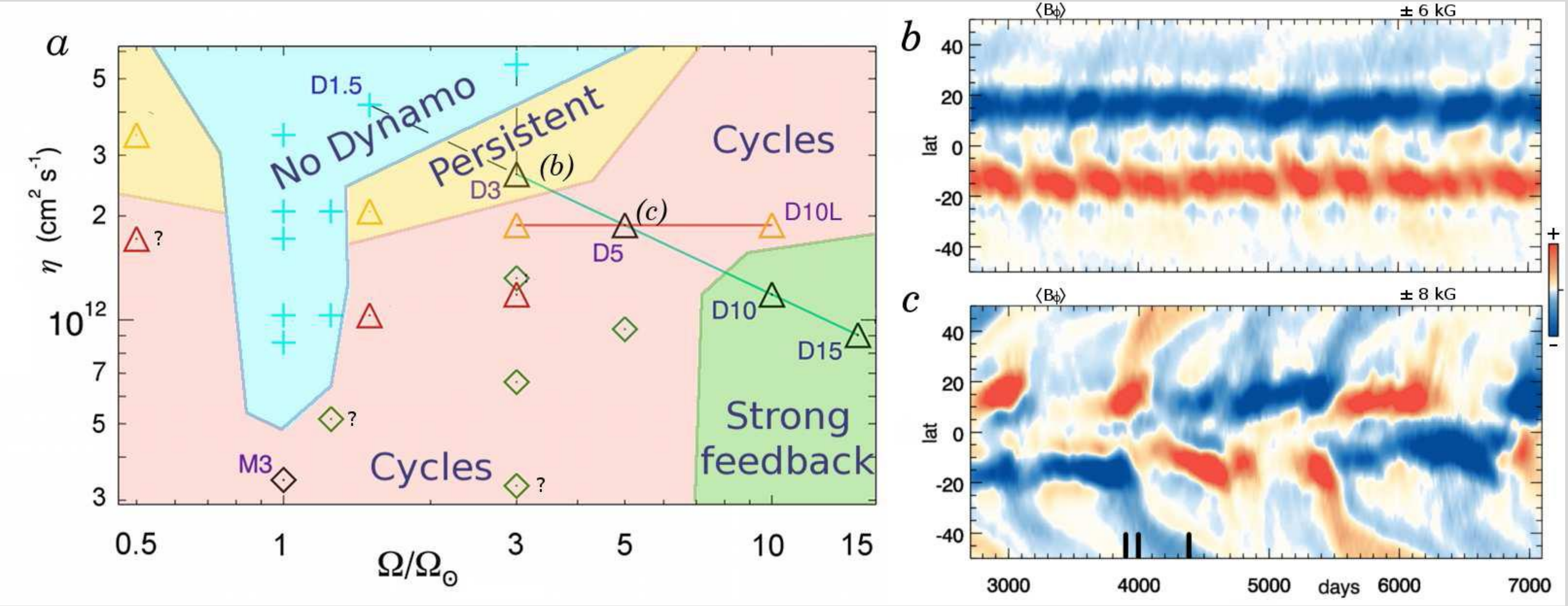
- 2006 Browning et al.
  - Addition of tachocline
  - Organized ~5 kG field in stably stratified region
- 2008+ Brown et al.
  - Faster rotating stars
  - Strong field (~10 kG) maintained within CZ
  - Cyclic behavior for certain parameter choices (faster rotation)





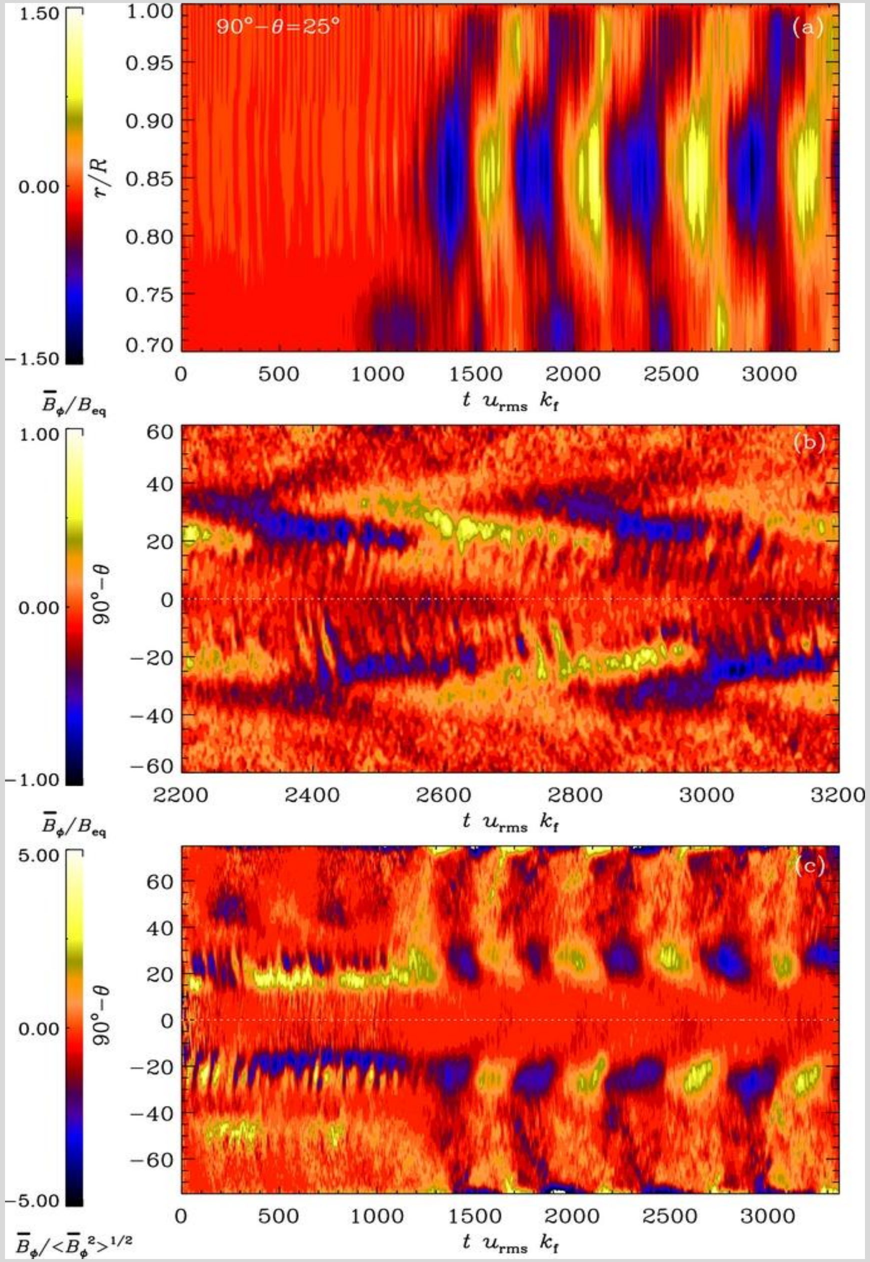
Cyclic dynamo regimes

- 2011 Brown et al.
  - Cyclic behavior typically found for sufficiently high Rm
    - Small diffusivity
    - Fast rotation
    - Difficultly to excite dynamo near solar rotation rate

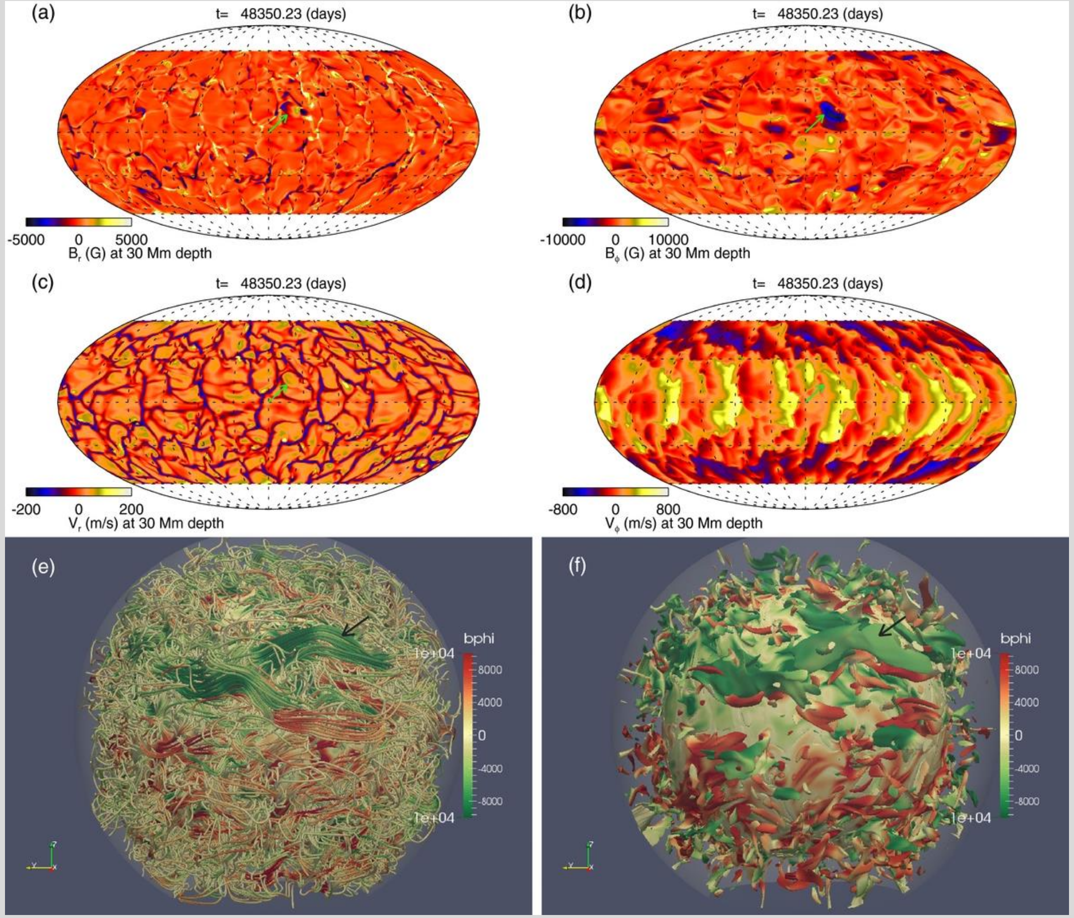


3D dynamo simulations

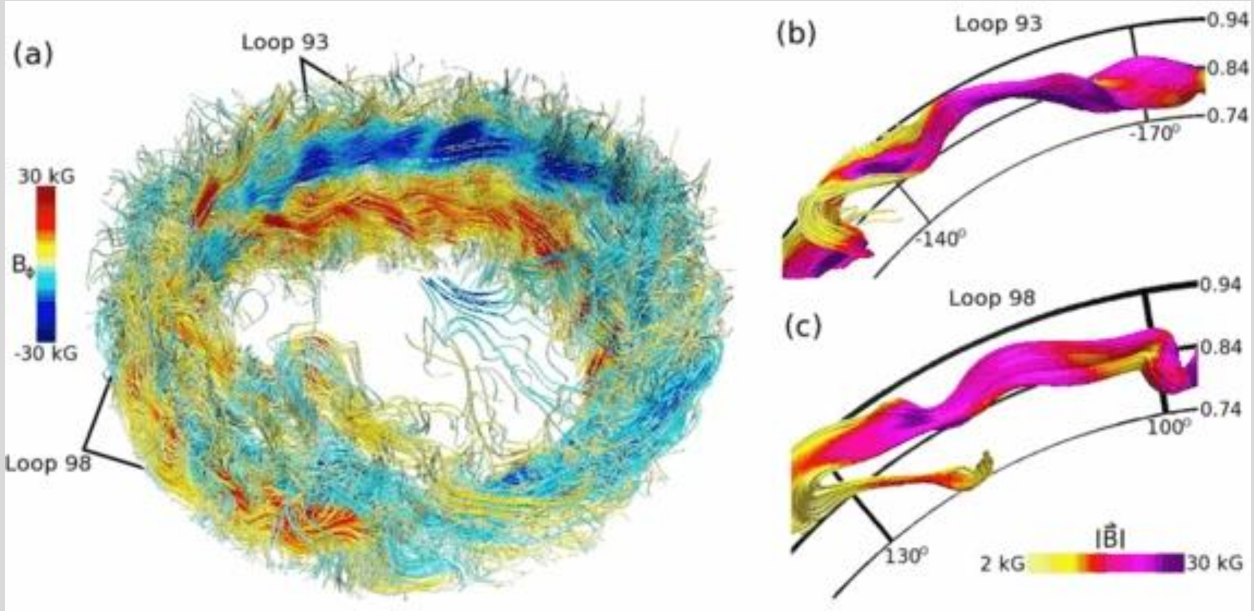
- Kapyla et al. (2012)
  - 33 year period
  - Field generated in bulk of CZ
  - Equatorward propagation below 40 deg latitude
    - Propagation due to non-solar-like differential rotation
  - Cycle length non-linear effect
    - Much shorter cycles during kinematic growth phase
    - “Phase transition” due to non-linear feedback



Dynamo simulations leading to flux emergence



Fan & Fang 2014



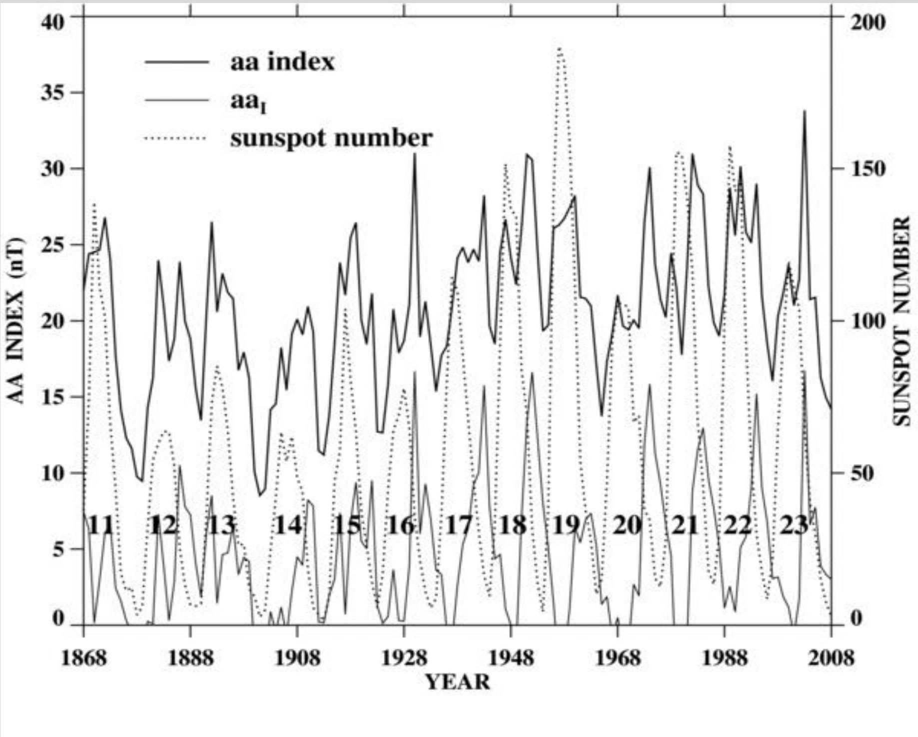
Nelson et al. 2014

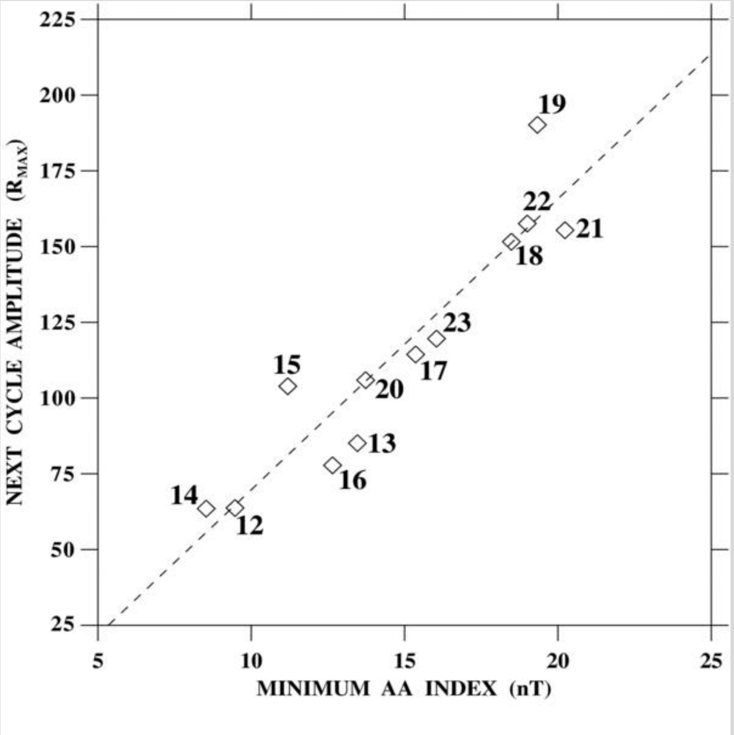
- Production of magnetic flux bundles ~10-30 kG in bulk of convection zone
- Amplified by non-axisymmetric zonal shear
- Buoyant rise towards top boundary
- Scale too large for typical solar active regions

3D dynamo simulations

- Recent developments:
  - Several independent groups find cyclic dynamos with periods in the 10-60 year range
  - Some models with equatorward propagation of activity
  - No simple explanation for cycle length and magnetic field patterns
    - Cycle length non-linear effect (longer cycles in saturated phase)
    - Not obvious if different models get similar solutions for the same reason
- Contrast to mean-field models:
  - In general no single dominant turbulent induction term (like a scalar α-effect) that could capture the behavior
  - Non-linear feedback more than just saturation effect (i.e. long cycle length only found in non-linear regime)
- Both, mean-field models and 3D simulations have serious challenges in providing a consistent model of the solar cycle!

What do the observations tell?





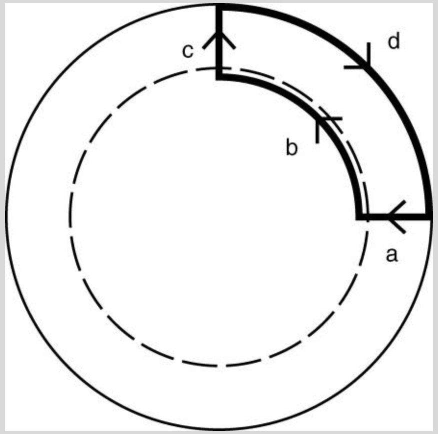
Wang & Sheeley 2009

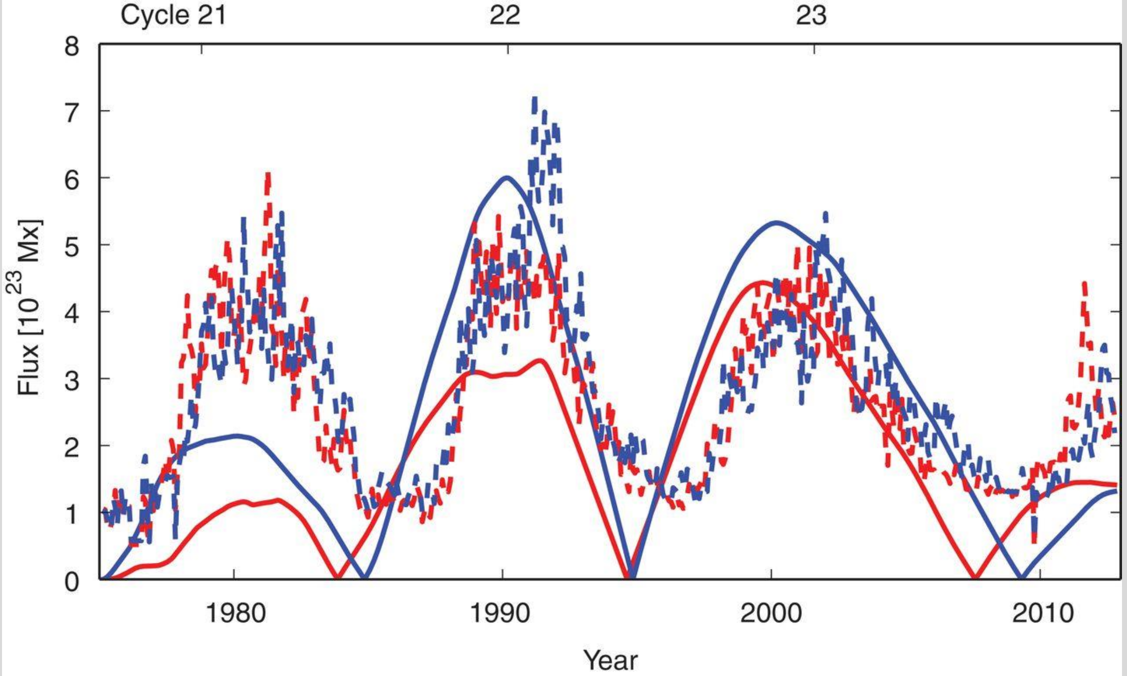
- Geomagnetic activity related to solar high speed streams (solar minimum) and CMEs (solar max)
- High speed streams during minimum related to flux of polar caps -> poloidal field of sun during minimum
- Shows strong correlation with upcoming cycle amplitude

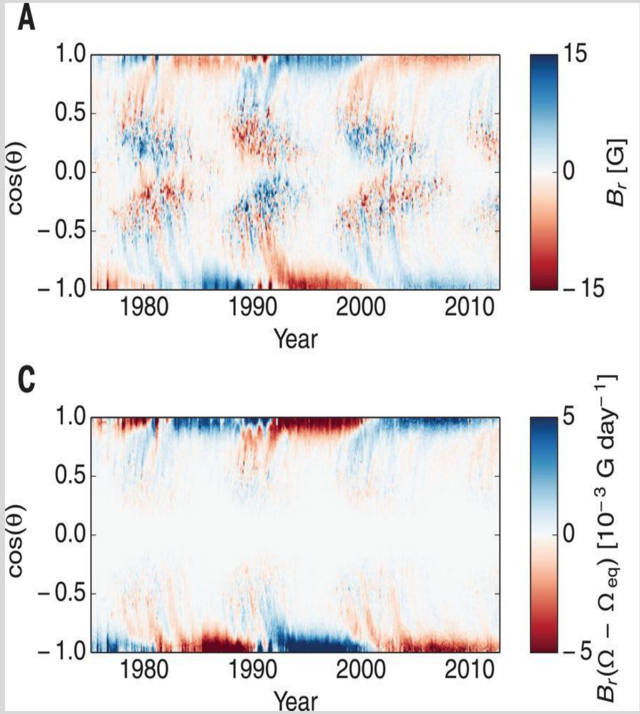
Surface flux evolution and net toroidal flux

$$\frac{\mathrm{d}\Phi^{\mathrm{N}}_{\mathrm{tor}}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\int_{\Sigma} B_{\phi}\,\mathrm{d}S\right) \quad = \int_{\delta\Sigma}\left(\mathbf{U}\times\mathbf{B} + \langle\mathbf{u}\times\mathbf{b}\rangle - \eta\nabla\times\mathbf{B}\right)\cdot\mathrm{d}\mathbf{l}$$

$$\frac{\mathrm{d}\Phi^{\mathrm{N}}_{\mathrm{tor}}}{\mathrm{d}t} = \int_{0}^{1}(\Omega - \Omega_{\mathrm{eq}})B_r R_{\odot}^2\,\mathrm{d}(\cos\theta) - \frac{\Phi^{\mathrm{N}}_{\mathrm{tor}}}{\tau}$$





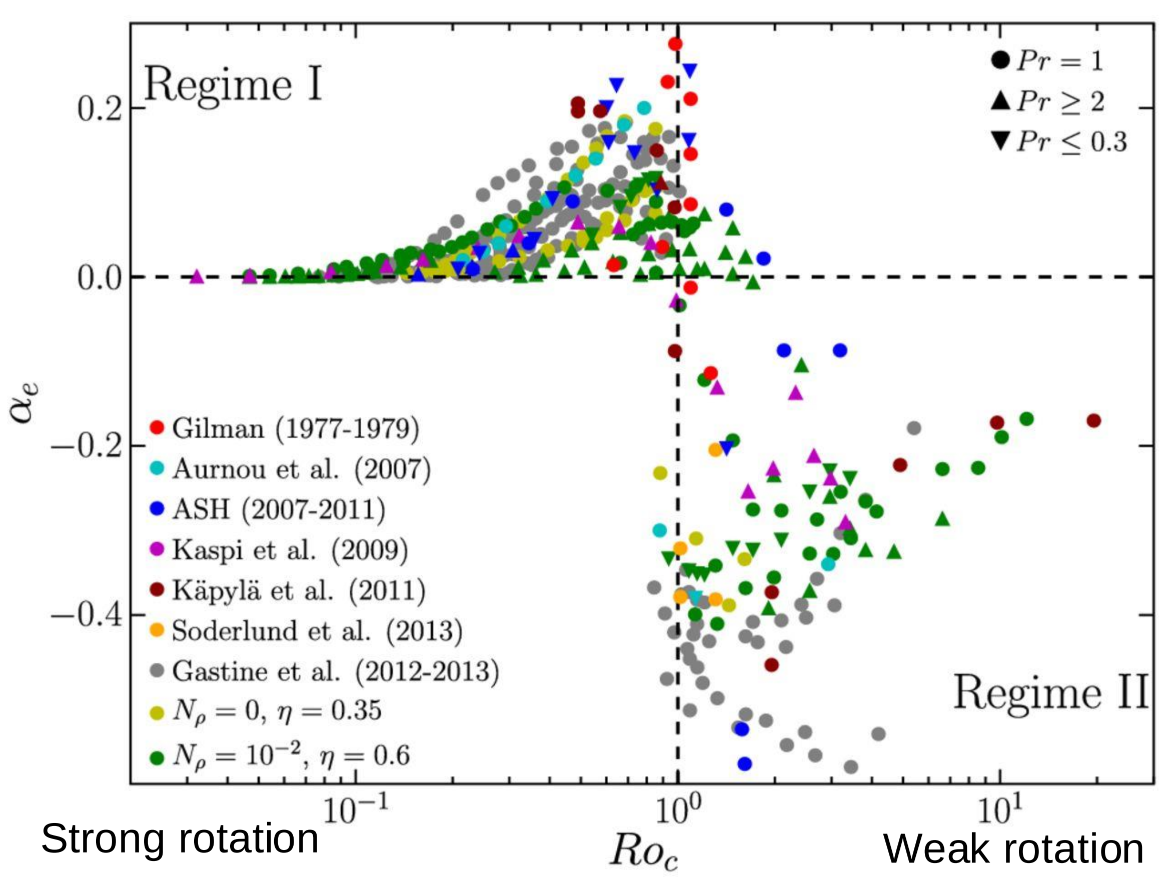


Robert Cameron, and Manfred Schüssler Science 2015;347:1333-1335

Minimalistic phenomenological model

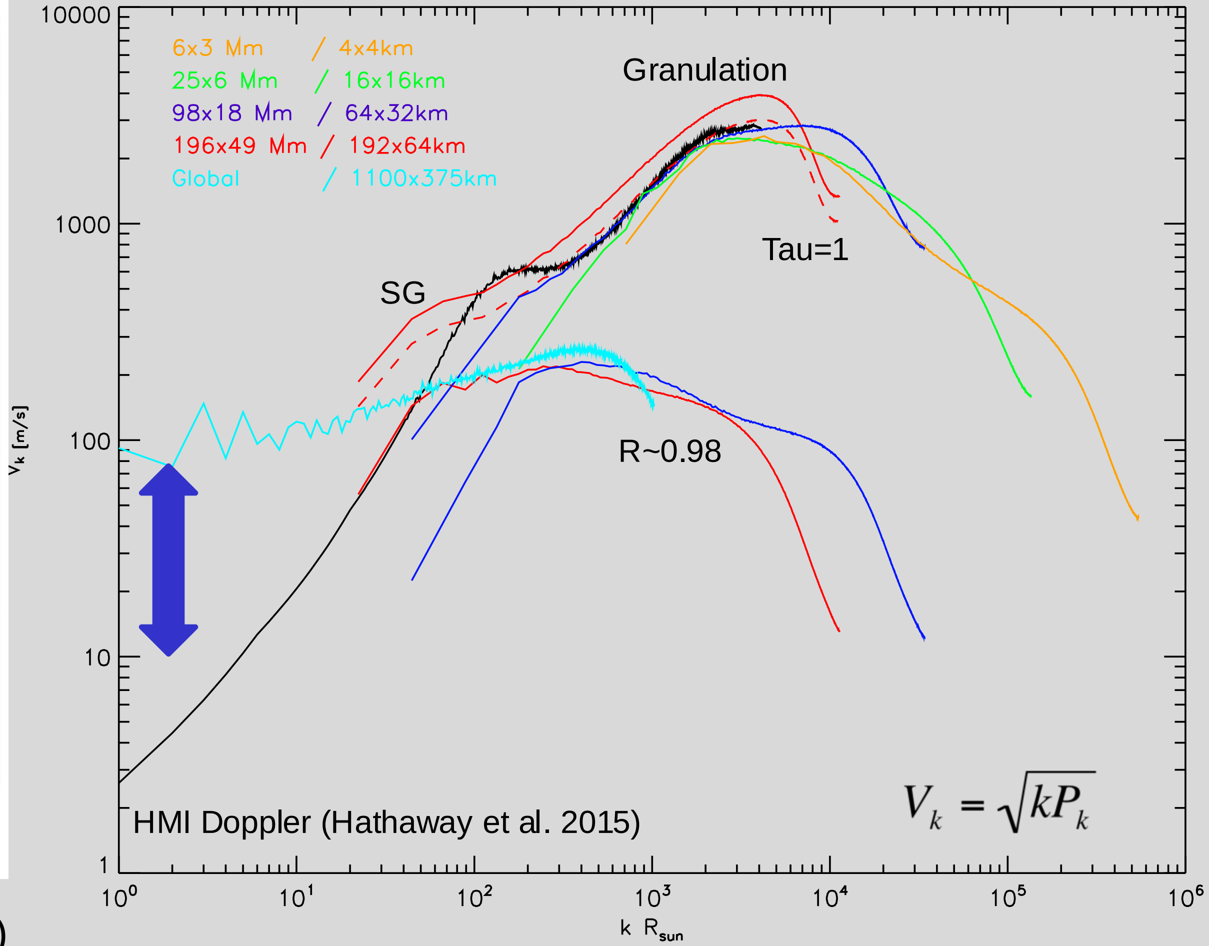
- The Sun tells us:
  - The polar field is strongly correlated with the strength of the next cycle
    - Must be connected to the poloidal field that that is converted to toroidal field
  - The Sun does generally obey Hale’s polarity rules, only few exceptions near beginning and end of cycle
    - Toroidal field in convection zone likely mostly unipolar, need to be able to produce a net toroidal flux
  - Surface term shows that (observationally constraint) BL source is sufficient to produce toroidal flux required for solar cycle
    - All other alpha-effects buried in convection zone would produce a mixed polarity toroidal field
  - Observed non-linearity: active region inflows
  - Regularity of cycle suggest weaky supercritical dynamo with noise
    - Actice region emergence is primary source of randomness, AR emergence late in cycle most critical (can strongly impact hemispheric poloidal flux)
    - Can explain long-term variability in statistical sense
- Equatorial transport by meridional flow + turbulent pumping
  - not observed, assumption
- Do we need anything else?
  - If the answer is “no”: Need to find reason why all the other dynamos in the convection zone don’t operate

A few additional “conundrums”



Gastine et al. (2013)

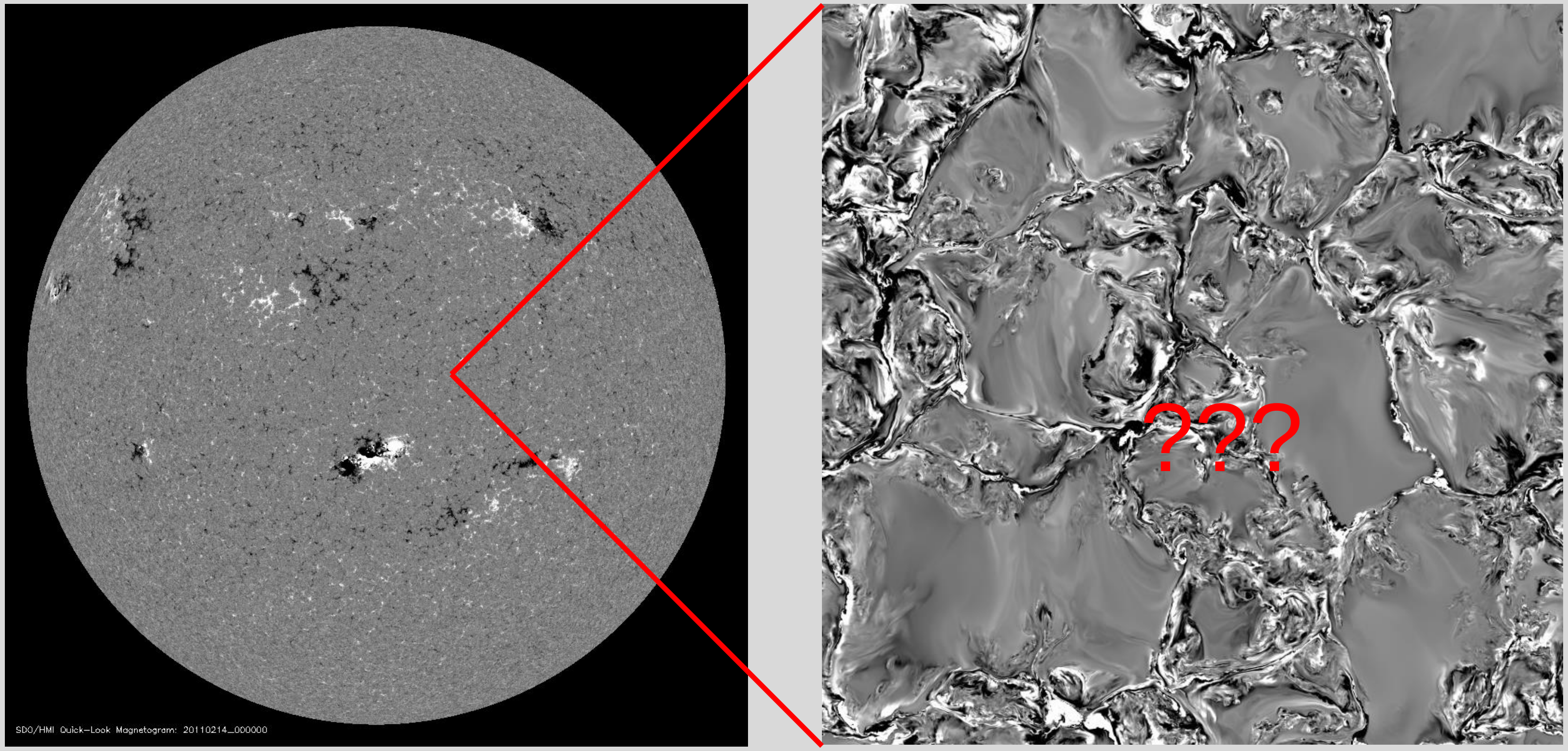
Sun close to transition from solar to anti-solar DR



On scales larger than SG (~30Mm, l~120) simulations have too much power compared to observations!

Is there something very fundamental about highly stratified convection we do not understand?

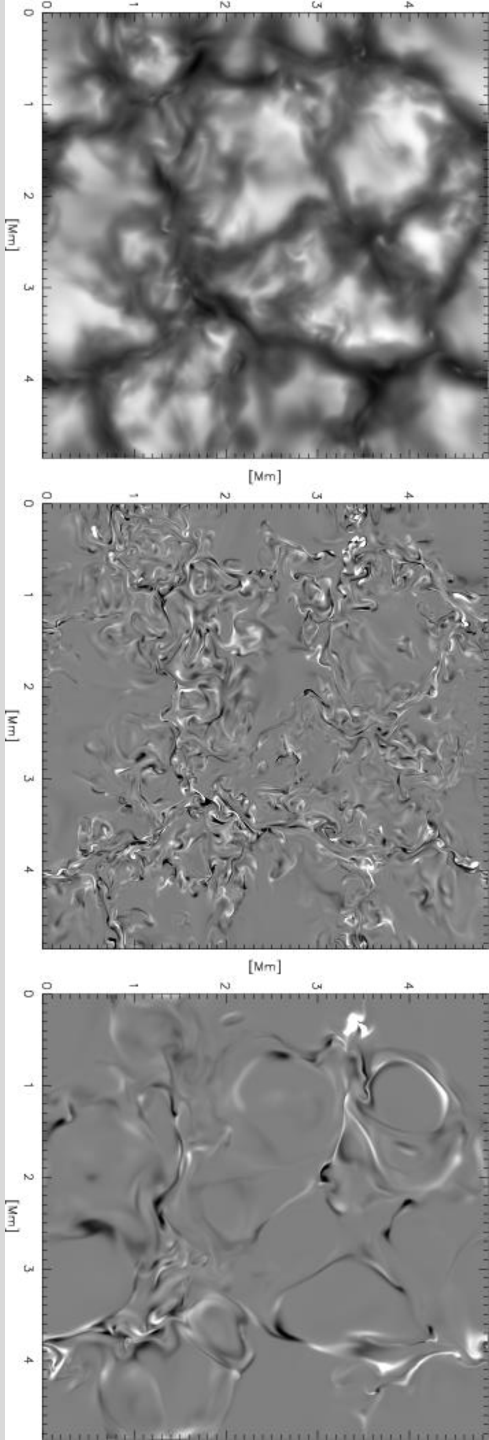
Quiet Sun magnetism



- Most of the solar surface is covered by “quiet Sun” at any time during the sunspot cycle!
- Where does this field come from?
- Does it have dynamic consequences for convection, differential rotation and the large scale dynamo?

Solar simulations of the quiet Sun

- Before 2000, mostly HD granulation simulation
- Idealized SSD simulations, Cattaneo (1999) (Boussinesq) Bercik et al. (2005) (anelastic)
- Vögler & Schüssler (2007), first “realistic” SSD simulation (compressible, EoS, RT)
- Discrepancy between simulations and observations
  - Danilovic et al. (2010): Zeeman, simulations 2-3 too weak
  - Trujillo-Bueno (2011): Hanle, stronger than Zeeman, simulation needs to be scaled up 12x in upper photosphere
- Many new recent models: Rempel (2014, 2018), Kitiashvili (2015), Khomenko (2017)
  - Higher resolution
  - Improved boundary conditions
- Good agreement between simulations, Zeeman and Hanle observations requires **$<|B_z|>\sim 60 – 80$ G** at optical depth unity
  - Danilovic et al. (2016) (Zeeman)
  - Del Pino Aleman et al (2018) (Hanle)



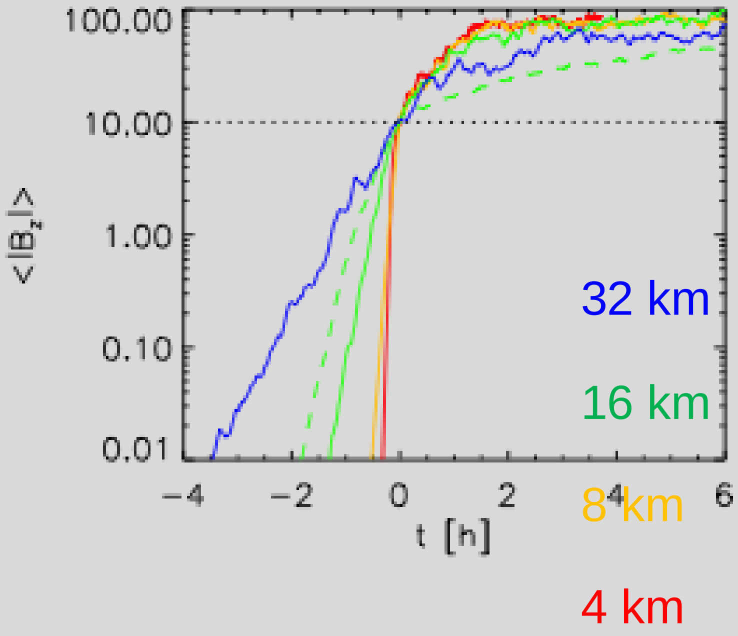
Vögler & Schüssler (2007)

Hidden unsigned flux in QS

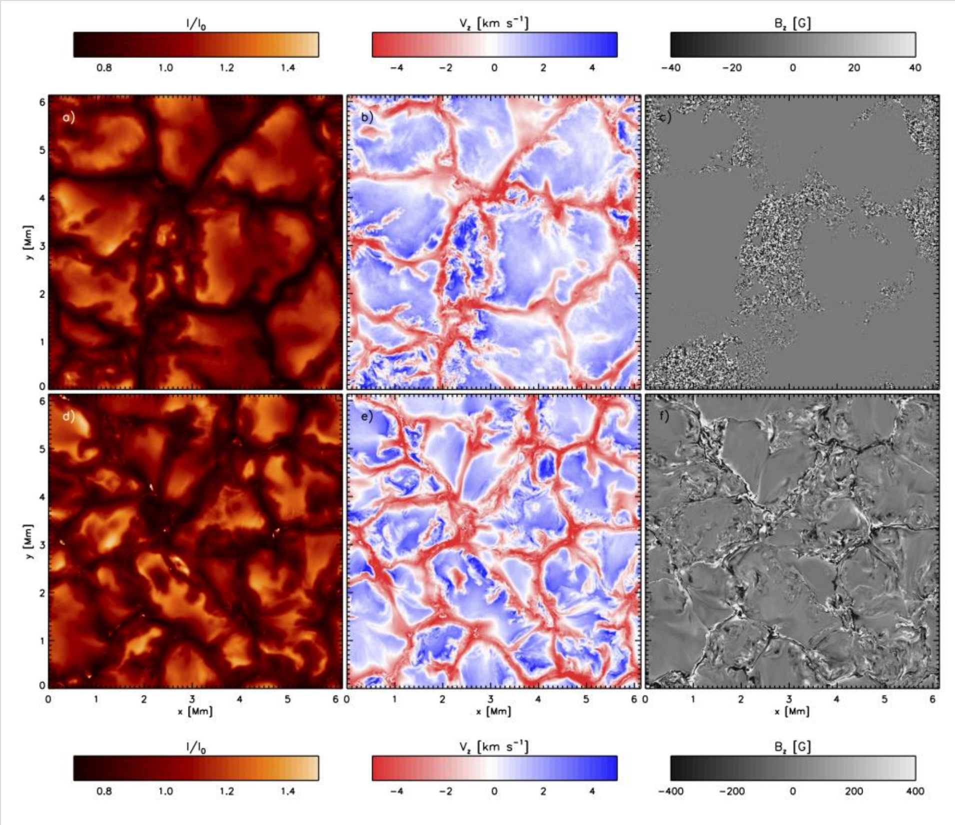
- Comparison of observations and simulations suggests:
  - <|Bz|> ~ 60-80 G at optical depth of unity
- Integrated over the entire solar surface:
  - ~ 4 x 10^{24} Mx
- Typical solar active region:
  - 10^{22} Mx

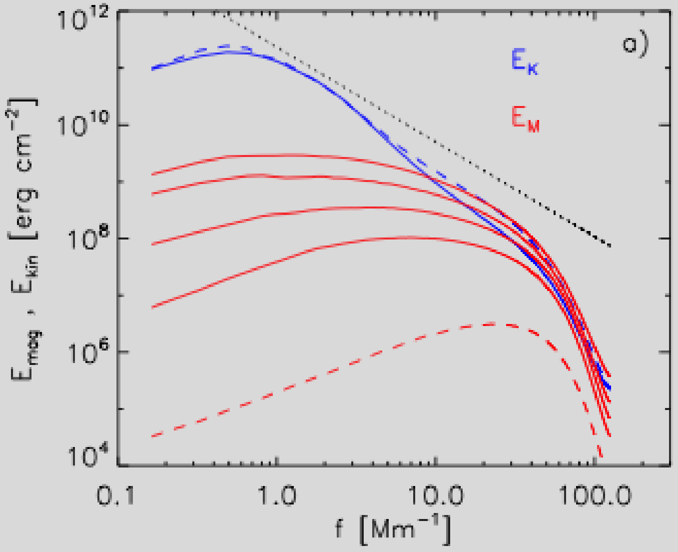
- **Unsigned flux content of QS comparable to that of all the active regions in an entire 11 solar cycle at any given time and gets replaced on a time-scale of minutes to hours!**
  - It is very unlikely that this is a remnant of the solar cycle!
  - We need an independent dynamo process that maintains the small-scale field!

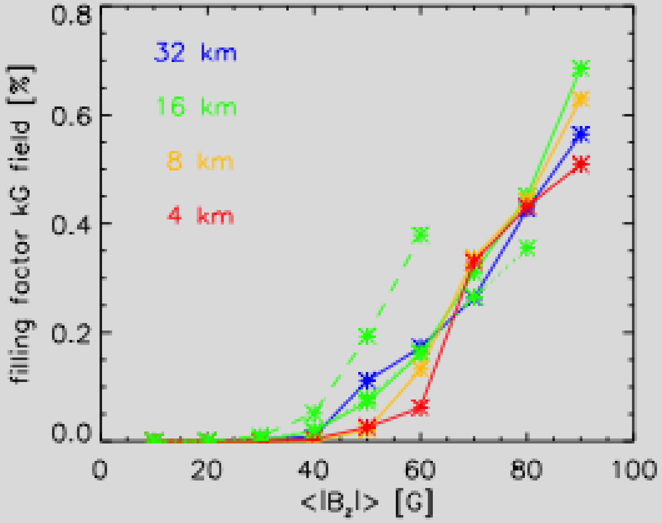
Kinematic regime to saturation



From Rempel (2014)



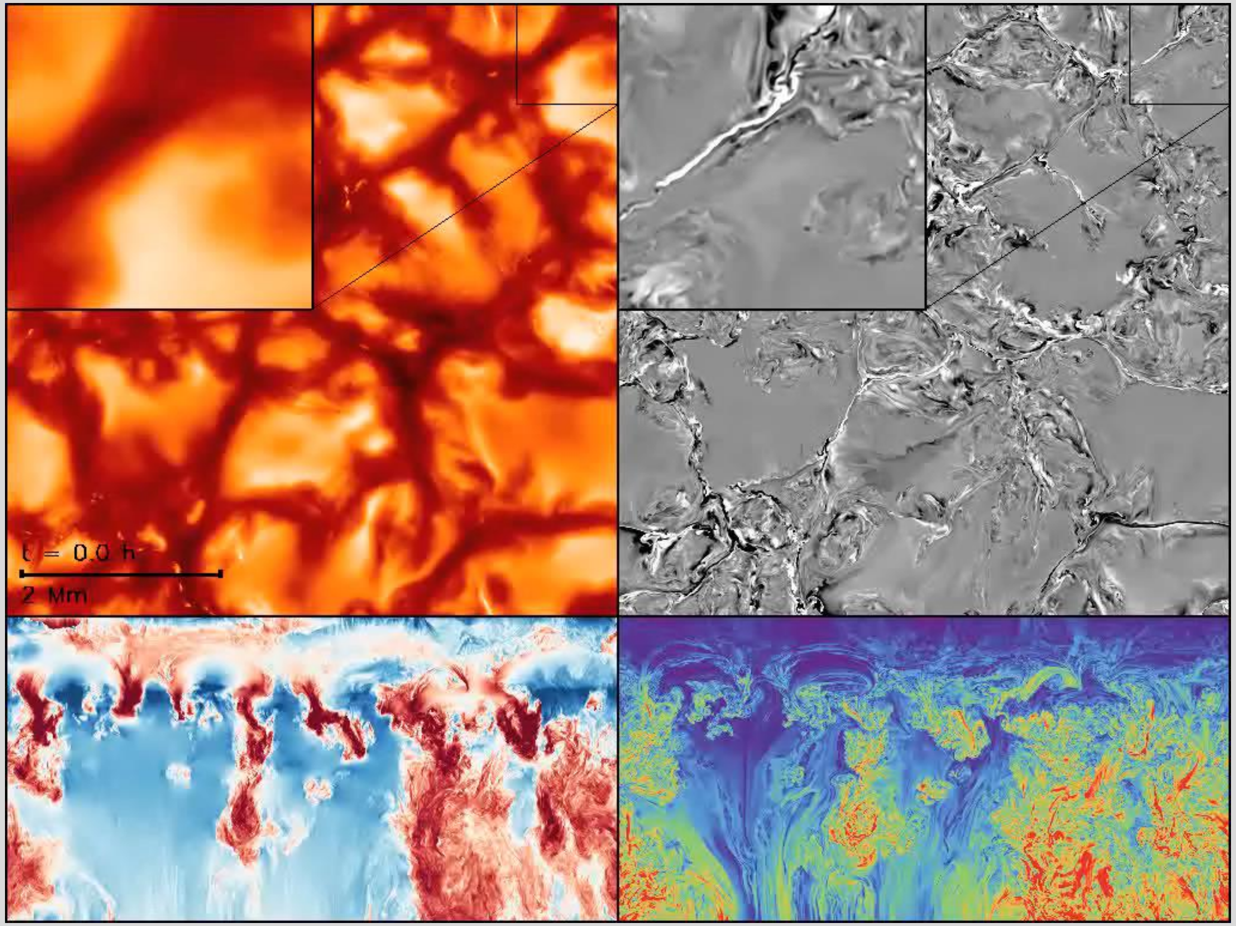




- Magnetic field organization changes dramatically during saturation
  - Non-linear saturation begins for <|B$_z$|>~10 G in photosphere
  - Sheet like appearance instead of “salt and pepper”
  - Peak of magnetic energy near granular scales
  - kG flux concentrations, bright points appear starting from <|B$_z$|>~30 G

Saturated SSD solution consistent with observational constraints

Domain: 6.144 x 6.144 x 3.072 Mm^3 4km grid spacing



Intensity

Bz (τ=1) [+/- 400 G]

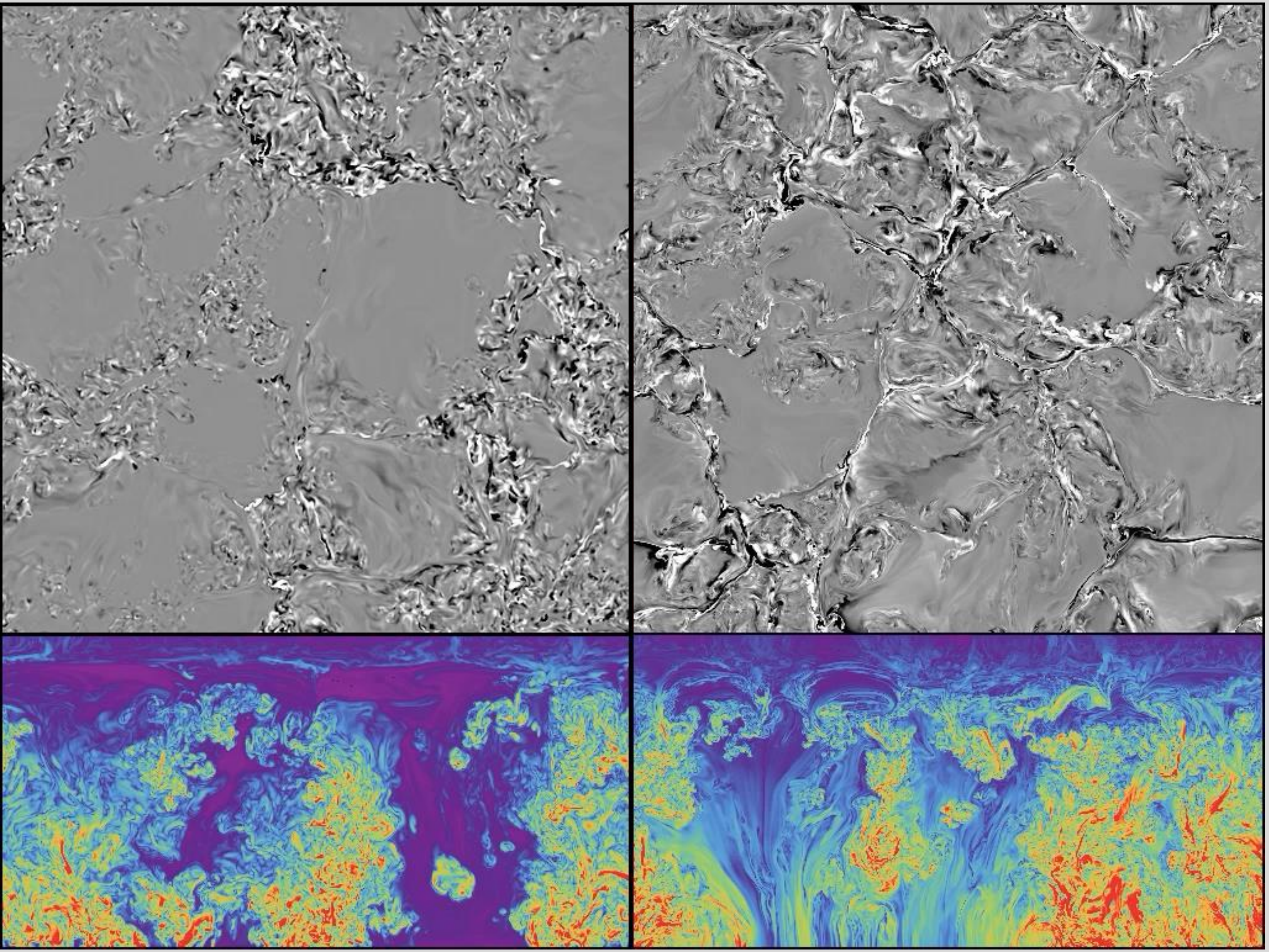
Vz [+/- 4 km/s]

|B| [< 2 kG]

Open bottom boundary mimics the presence of a deep magnetized convection zone

Rempel (2014)

Shallow vs. deep recirculation



Left: shallow recirculation

<|Bz|> ~ 30G

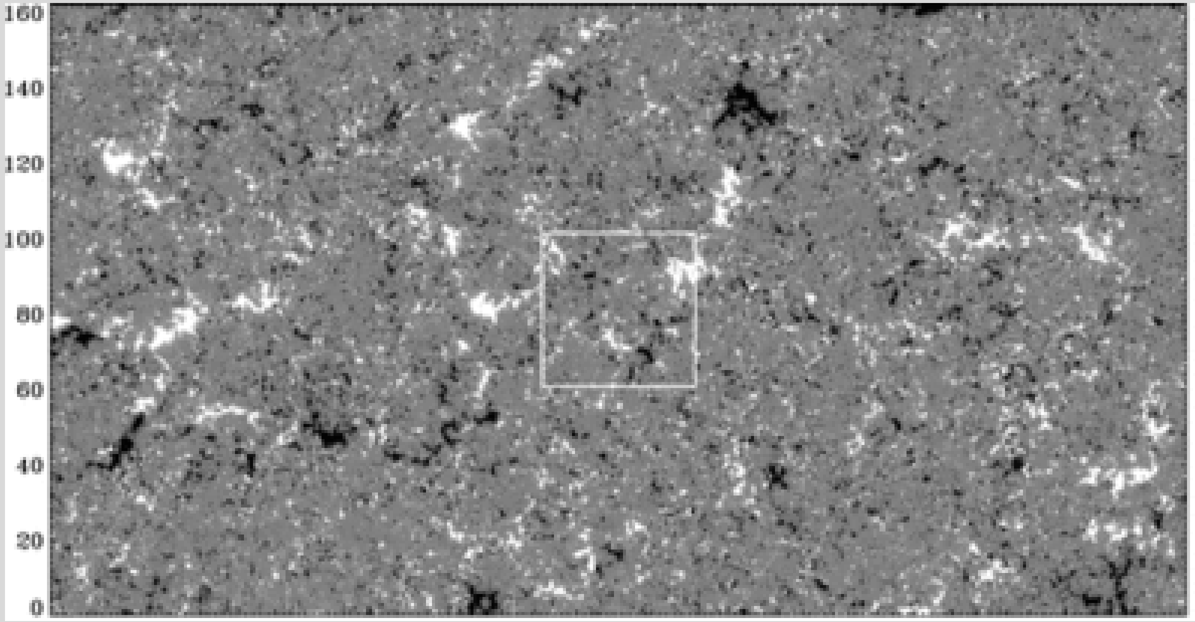
Right: deep recirculation

<|Bz|> ~ 60G

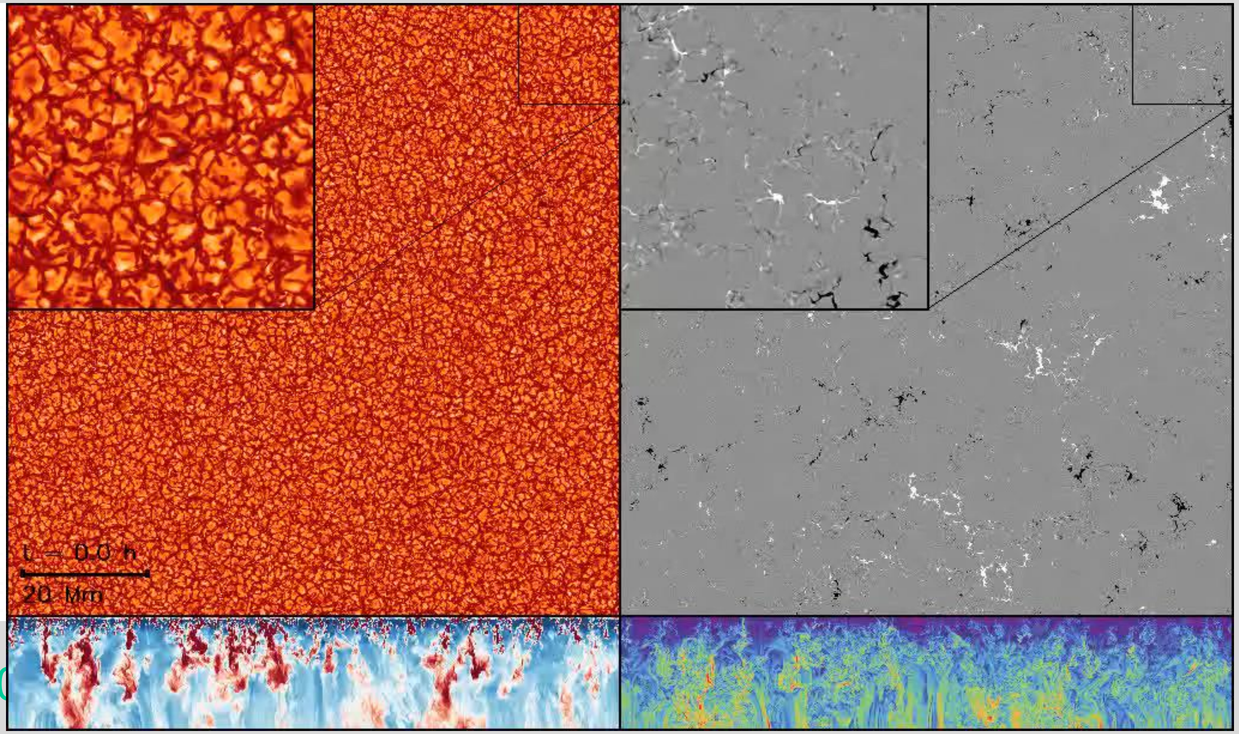
Solar SSD is operating over a wide range of scales in the convection zone

  - **Stratification leads to organization of field on scales larger than granulation**

Origin of Quiet Sun Network field



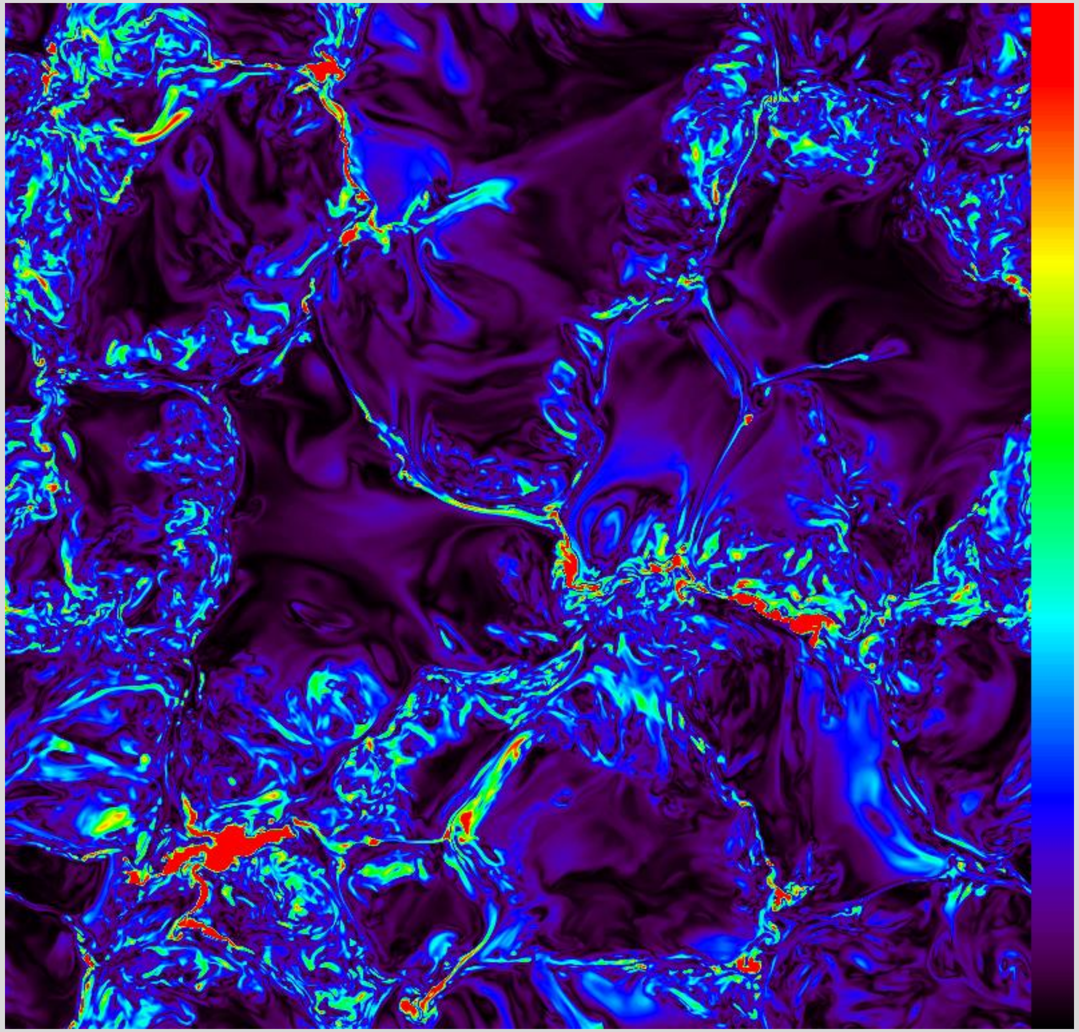
From Lites et al (20



- ➢ What is the origin of the QS network field?
  - – Is it part of the quiet Sun?
  - – Still a remnant of the solar cycle?

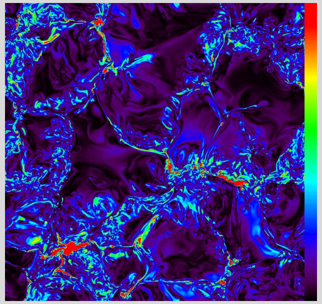
SSD can produce mixed-polarity network in sufficiently large domains, here 100x100x18 Mm

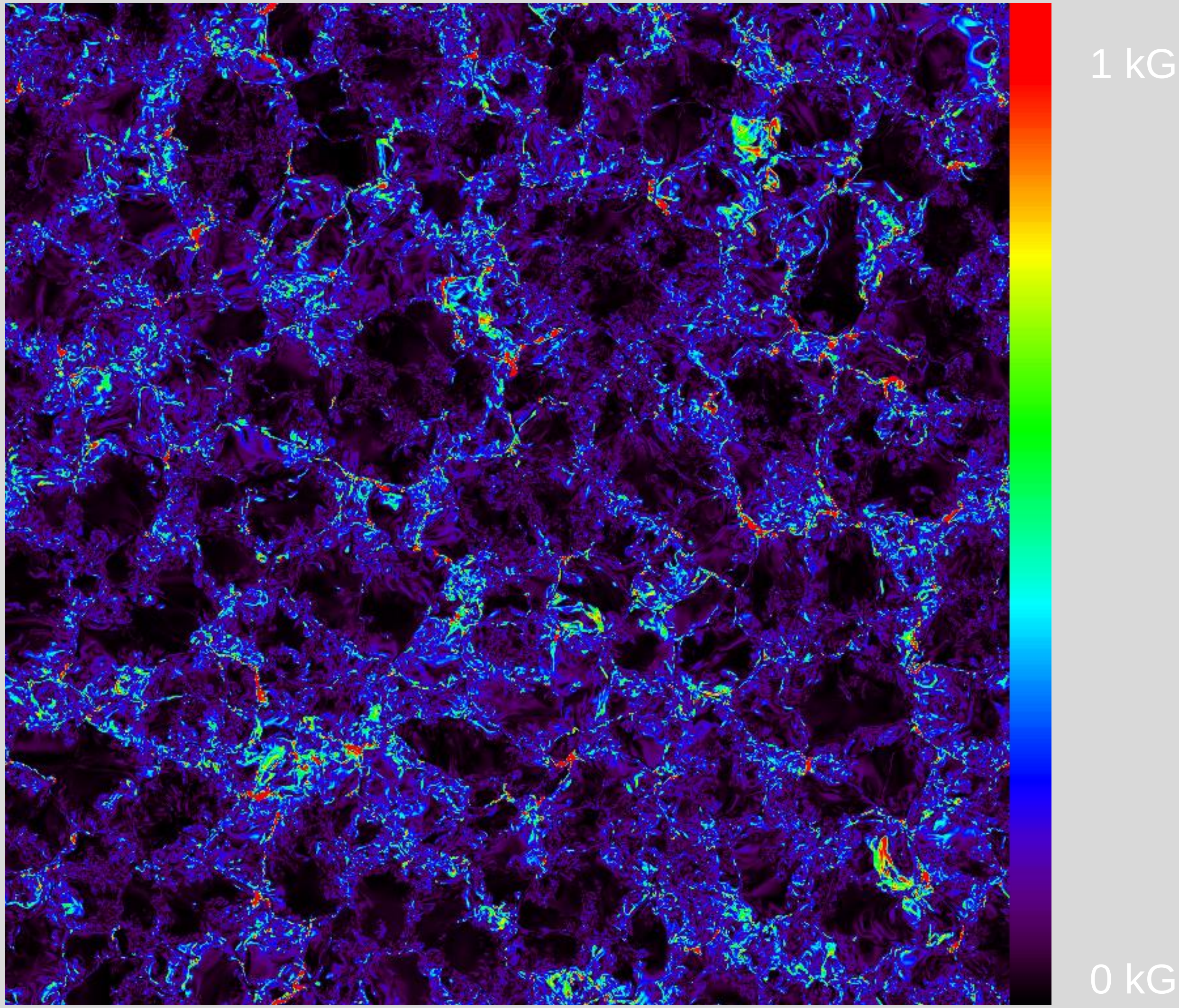
Larger scale organization and “voids”



6x6x2.3 Mm

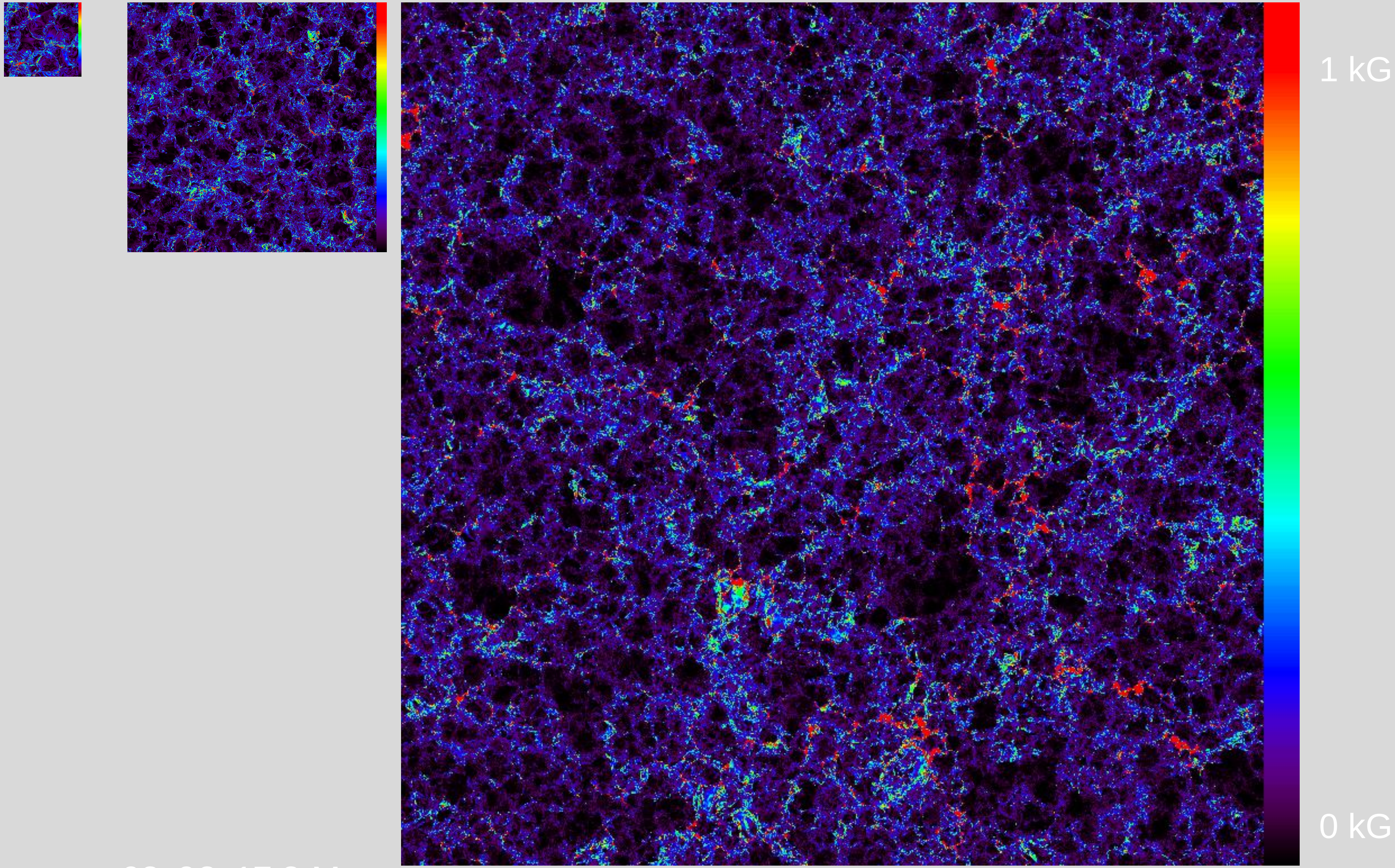
Larger scale organization and “voids”



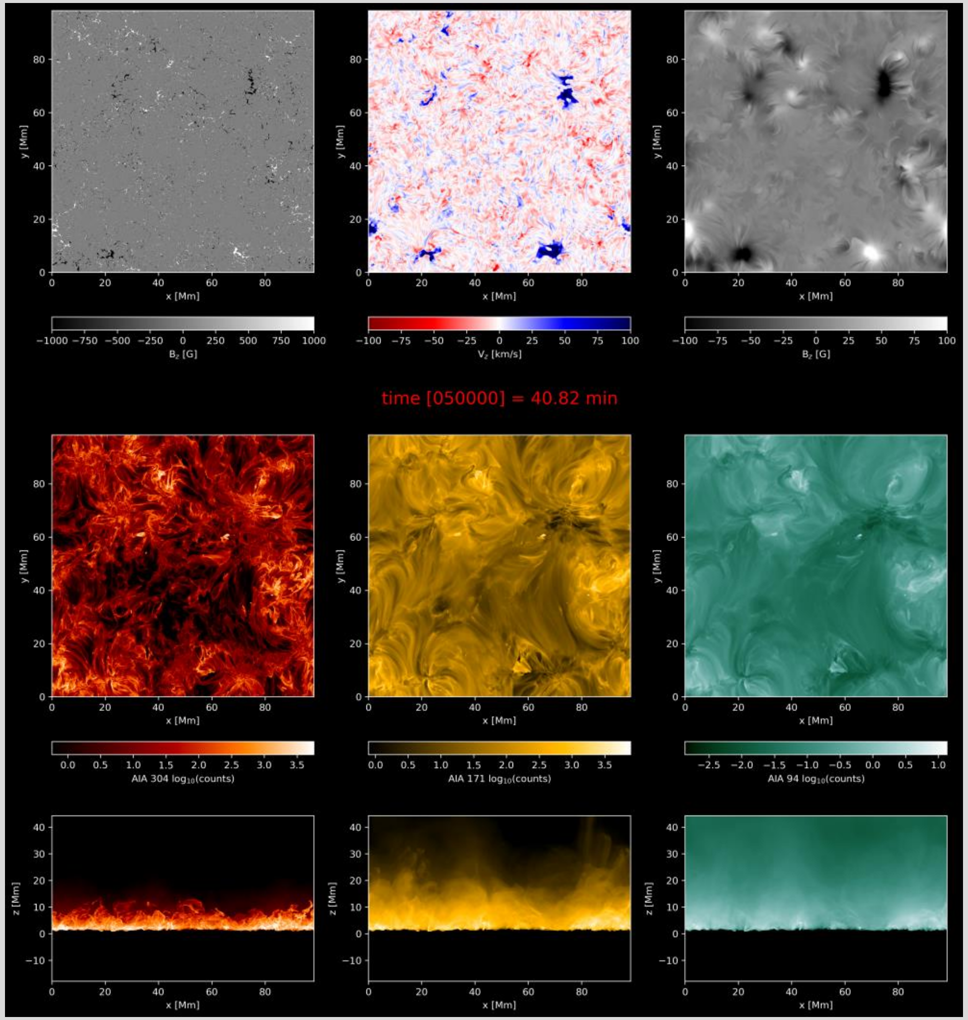


25x25x6.2 Mm

Larger scale organization and “voids”



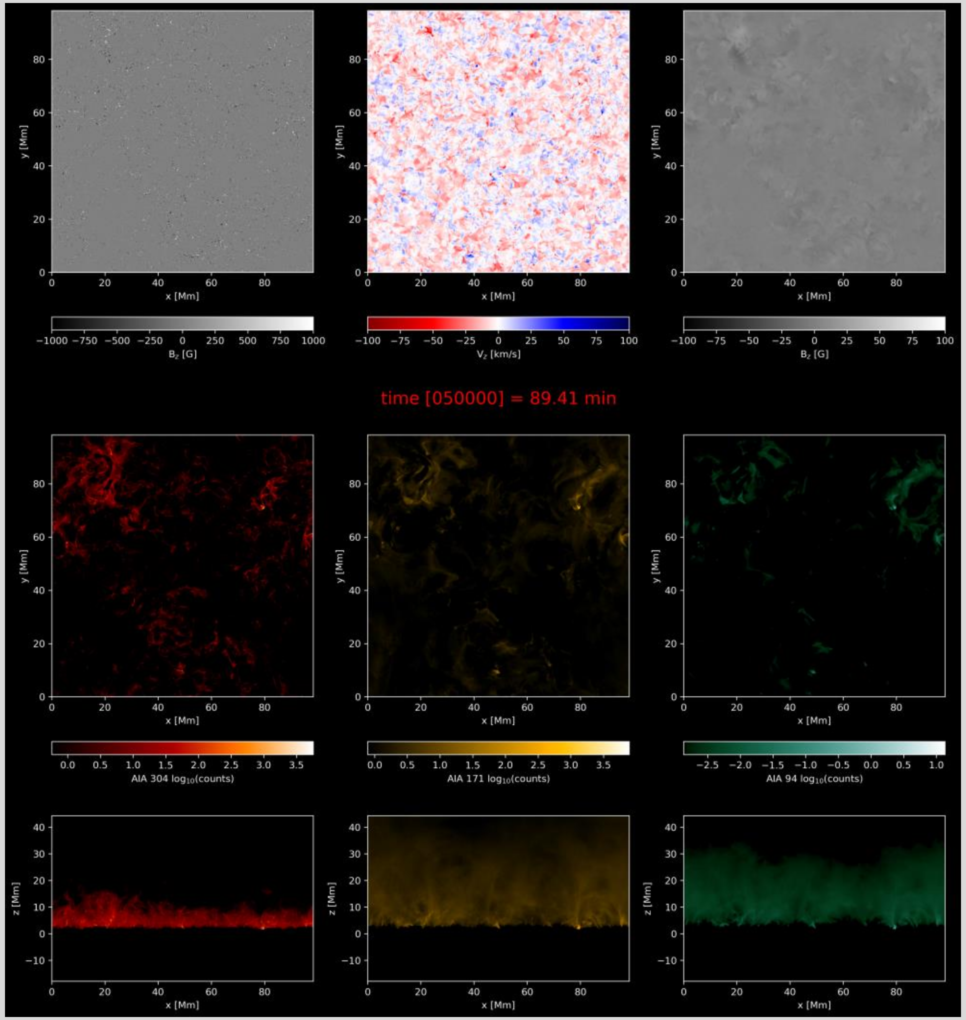
98x98x17.8 Mm



Corona with deep recirculation:

Total radiative loss ~ 6x10^5 erg/cm^2/s

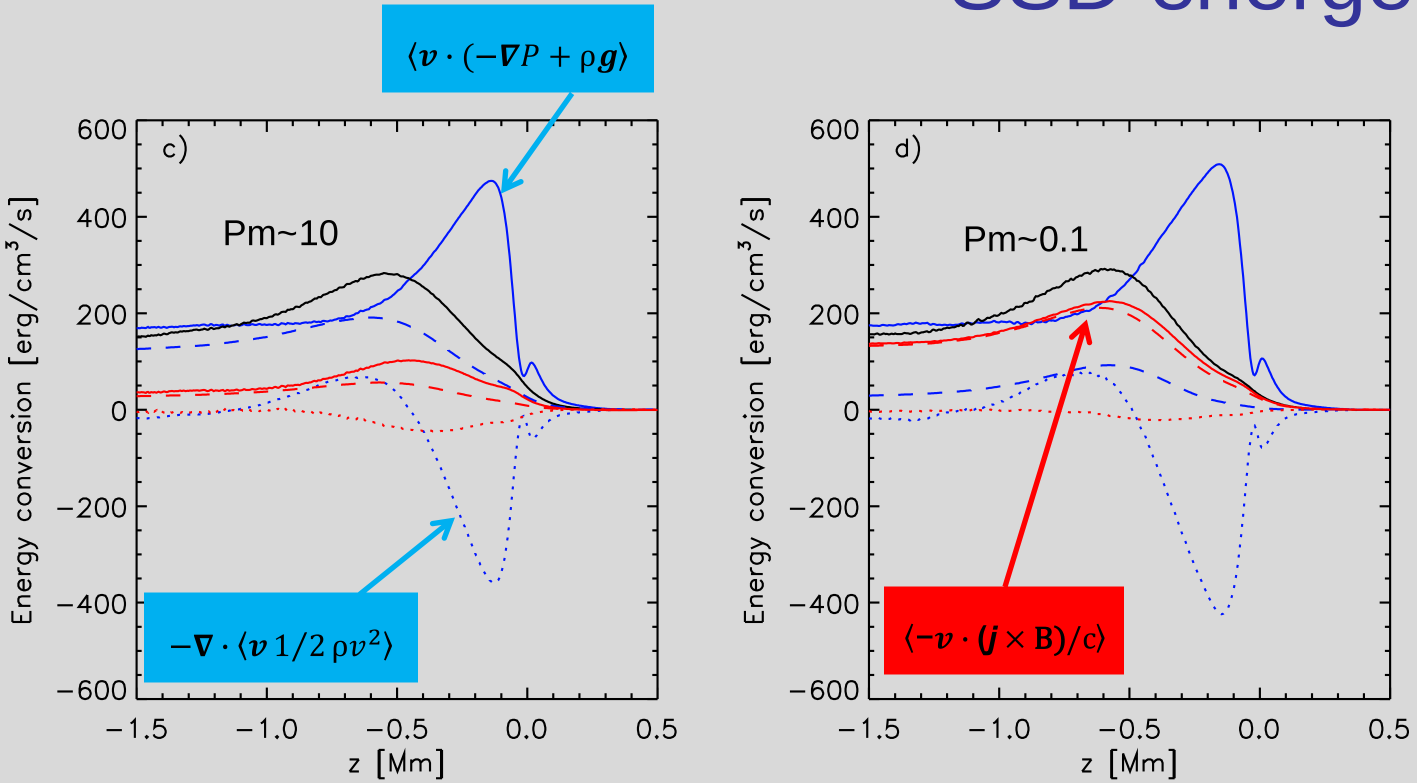
Withbroe & Noyes (1977) ~3x10^5 erg/cm^2/s

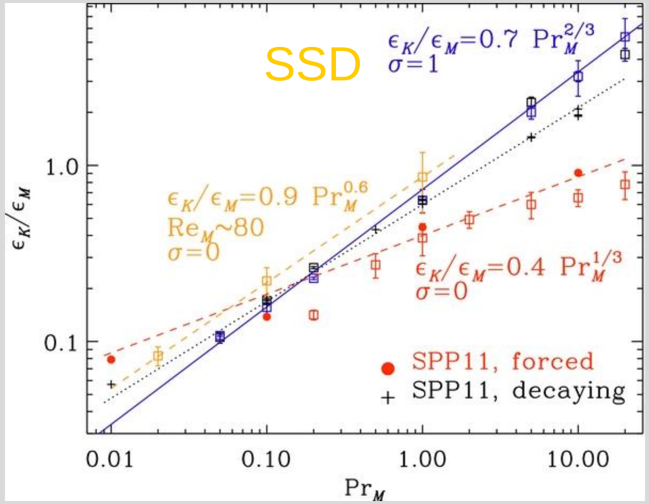


Corona without deep recirculation

Total radiative loss ~10^4 erg/cm^2/s

SSD energetics

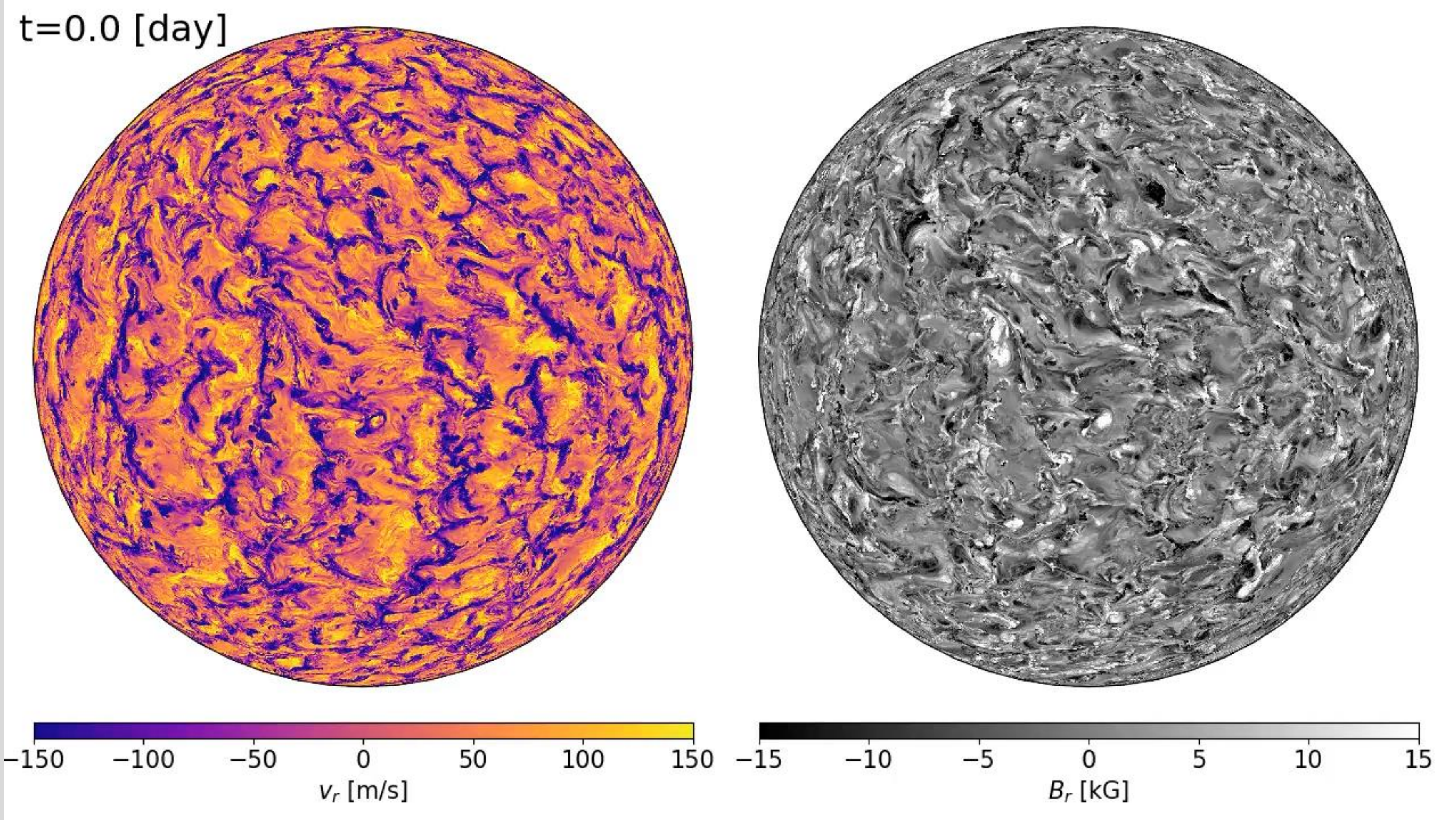




Brandenburg (2014)

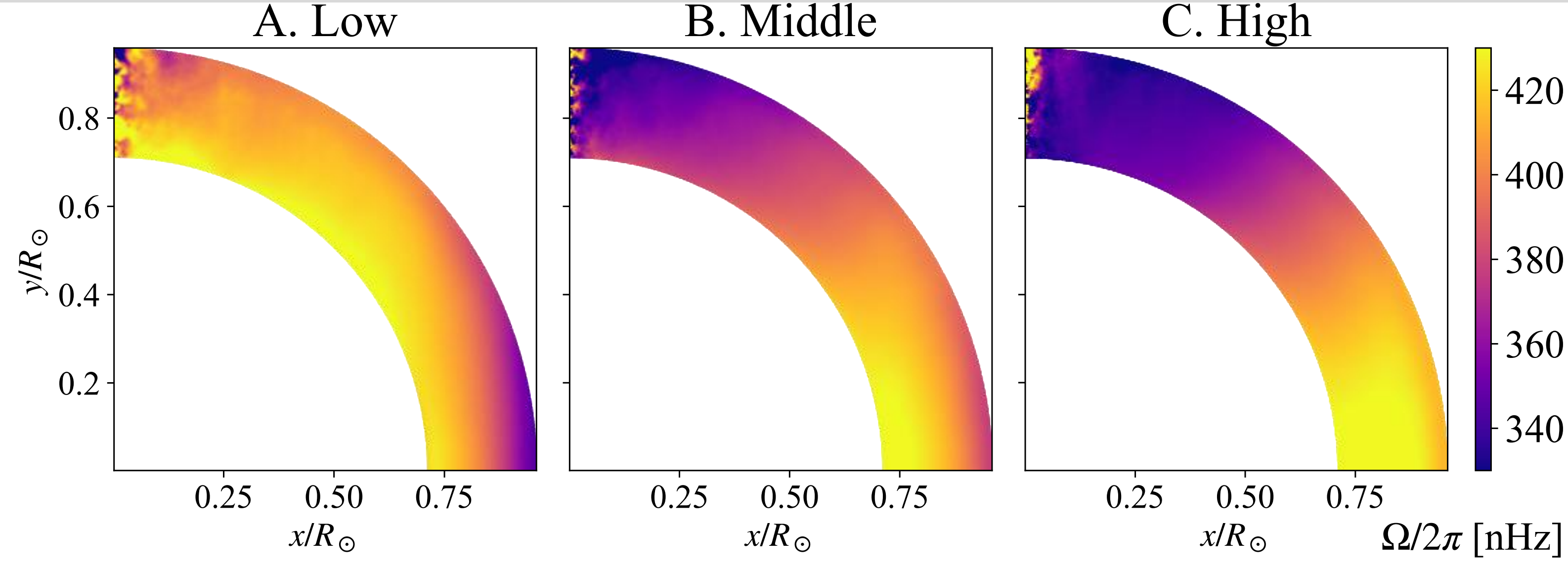
- About 150 erg/cm^3/s “convective driving” available in upper CZ/photosphere to drive dynamo
- Energy transfer to magnetic energy strongly Pm dependent (Brandenburg 2011, 2014, Brandenburg & Rempel 2019)
- Most efficient dynamos (in terms of energy conversion) found for low Pm regime
- **Uppermost 1.5 Mm of convection zone: About 0.3 L$_{\mathrm{Sun}}$ converted to B**
- **Total pressure/buoyancy driving in CZ ~ 3 L$_{\mathrm{Sun}}$**

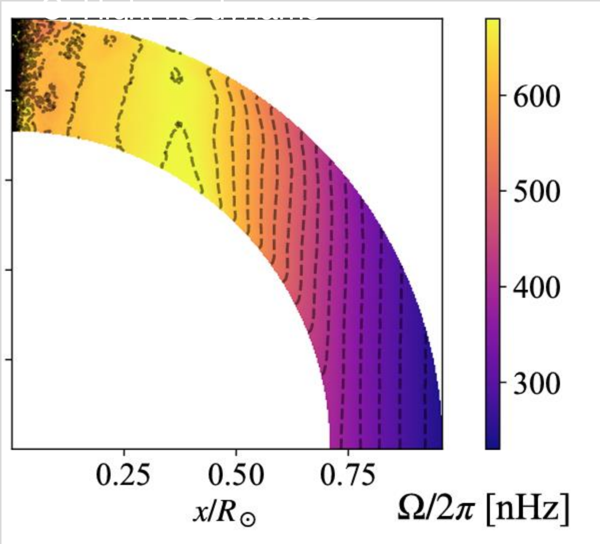
Differential rotation/convectice conundrum



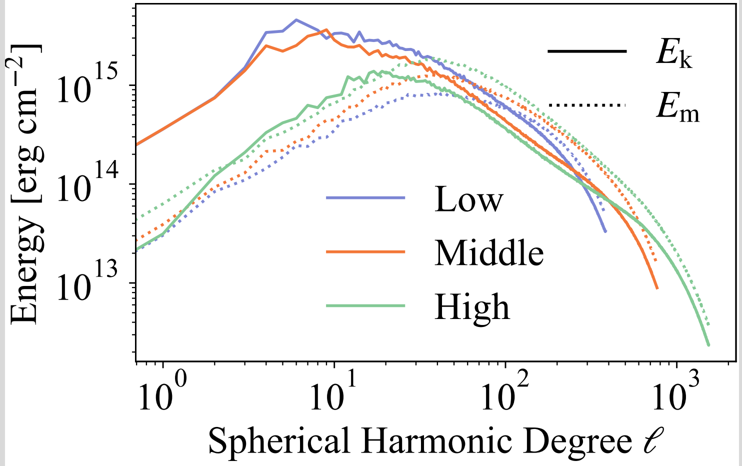
From Hotta & Kusano (2021)

Differential rotation/convectice conundrum



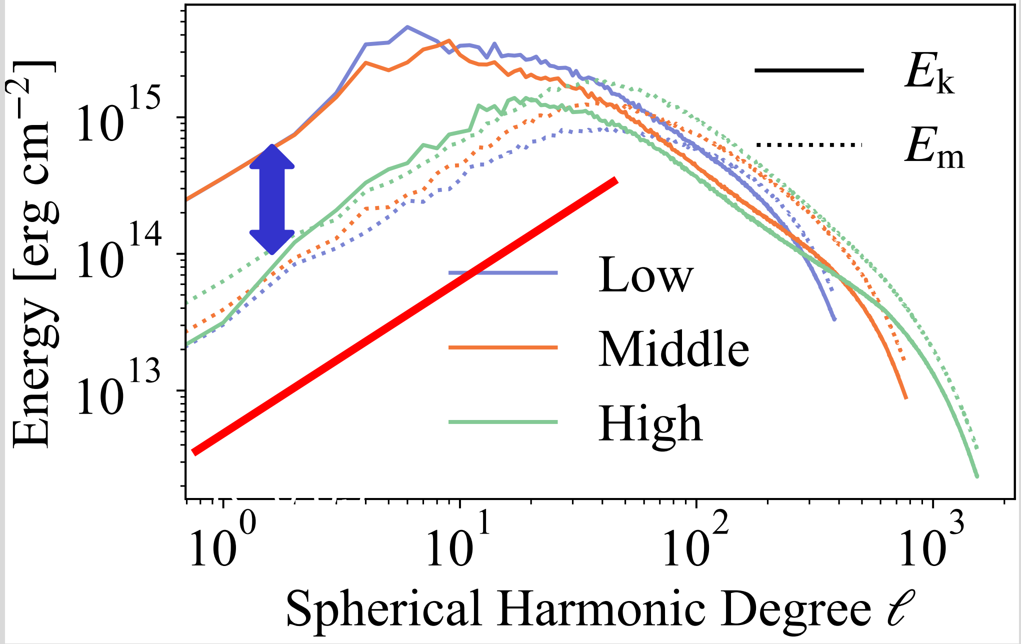


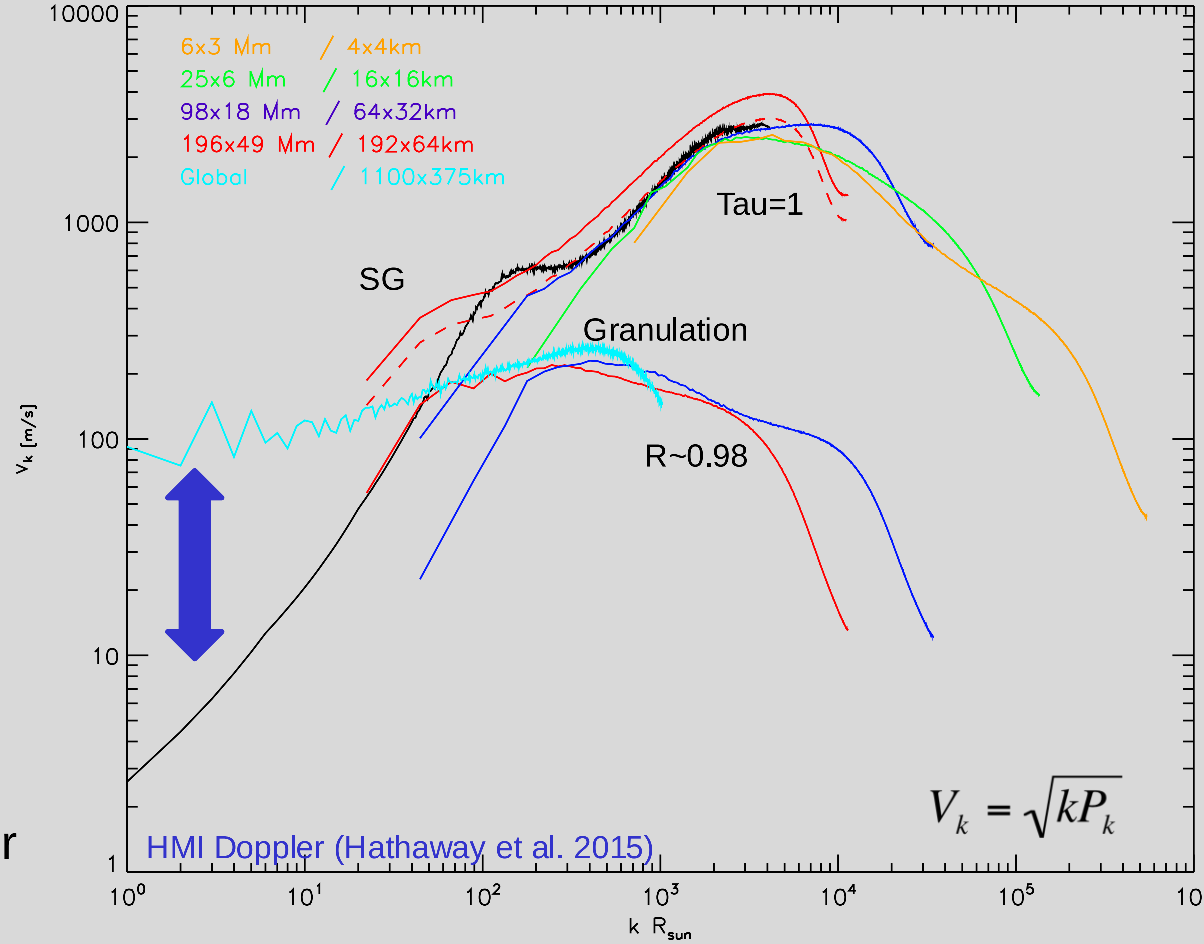
From Hotta & Kusano (2021, 2022)



- Flip from fast pole to fast equator for high resolution simulation ~384x3072x6144, happens only in presence of magnetic field
- Suppression of flows on large scales, peak of power shifts from l=6 to l=30
- Did not (yet?) produce a large-scale field, possibly due to total simulation time (9000 days)

Solar velocity spectrum at large scales ("Convective Conundrum")





Reduction of kinetic energy by an order of magnitude on large scales by SSD, need a factor of 100x to be consistent with observations.

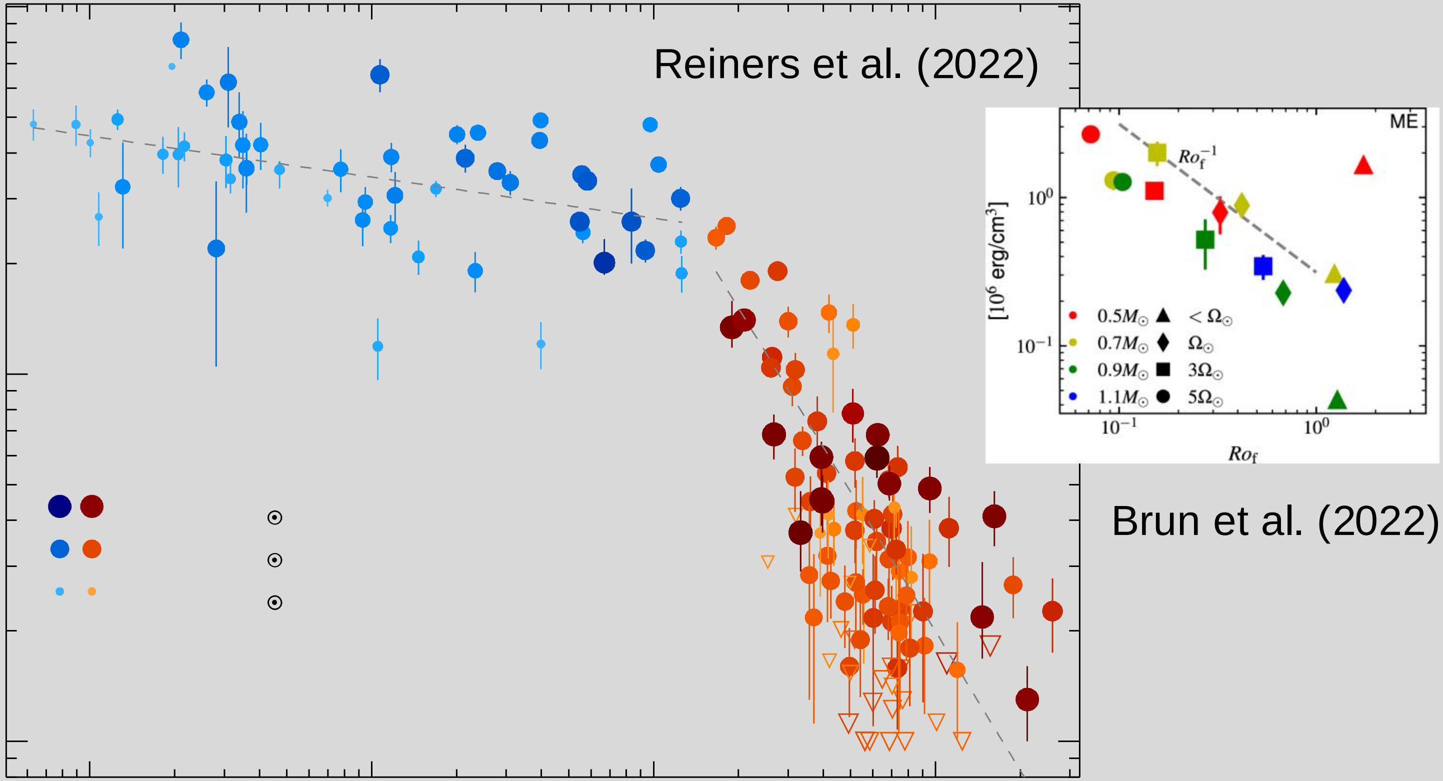
Understanding convection, angular momentum transport and large-scale dynamos may require capturing the SSD component

From Sun to stars

- We still have substantial uncertainty about the detailed processes of the solar dynamo
  - Sun is a single realization of a stellar dynamo – how typical is it?
  - We cannot easily extrapolate from Sun to other solar-like stars

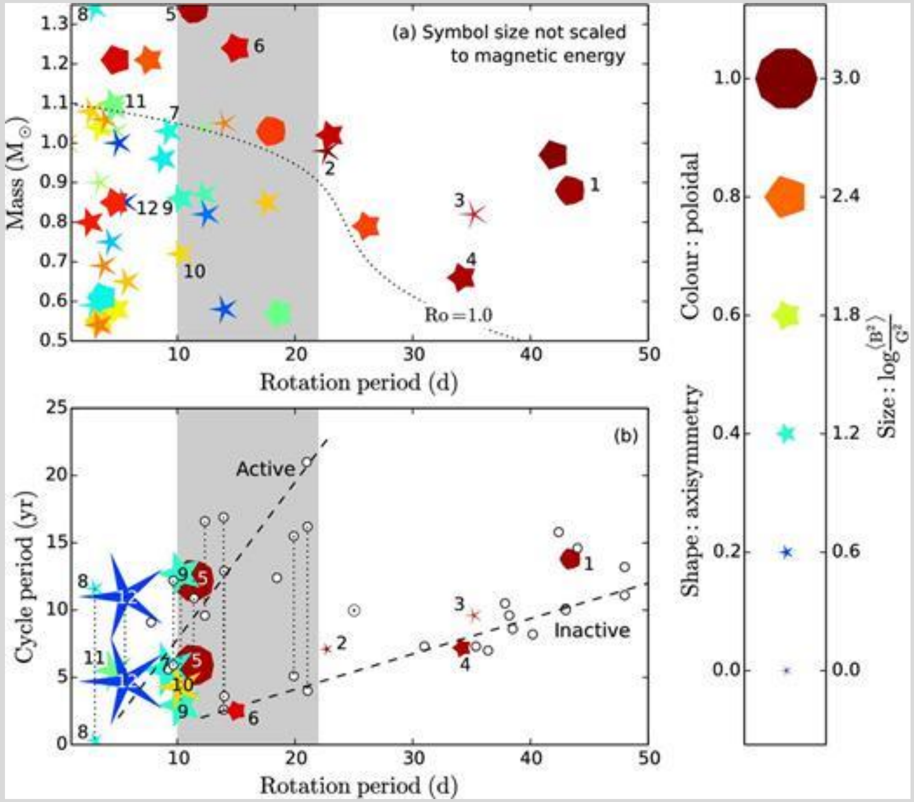
- Solar-like stars provide a large sample
  - Dependence on stellar structure (convection zone depth, transition to fully convective)
  - Dependence on rotation
    - Evolution of stellar rotation and dynamos
    - Young stars rotate fast, old stars slow

Rotation-activity relation



Reiners et al. (2022)

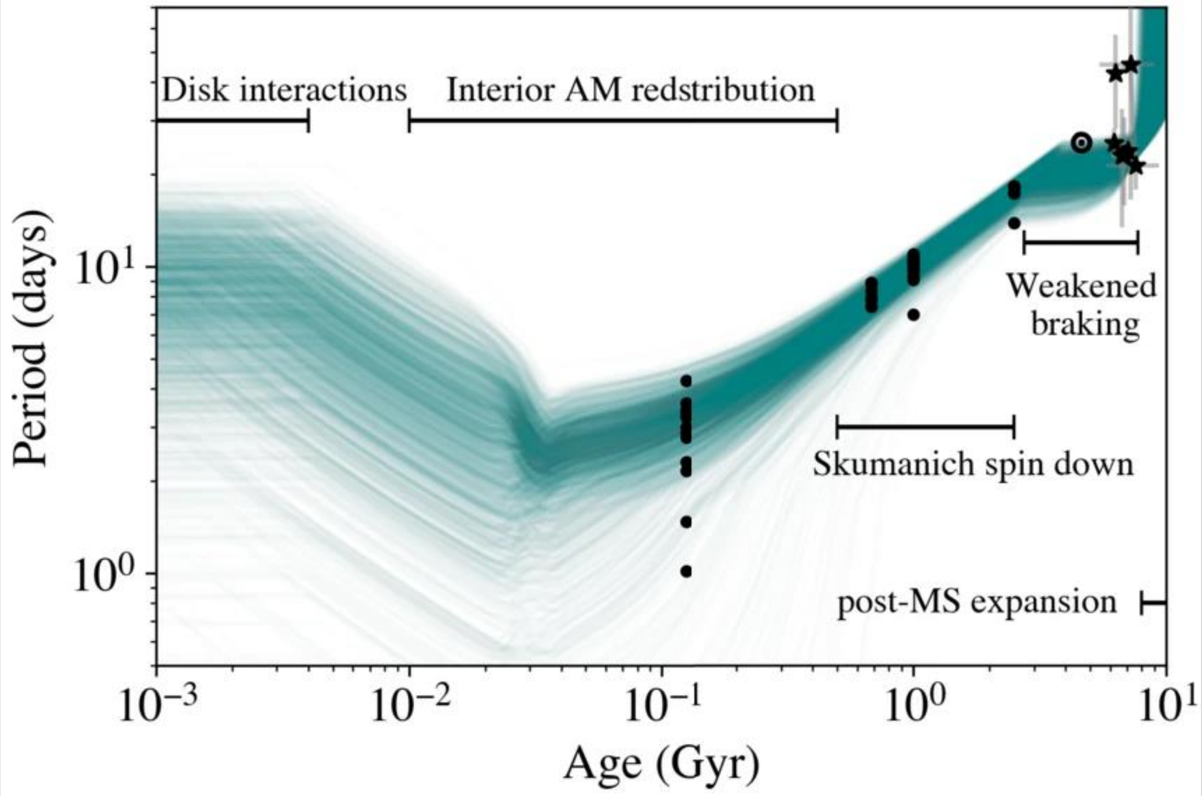
Brun et al. (2022)

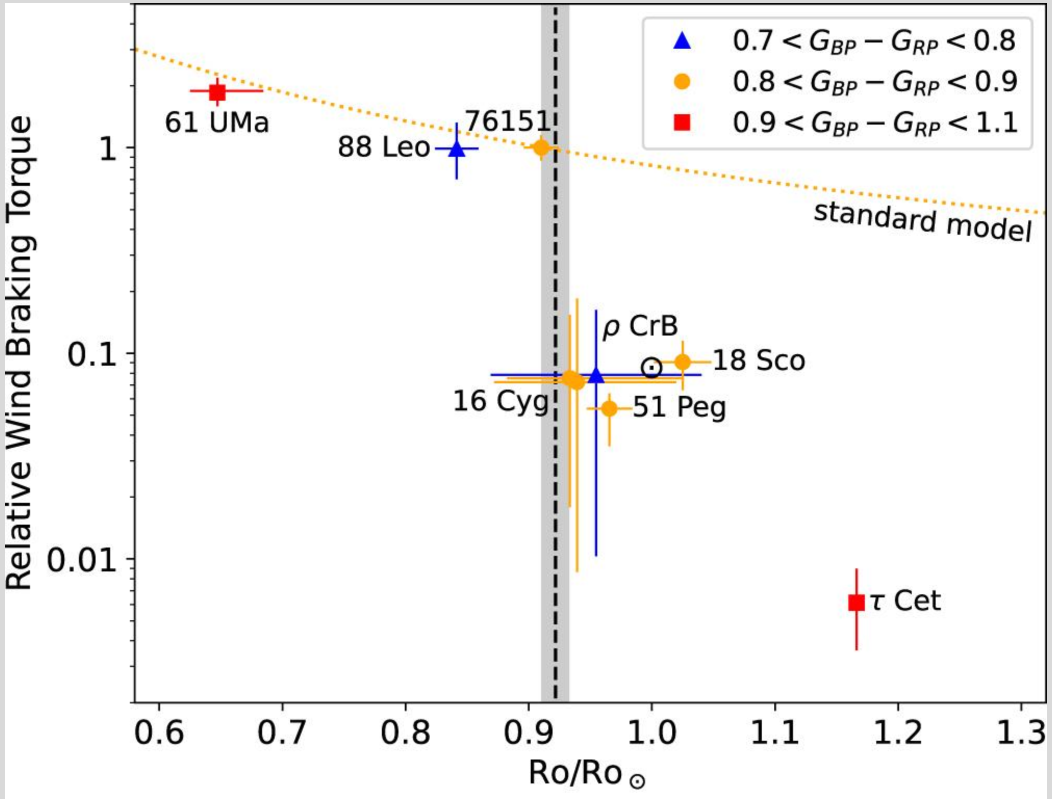


See et al. (2016)

- ➢ Dynamos are more efficient in maintaining large-scale fields for stronger rotation
  - – General trend reproduced in simulations
- ➢ Saturation regime for stars rotating about 10x faster than sun
- ➢ More complex magnetic field topology for faster rotators

Rotation evolution, breaking laws



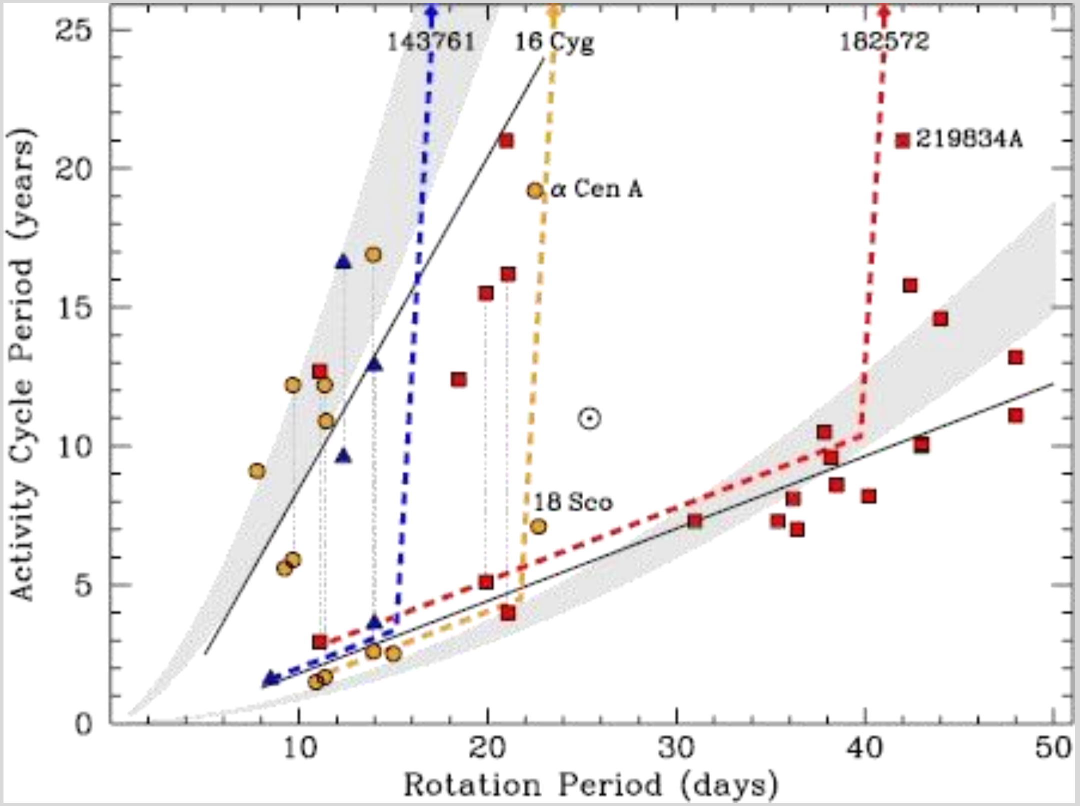


Isik et al. (2023)

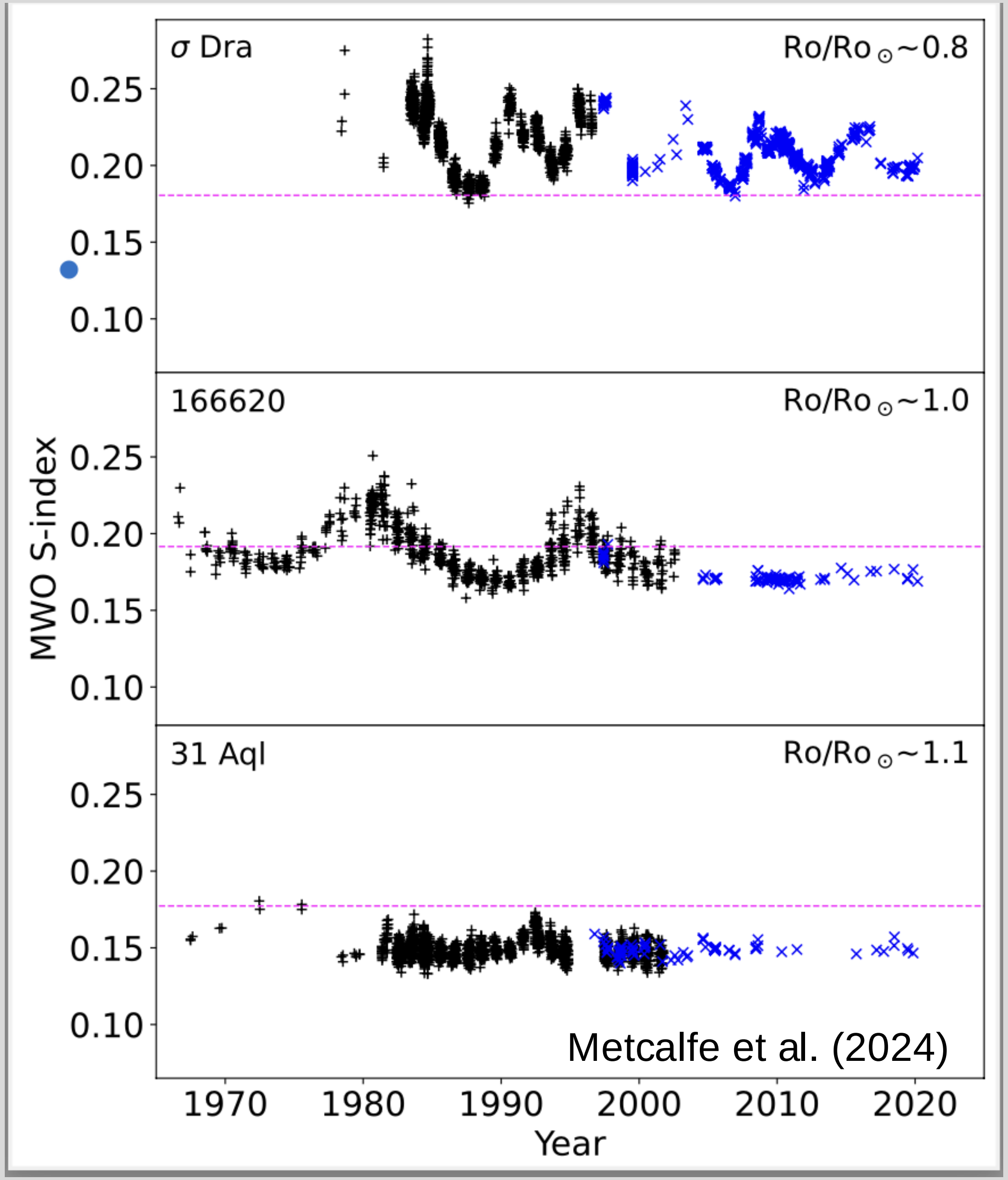
Metcalfe et al. (2024)

- Stars enter phase of weakened breaking when approaching Rossby numbers ~1
- Loss of strong large-scale field, transition towards small-scale field dominated regime
- Sun appears to be in this transition regime

Changes in cycle period



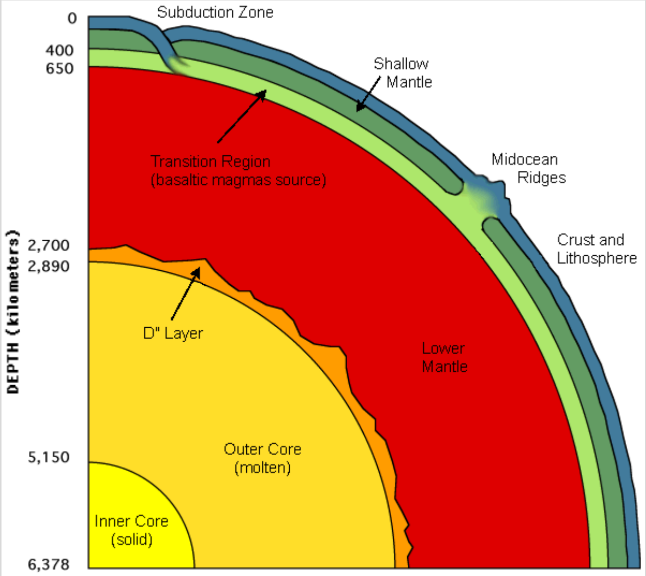
Metcalfe & van Saders (2017)

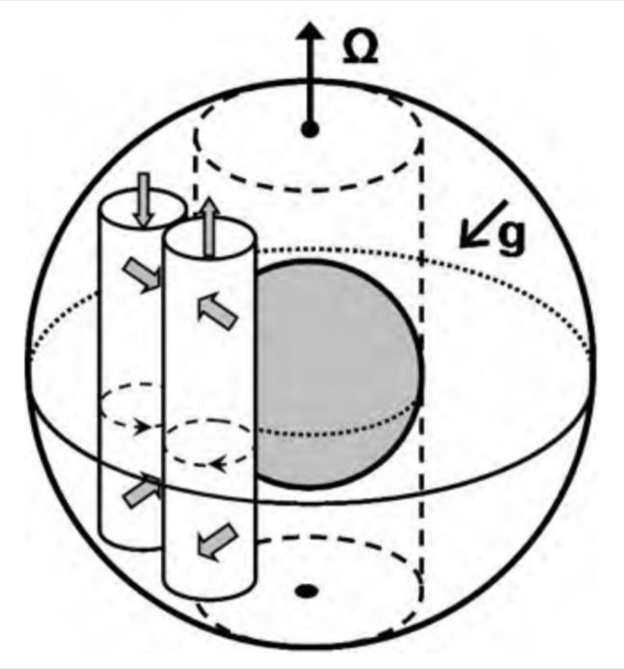


Metcalfe et al. (2024)

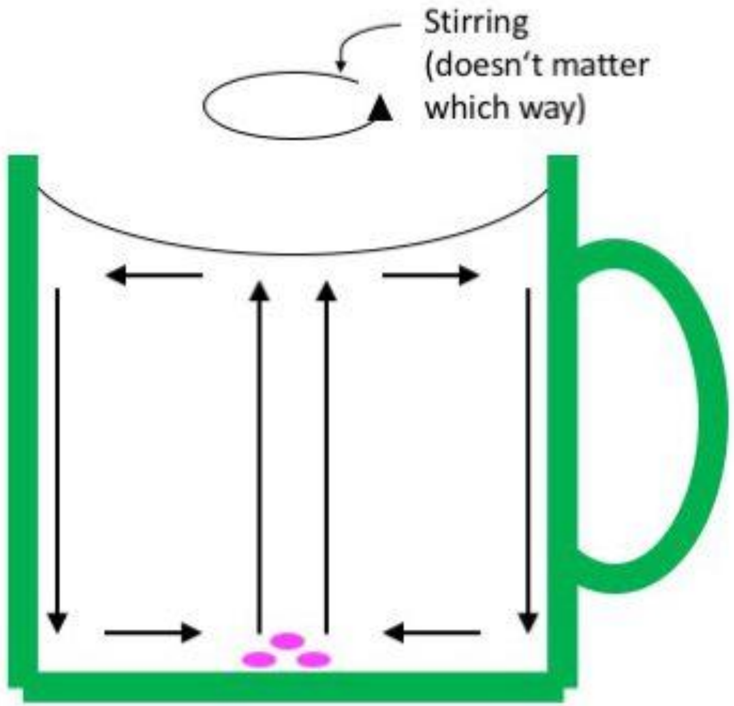
- Lengthening of cycle during transition
- Transition from cyclic to flat activity
  - Intermittent regime with indication of grand minima

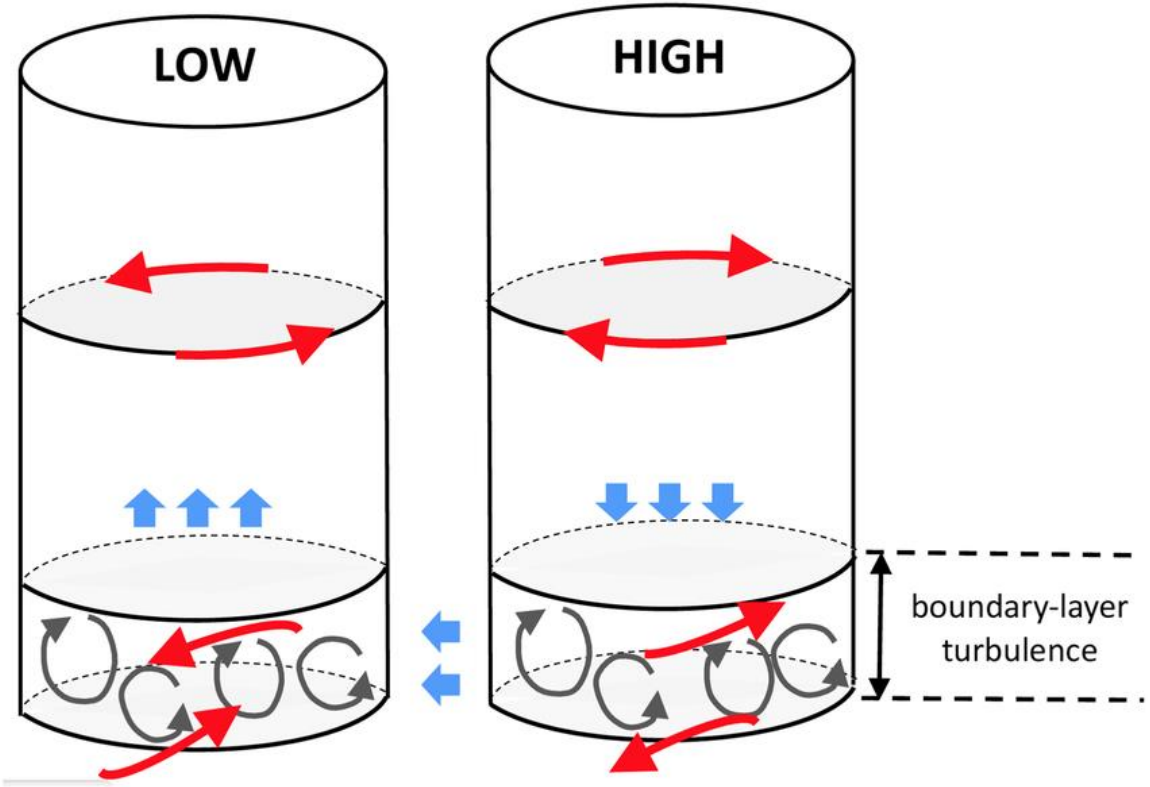
Geodynamo models





- Dynamo region
  - Outer earth core (liquid iron)
  - Compositional convection (phase transition to solid inner core leaves lighter elements behind)
- Dynamo parameter
  - Rm ~ 300 (can be captured in current 3D simulations)
  - Ro ~ 10^{-6} (strongly rotationally constrained)
- Helical flows due to Ekman pumping
  - Breakdown of geostrophic flow balance near core-mantle boundary (Ek ~ 10^{-10}, difficult to capture)
  - D-layer complicated structure, may evolve on time-scales of mantle convection (10-100 million years)
  - Columns of helical flows outside tangent cylinder
- Little differential rotation
  - α^2-dynamo
- Strong field regime
  - Balance between Lorentz and Coriolis force

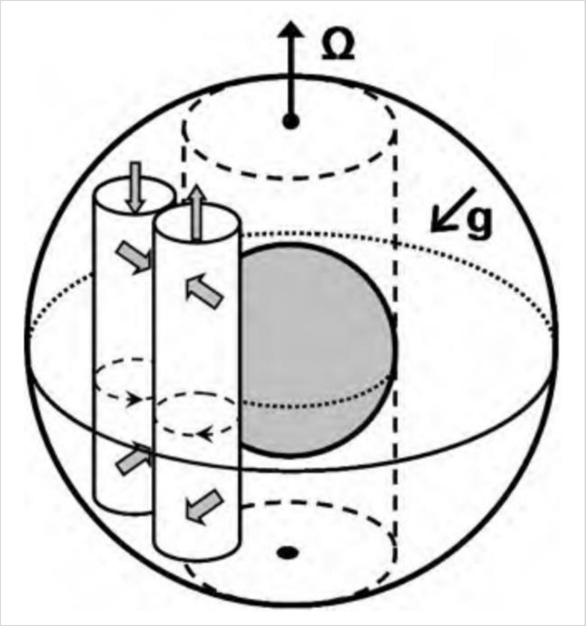


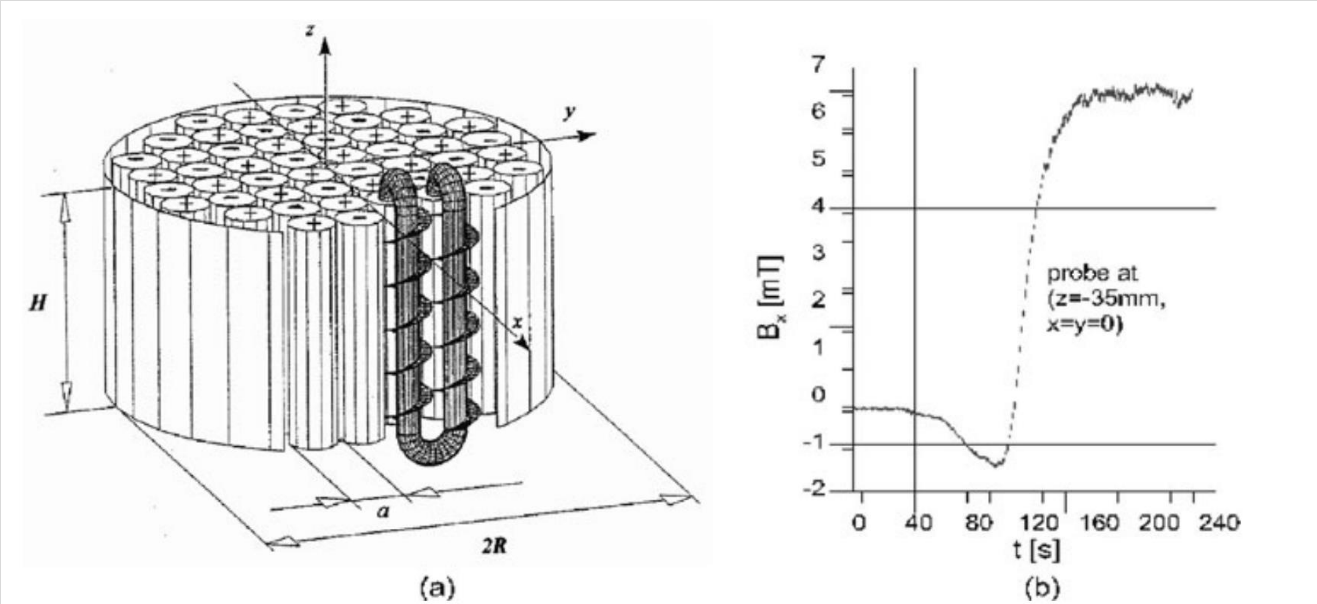


Ekman pumping

- Balance between Coriolis force and pressure force breaks down in boundary layer due to viscous stress
- This leads to axial flow along rotating columns

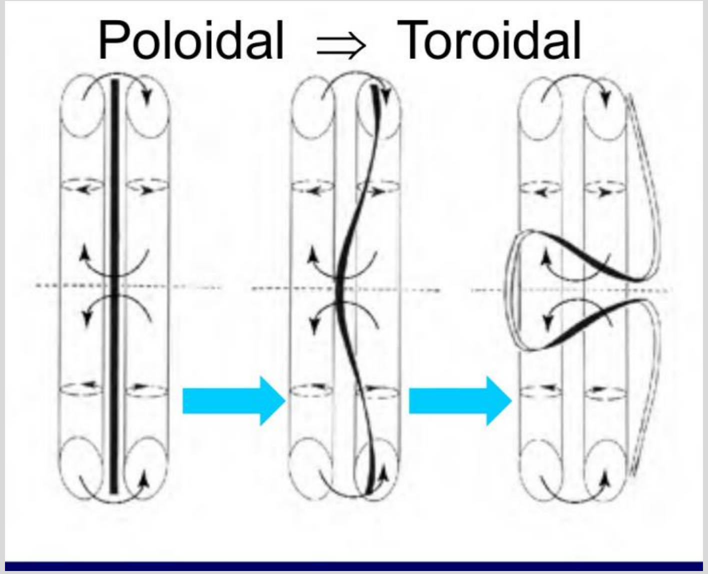
Liquid Sodium Experiments

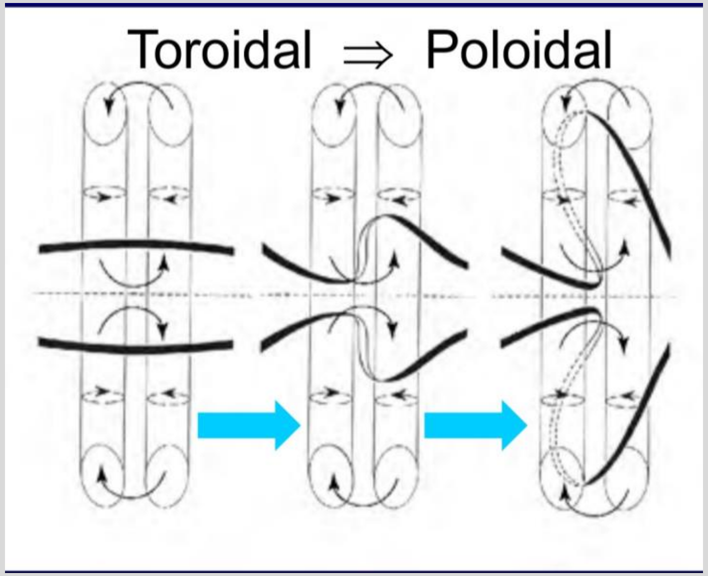




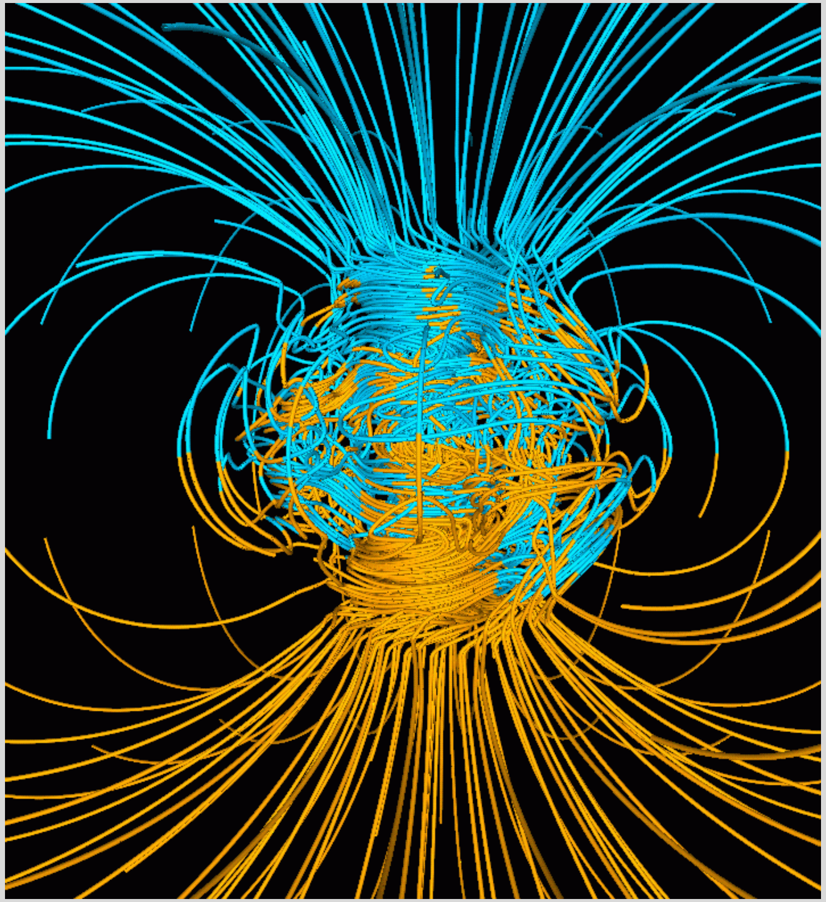
From Stefani (et al. 2008)

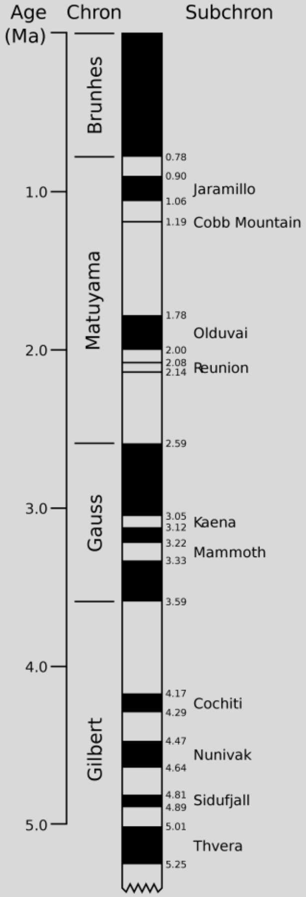
Array of helical columns with alternating up- and downflows filled with sodium (Karlsruhe dynamo experiment)

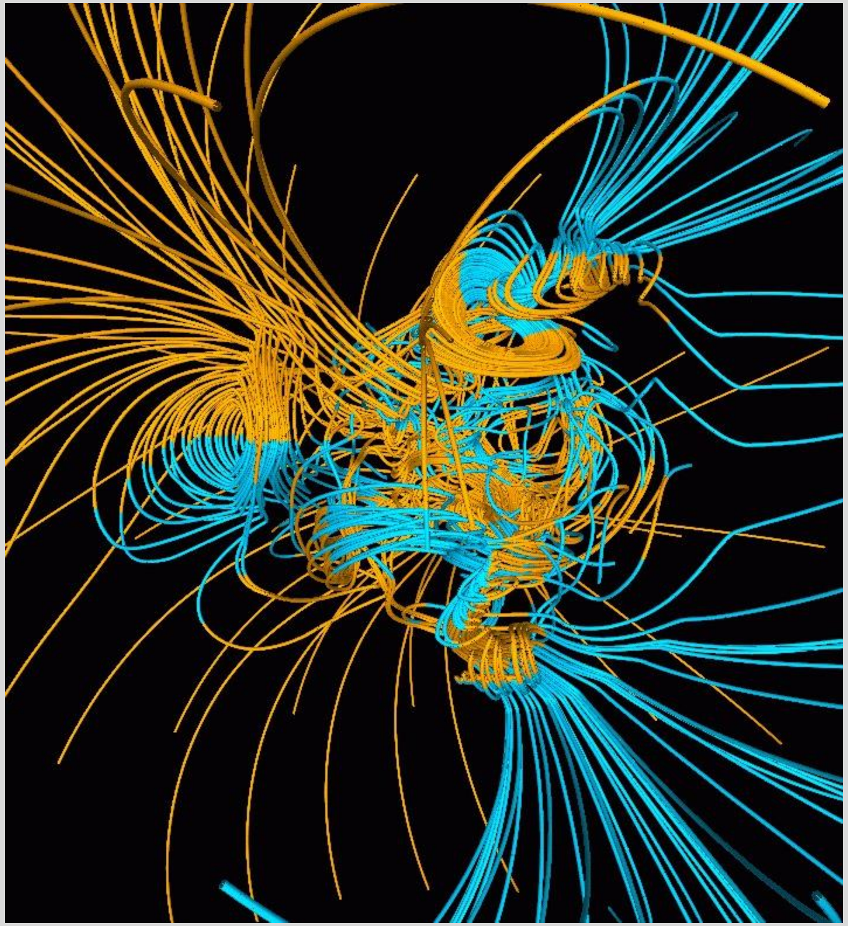




Geodynamo field reversals







Glatzmaier et al. (1995)

- Primarily dipolar field
- Multi-polar field during field reversals

Concluding remarks

- Fundamental understanding of dynamo processes through dynamo theory
- Specific applications to the Sun and stars have had limited success
  - Mean-field models can capture many aspects of solar cycle after careful “tuning” of degrees of freedom
  - Limited success with 3D dynamo solutions:
    - Found many examples of dynamos, but most do not look solar-like
    - Fundamental challenge in getting differential rotation correct, too large flow amplitudes on large scales
  - Sun appears to be close to 2 critical transitions that happen near Ro~1:
    - transition from solar to anti-solar DR
    - stellar dynamos become weak, reduced angular momentum loss
  - Observations strongly suggest a weakly supercritical Babcock-Leighton model for the Sun
- 3D geodynamo simulations have produced acceptable solutions 30 years ago
  - Some debate whether we get the “right” answer for the correct reason (cannot do Pm<<1, very low Ekman number)
  - Only modes up to l=13 are constrained by observations (higher modes are hidden by permanent magnetism in Earth crust)