Introductory discussion of selected activities from PRINCIPLES OF HELIOPHYSICS

by Karel Schrijver

based on Principles of heliophysics, V1.3, August 1, 2022

1
Instructions and notes

- Pre-school homework: For part 1 of this 'Discussions' session, read Section 2.2 (10 pages). For part 2, read Sections 5.5.5 and 5.5.6 (6 pages). Then, in groups, work through the Activities in this document (four from 'Principles ...', others new or variations to those in the book formulated for these introductory discussions).
- During the discussions: make sure you have access on your screen to 'Principles of Heliophysics V1.3'.
- Sections and equations from the book are shown as '#.#'; for this document either letters (for Activities) or roman numerals (for equations) are used.
- As in the book, all units are cgs (i.e., G, cm, K, etc.).

2

Approximations, scales, and models

Focus: Sun-heliosphere coupling, heliosphere-geospace coupling, scales, common physics, different parameters; some simplifications work unexpectedly well; and different formulations of the same equation give different perspectives, ...
Learning objectives: get a feel for system scales; introduction to a few fundamental concepts; distill a process to its essentials; question (often implicit) assumptions

2.1 Parker solar wind: basics

Activity 15 (or A for these discussions): **What powers the solar wind in the basic model discussed here? To see the answer, rewrite [Eq. (2.7) or] Eq. (2.10) to an energy equation** *(a version of Bernoulli's law) with the terms for the kinetic and potential energy in the Sun's gravitational field, plus a term that reflects the work done by the expanding gas both geometrically and by acceleration; the energy for that expansion in the isothermal approximation is provided by the efficient thermal conduction by the electron population (see Eq. (9.2) and footnote xxiii.). The real-world solar wind is not isothermal, certainly not far from the Sun (compare the coronal temperatures in Table 2.3 with near-Earth wind properties in Table 2.4), and moreover is provided some additional power (in the form of heating and pressure) by waves and turbulence.*

Groups 5, 7

Solution: Start with Eq. (2.10):

$$\frac{1}{v}\frac{\mathrm{d}v}{\mathrm{d}r}\left\{v^2-\frac{2kT}{m_\mathrm{p}}\right\}=\left\{\frac{4kT}{m_\mathrm{p}r}-\frac{GM_\odot}{r^2}\right\}. \tag{2.10}$$

With $e_\mathrm{kin}=\frac{1}{2}\rho v^2$, $e_\mathrm{grav}=-GM_\odot\rho/r$, and $p_\mathrm{gas}=2\rho kT/m_\mathrm{p}$, this can be rephrased as a balance of specific energies and work by isothermal expansion:

$$\frac{\mathrm{d}}{\mathrm{d}r}\left\{\frac{e_\mathrm{kin}}{\rho}+\frac{e_\mathrm{grav}}{\rho}\right\}=\frac{p_\mathrm{gas}}{\rho}\left\{\frac{1}{v}\frac{\mathrm{d}v}{\mathrm{d}r}+\frac{2}{r}\right\}=-\frac{p_\mathrm{gas}}{\rho}\left\{\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}r}\right\}, \tag{i}$$

where the rightmost term follows from the central expression using the time-independent continuity equation absent sources and sinks:

$$\rho\nabla\cdot\mathbf{v}=-(\mathbf{v}\cdot\nabla)\rho \qquad \text{(stationary 3.4)}$$

In words: the changes in kinetic and potential energy are compensated by the work done by the change in volume of the isothermal gas (expressed in the central expression in terms of radial expansion by the velocity gradient and lateral expansion because of the geometry). And that work/energy is supplied by what keeps the plasma (near-)isothermal: electron heat conduction from the coronal heat source.

Solar corona: $T\approx 1-3\,$MK; solar wind at Earth: $T_\mathrm{e}\approx 0.1\,$MK, $T_\mathrm{p}\approx 0.04-0.2\,$MK.

Problem: for an isothermal wind, the terminal velocity is essentially unbounded, and thus the required energy to power it is also unbounded. That issue at least goes away if we would allow the temperature to decrease with distance so that actual heat conduction can do its job properly (and satisfy the energy equation) at the expense of somewhat more complicated math. If you did that, then the energy available to power the solar wind would be set by $\kappa(T)\mathrm{d}T/\mathrm{d}r$ at the coronal base (with $\kappa(T)$ the Spitzer thermal conductivity).

N.B. Some people draw a comparison between the solar wind subject to gravity and a *de Laval* nozzle absent gravity. The continuity and momentum equations are:

$$\frac{\partial \rho}{\partial t} + (\vec{v} \cdot \vec{\nabla})\rho = -\rho\vec{\nabla} \cdot \vec{v} + (S - L), \tag{3.4}$$

$$\rho\frac{\partial \vec{v}}{\partial t} + \rho(\vec{v} \cdot \vec{\nabla})\vec{v} = +\rho\vec{g} - \vec{\nabla}p + \frac{1}{4\pi}(\boldsymbol{\nabla}\times\vec{B}) \times \vec{B} + \vec{\nabla} \cdot \vec{\tau} - \vec{v}(S - L) + (\mathbf{S}_{\rm p} - \mathbf{L}_{\rm p}). \tag{3.5}$$

For a stationary, one-dimensional, isothermal flow through a geometry with cross section $A(r)$, and without magnetic field, gravity, viscosity, or sources or sinks for particles these simplify to:

$$\frac{\mathrm{d}}{\mathrm{d}r}(\rho v A) = 0 \ ; \ \ \rho v \frac{\mathrm{d}v}{\mathrm{d}r} = -\frac{2kT}{m_{\rm p}}\frac{\mathrm{d}\rho}{\mathrm{d}r}, \tag{ii}$$

which can be combined to read

$$\frac{1}{v}\frac{\mathrm{d}v}{\mathrm{d}r}\left\{v^2 - \frac{2kT}{m_{\rm p}}\right\} = \frac{2kT}{m_{\rm p}}\frac{1}{A}\frac{\mathrm{d}A}{\mathrm{d}r}. \tag{iii}$$

Comparison with Eq. (2.10) suggests:

$$\frac{2kT}{m_{\rm p}}\frac{1}{A}\frac{\mathrm{d}A}{\mathrm{d}r} = \left\{\frac{4kT}{m_{\rm p}r} - \frac{GM_\odot}{r^2}\right\}, \tag{iv}$$

which has a solution

$$A(r) \propto r^2 \exp\left(-\frac{e_{\rm grav}}{e_{\rm gas}}\right), \tag{v}$$

which has a local minimum where $-2e_{\rm grav} = e_{\rm gas}$ below which gravitational potential energy acts like a constricting nozzle and beyond which the 'nozzle' eventually 'expands' as r^2.

Insights: (1) The solar wind is enabled by the high coronal temperature (electron conduction and high conductive flux - see Eq. (9.2) and Note xxiii: high mean-free path in Coulomb collisions), which supplies energy to beyond the point where the thermal energy equals the (unsigned) gravitational potential energy (= kinetic energy of the escape velocity); (2) there is a hidden energy transport in the assumption that the gas is isothermal, which violates the equation for internal energy (Eq. 3.6) because you cannot have conductive energy transport without a thermal gradient; (3) a different formulation of the same equation(s) provides different insights, sometimes more helpful than others.

2.2 Collisions and collective behavior

Activity B: **Estimate the mean-free path for collisions between electrons in the fast and slow solar wind near Earth** *based on Tables 2.4 and 2.5. Despite these large numbers, the use of pressure and temperature as defined from Maxwellian velocity distributions is useful in the heliosphere. Discuss how this may come about.*

Groups 3, 1

Answer: The 'mean' or 'characteristic' mean-free path length (in cm) for heliospheric electron-electron interactions (because we are talking about electron thermal conduction) as derived from quantities in Table 2.5:

$$\overline{\lambda}_{\rm mfp,e} \approx \frac{v_{T\rm e}}{\nu_{\rm ee}} \sim 1.1\,10^4 \frac{T_{\rm e}^2}{n_{\rm e}}. \qquad \text{(cf. 3.49)}$$

Near Earth, with Table 2.4, for slow and fast wind:

$$\overline{\lambda}_{\rm mfp,e} \approx 2\,10^{13} - 4\,10^{13}\,{\rm cm\,or}\,1.3 - 2.6\,{\rm AU}.$$

Push or pull? Push and pull?

The critical point in the isothermal Parker solution at 10^6 K lies at $\approx 6R_\odot$. That is roughly where the density scale height equals the electron mean-free path, i.e. the beginning of the largely collisionless exosphere (so roughly at the 'exobase' of the actual solar wind).

An (extreme) alternative to the (equally extreme) hydrodynamic (or fluid) approximation is the *collisionless approximation*. In the latter, an electrostatic potential builds up between the low-mass electron population and the high-mass ion population. One way to think about the resulting wind is this: The fastest electrons (which, by the way, are least affected by collisions) can overcome the potential barrier, but they cannot flow out in bulk without taking the ions (and lower-energy electrons) lest they increase the electric potential as they would leave a charged Sun behind. So any bulk electron flux must be balanced by an ion flux (which has to deal with its much higher gravitational potential that is partially countered by the electric potential), which happens when the electrostatic field that builds up sufficiently counters the gravity on the ions to pull them along.

A very rough approximation (see the reference below) and the assumption of Maxwellian tails (maintained by the hot electron reservoir in the corona below the 'exosphere') shows that the electric potential energy at the exobase would be roughly double the enthalpy of a Maxwellian electron-proton plasma: $\sim 5kT$ vs. $\sim 5kT/2$.

You can expect that the resulting solar wind (mass flux and speed) is determined by the high-energy tail of the velocity distribution ... which is not likely to be that of a true Maxwellian precisely because the fastest particles have an even lower cross section for Coulomb collisions and thus interact even less: plasmas are notorious for having non-thermal high-energy tails.

The reality of the solar wind is neither fully hydrodynamic nor fully collisionless, that of course also includes a magnetic field and its perturbations. But the net behavior – see 'Parker's lesson' – is much the same because that is governed by unavoidable conservation laws.

For a very readable introduction of the collisionless 'exospheric' model of the solar wind, see Nicole Meyer-Vernet (Eur. J. Phys. 20 (1999) 167).

Insight: closer to the Sun, $\lambda_{\rm mfp,e}$ is much smaller (because of the higher density) and Maxwellian distributions are a fair approximation; further out, you need to consider collective electromagnetic interactions and magnetic perturbations (gradients, including waves: turbulence) that can help scatter particles and maintain fair validity of the concepts of temperature and pressure as in thermodynamics.

N.B. Parker (1960) took the wind density at Earth to be $100\,\mathrm{cm}^{-3}$ (interestingly much lower than in his 1958 paper where he says that "Biermann infers densities at the orbit of earth ranging from 500 hydrogen atoms/cm^3 on magnetically quiet days [to much higher during storms]"). In his 1960 paper, he simply noted that "the mean-free path for interparticle collisions is small compared with the dimensions of the flow" and used standard hydrodynamic equations, assuming collective behavior and Maxwellian statistics. Also, at the time, he could assume "that the extension of solar gas into interplanetary space comes from the entire corona. Hence the observations altogether suggest that the whole corona flows hydrodynamically outward into space".

Note that in his reminiscences of 2014 (Res. Astron. Astrophys. 14 1), he writes: "Hardly anyone believed the trans-sonic expansion of the solar corona. So I had the field to myself for about four years, elaborating the analytic theory of the expanding corona, producing two hydrodynamic models of the heliosphere depending on the existence or absence of an interstellar wind."

One of Parker's lessons: "To begin, then, it is widely believed that the large-scale bulk motion within a body of collisionless gas or plasma is not described by the Newtonian equations of hydrodynamics ... But whether interparticle collisions happen or not, the bulk flow conserves particles, momentum, and energy, and when those three conservation conditions are written down, they provide the equations of hydrodynamics, with the familiar gradient of [pressure], compressibility, etc. Most textbooks derive these hydrodynamic equations by computing the zero, first, and second velocity moments of the collisionless Boltzmann equation, but the simple idea of flux conservation of particles, momentum, and energy can be used directly (Parker 2007)."

2.3 Gas and field

Activity C: **Take the momentum equation of Eq. (3.5) – for a stationary state of the solar wind, ignoring viscosity, sources, and sinks – to show the usefulness of the plasma** β *(Eq. 3.24, and Table 2.5). Estimate* β *for the solar wind (a) near Earth for the slow and fast solar wind as in Table 2.4 (use the electron temperatures) and (b) around* $10R_\odot$ *(roughly the closest approach of the Solar Orbiter; use a temperature of 1 MK there). For (b), assume a radial field and a radial flow (which is not too bad within Earth's orbit for these rough estimates, see Section 5.4). Note that the simplest Parker solar wind model as discussed in Section 2.2 ignores the magnetic field; where is that appropriate in the Parker model? Use the same momentum equation as above to show that the ratio of dynamic (or ram) pressure to magnetic pressure also arises naturally (to which we return for the magnetopause standoff distance).*

Groups 4, 6

Answer: For a stationary state and ignoring viscosity and plasma sources/sinks:

$$\rho(\mathbf{v}\cdot\nabla)\mathbf{v} = +\rho\mathbf{g} - \nabla p + \frac{1}{4\pi}(\nabla\times\mathbf{B})\times\mathbf{B} + \ldots \qquad \text{(modified 3.5)}$$

which for orders of magnitude reads like:

$$\mathcal{O}(\frac{1}{2}\rho v^2/\ell) = \ldots - \mathcal{O}(p/\ell) + \mathcal{O}(B^2/8\pi\ell) + \ldots \qquad \text{(vi)}$$

Here, β naturally arises from a comparison of the final two terms on the right:

$$\beta_{\rm gas} = \frac{p/\ell}{B^2/8\pi\ell} = 3.5\,10^{-15}\frac{nT}{B^2}, \qquad \text{(cf. 3.24)}$$

(where n is the total number of particles, i.e., ions plus electrons) which (for a fully-ionized hydrogen plasma) yields $\beta_{\rm gas} \approx 2.5 - 0.6$ near Earth and (with $n \approx (4600 - 1400)\,\text{cm}^{-3}$ and $B \approx 0.03\,\text{G}$), $\beta_{\rm gas} \approx 0.04 - 0.01$ at $10R_\odot$. That might suggest that the magnetic field is not important near Earth orbit but not quite ignorable close to the Sun. However, ...

Alternatively, a plasma β can be based on the dynamic (or ram) pressure of the flow, i.e. in a comparison of the term on the left with the final term on the right in Eq. (3.5):

$$\beta_{\rm ram} = 1.1\,10^{-23}\frac{nv^2}{B^2}, \qquad \text{(vii)}$$

Using the same velocities (ignoring acceleration here), this yields $\beta_{\rm ram} \approx 6. - 20.$ near Earth and $\beta_{\rm ram} \approx 0.1$ at $10R_\odot$, leading to the same conclusion as above. However, ...

In Parker's initial approximation (1958, ApJ 28, 664), the flow is strictly radial, and he ignores rotation. Thus $\nabla\times\mathbf{B} \equiv 0$, so there is no effect of the magnetic field: the approximation is internally consistent. In reality, the field is not ignorable, hence models such as that discussed in Section 5.4 (in which a wind can act as a magnetic brake).

The plasma $\beta_{\rm ram}$ based on the ram pressure is commonly used to assess the balance between, for example, the solar wind and a planetary magnetic field (see Sect. 5.5.5, and Activity J).

Insight: dimensionless numbers provide insight into the relative importance of terms, but note: you have to assess them *at a given scale*, so be careful with them in, e.g. turbulent spectra, such as in solar convection where a term may seem ignorable at one scale but is important nonetheless through scale couplings, and also where small scales enable large-scale evolution, as in reconnection.

2.4 Solar-surface to heliospheric field

Activity 63 (corrected) (or D for the discussion session): The solar wind stretches the high-coronal magnetic field into the heliosphere into a roughly radial field below the Alfven radius. This enables an analogy with electrostatics: the field of electric charges placed above a at perfect conductor can be computed by placing mirror charges opposite to the conducting surface, which then naturally has the electric field perfectly normal to the conducting surface. Analogously, in a magneto-static consideration above the spherical Sun (of unit radius) called the 'source surface model,' the magnetic field can be approximated by placing mirror 'charges' on a sphere at distance $d_{\rm SS}^2$ which then has the field perfectly radial at $d_{\rm SS}$. Empirically, $d_{\rm SS} \approx 2.5$ (where that 'source surface' is taken as the foundation of the heliospheric field; the virtual surface with mirror charges used to compute the potential field below $d_{\rm SS}$ is then at $d_{\rm SS}^2$). This model (introduced by Schatten et al. (1969)) works remarkably well below $d_{\rm SS}$ on large scales. The heliospheric field is approximated by a continuation from that source surface, subject to the Parker spiral.

(1) For illustration, **simplify the source-surface model by a 2-d sketch** *involving charges (of alternating polarity: N-S-N-S-N-S) placed on a straight line and another line parallel to it involving mirror charges of opposite polarity. Sketch the equivalent of the foundation of the heliospheric current sheet and examples of 'closed' field lines (the equivalent of coronal loops closing back onto the solar surface) and 'open' field lines (the equivalent of field stretched out into the heliosphere), at the base of which we find dark 'coronal holes' in X-ray images of the Sun. Consider how the X-points that form on the midpoints between these lines are like the 'helmet streamers' seen in coronagraphs and during eclipses.*

(2) For a sphere, **show that the mirror charges on the surface at $d_{\rm SS}^2$ are $-d_{\rm SS}$ times the strength of those on the solar surface.**

Groups 2 (part 1), 8 (part 2)

On part (1): What does the streamer belt look like in source-surface models for the Sun? See Fig. 2.2.

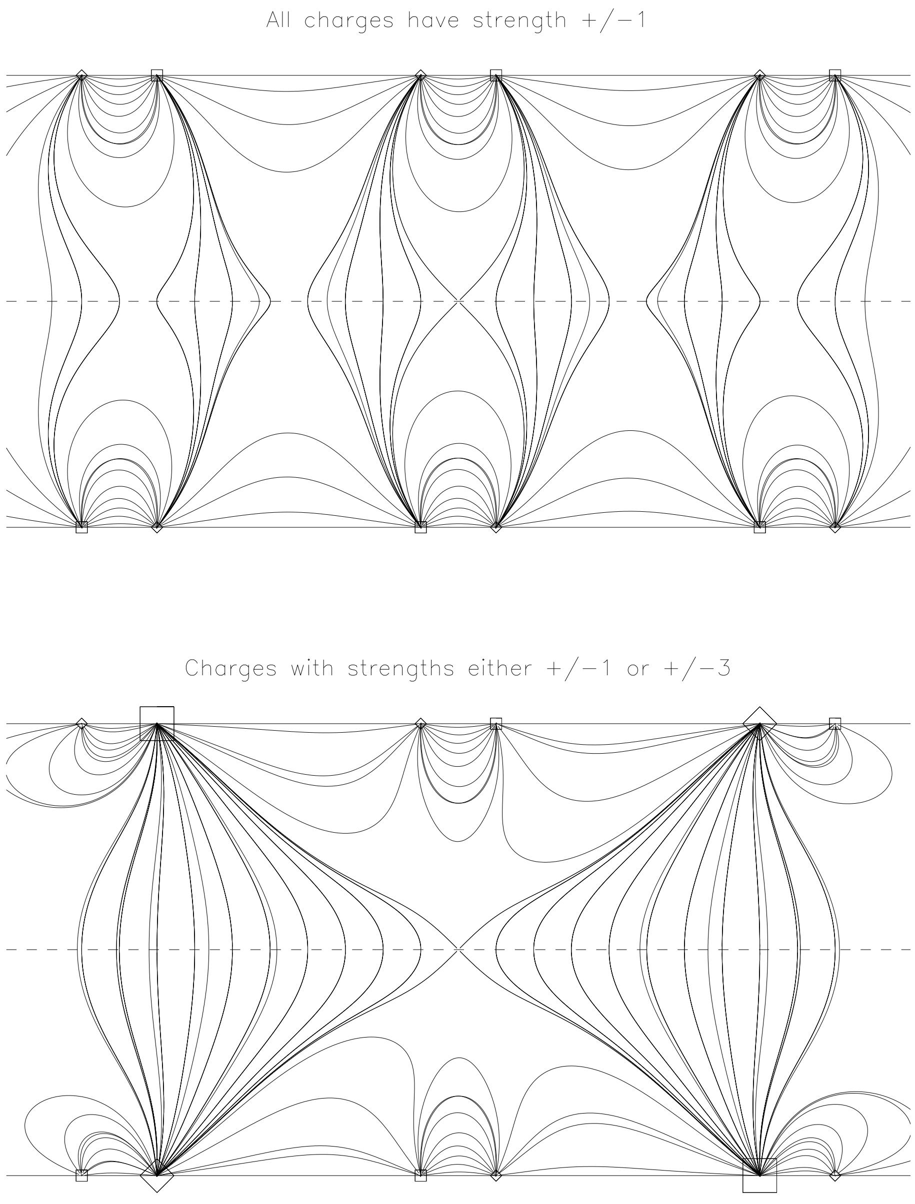


Fig. 2.1. Concept of mirror charges leading to nulls and 'helmets.'

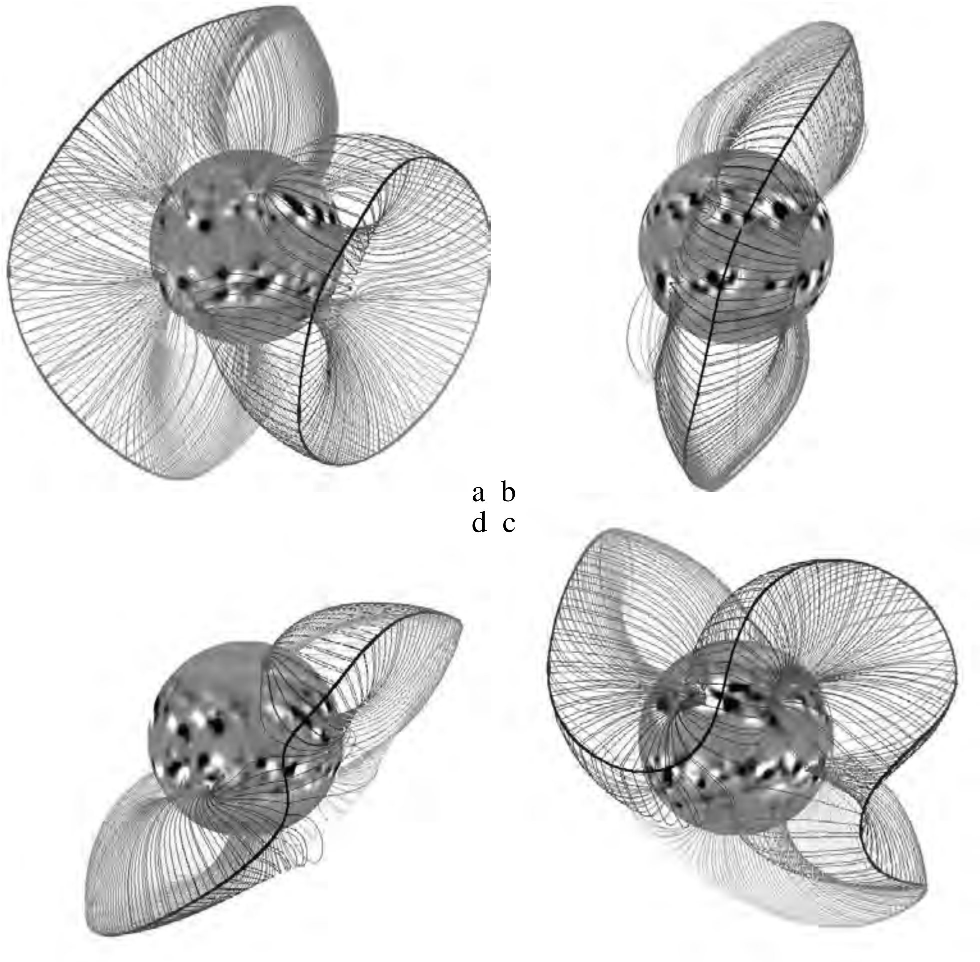


Fig. 2.2. The Sun's surface magnetic field is comprised of a multitude of dipolar regions of widely different fluxes, whose numbers wax and wane with the solar cycle. The large-scale coronal magnetic field, the foundation of the heliospheric field, expands from regions of partly open magnetic field that enclose the closed-field corona. This diagram shows the global topology of the Sun's field in a so-called potential-field source-surface approximation. In particular, it shows four realizations of the 'streamer belt' for a solar magnetic model. Shown are four phases of the simulated magnetic cycle: clockwise from the top left, $t = 3.1,\ 3.6,\ 4.5,\ 6.0$ *years into a sunspot cycle of 11. years. Each panel shows a magnetogram of the solar surface, the neutral line(s) at the source surface, and the highest closed field lines that reach up to the neutral line(s); the lines are colored so that the darkest colors are nearest to the 'observer.' The panels show, clockwise, an example of a near-quadrupolar situation; a strongly tilted dipolar case; a strongly warped current sheet; and another nearly dipolar case with less tilt relative to the solar equator. [Fig. H-I:8.1]*

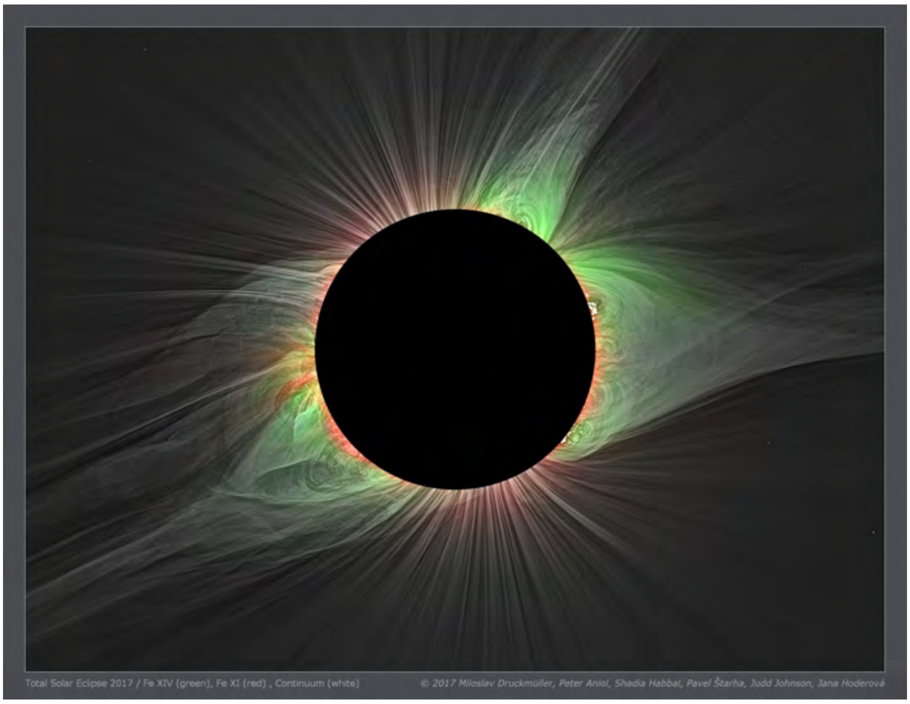


Fig. 2.3. Special filters enable scientists to measure different temperatures in the corona during total solar eclipses, such as this one seen in Mitchell, Oregon, on August 21, 2017. The red light is emitted by charged iron particles at 1.2 MK (Fe IX*; coronal "red line") and the green are those at 1.8 MK (Fe* XIV*; "green line"). Credit: Image produced by M. Druckmuller and published in Habbal et al. (2021).*

Wikipedia: "Helmet streamers are bright loop-like structures which develop over active regions on the Sun." Is the Wikipedia entry correct?

NO! "... the most prominent white-light features (i.e., the bright streamer stalks) occur at those longitudes where the latitude of the plasma sheet reaches a local maximum or minimum $\mathrm{d}\lambda/\mathrm{d}\phi \approx 0$; when these "stationary points" are located close to the sky plane, the sheet is viewed edge-on, and the number of scatterers in the line of sight is greatest." (From Wang et al. 2000, JGR105, A11, p. 25133)

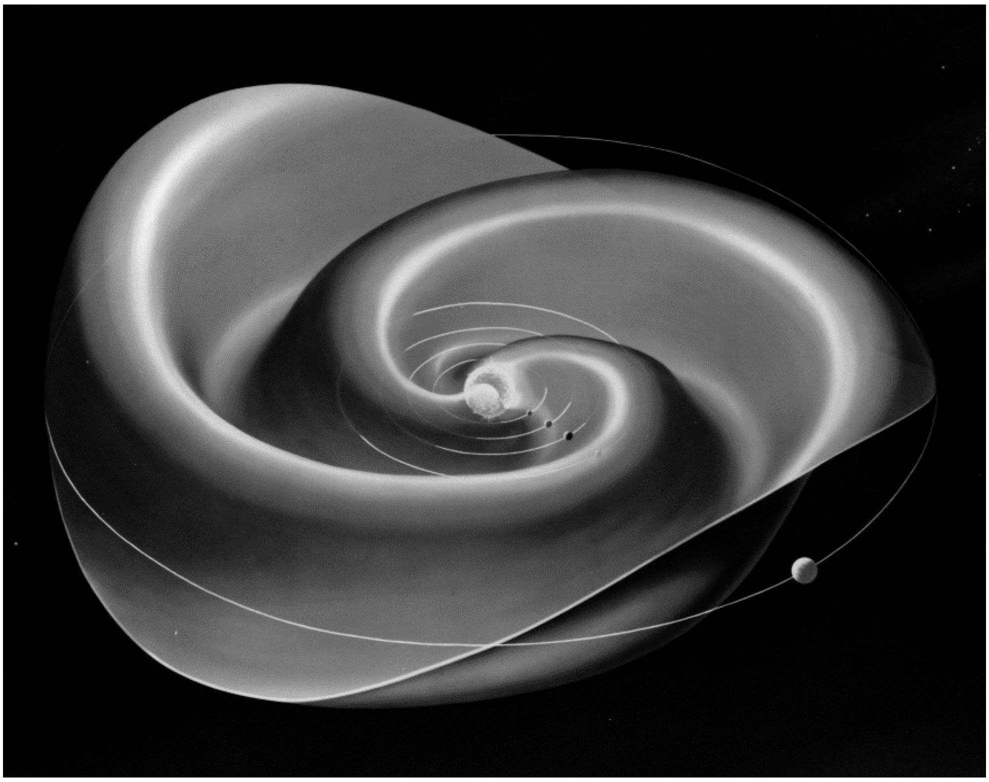


Fig. 2.4. The heliospheric current sheet forms a warped, undulating structure extending from the top ridge of the helmet streamer belt [. . .] that sweeps by the planets as the Sun rotates once per 27 days (synodic period). The magnetic field changes direction across the current sheet. [image source; see also Fig. H-I:9.3]

On part (2): Simplifications: (1) only one pair of mirror charges needs to be considered: if the field is radial on some sphere for that pair, than any other similar pair added will conserve that property; (2) we can look at this in the x, y plane and can rotate the charges to lie on the x axis; (3) we can think of this as a problem in electrostatics (allowing monopoles, as long as the Sun does not). And we simplify it to a 2d analysis here (you're welcome to go to 3d).

Field of a point charge of charge q at $\vec{\mathbf{r}}_i$ is given by:

$$\vec{\mathbf{E}}_i = q_i \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}_i}{|\vec{\mathbf{r}} - \vec{\mathbf{r}}_i|^3}. \tag{viii}$$

Let the charges 1 and 2 be located at $(1, 0)$ and $(\alpha, 0)$, respectively.

The requirement that the field becomes radial at a distance $r_{\rm SS}$ from the origin can be expressed as the requirement that the vector field anywhere on that circle (sphere) is normal to the tangent to that circle (sphere), i.e. normal to the vector pointing to any point on that circle.

For a vector and its unit-length normal

$$\vec{\mathbf{r}} = \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix}; \hat{\mathbf{r}}_\perp = \begin{pmatrix} \sin\theta \\ -\cos\theta \end{pmatrix} \tag{ix}$$

this translates to the requirement that

$$\left(\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) + \vec{\mathbf{E}}_2(\vec{\mathbf{r}})\right) \cdot \hat{\mathbf{r}}_\perp = 0. \tag{x}$$

Use a unit radius for the Sun and a TBD radius α for the mirror surface and put the charges at $(1, 0)$ and $(\alpha, 0)$. Then, working through the math, you end up with:

$$(\alpha^2 + 2\alpha r\cos\theta + r^2)^{3/2} q_1 + (r^2 + 2r\cos\theta + 1)^{3/2} \alpha q_2 = 0, \tag{xi}$$

which holds true at any θ provided that $r = \sqrt{\alpha}$ and $q_2 = -\sqrt{\alpha} q_1$. So, for a source surface radius (where the field becomes radial – sorry for the confusing term here!) at $r_{\rm SS} = 2.5R_\odot$, the mirror charges need to be placed at $6.25R_\odot$ and be of strength -2.5 times the surface charge.

Insights: (1) natural formation of streamer cusps (see Fig. 5.7); (2) superradial expansion of magnetic field (important factor in determining wind speed). But: no dynamics, no currents, . . .

2.5 Length and time scales

Activity 13 (or E): With the values in Table 2.4, **how long do the slow and fast solar-wind streams take to reach Earth?** *How many degrees does the Sun rotate between the moment these wind streams leave the Sun and the moment they arrive at Earth? How long for Neptune? Given that the wind flows out essentially radially, what is the apparent direction of the wind relative to the direction of the Sun as observed from the orbiting Earth (with an orbital velocity of about 30 km/s)?*

Groups 1, 4, 7, 5

Answer: $v_{\rm sw} = (4.3\,{\rm to}\,9)\,10^7$ cm/s, $d_\oplus = 1.5\,10^{13}$ cm, so $\Delta t_\oplus = 4.0\,{\rm to}\,1.9$ days; with $d_{\rm Neptune} = 30$ AU, so $\Delta t_{\rm Neptune} = 30 t_\oplus = 120\,{\rm to}\,57$ (or $\approx 5\,{\rm to}\,2$ solar rotations).

The angle of the incoming wind at Earth relative to the Sun-Earth line is given by $arctan(v_{\rm orb}/v_{\rm sw})$: $4.0^\circ\,{\rm to}\,1.9^\circ$.

Note that in the 2 to 4 d that it takes for the solar wind to reach Earth, the Sun has rotated $25^\circ\,{\rm to}\,53^\circ$, so that the magnetic connection from Sun to Earth starts around that angle toward the solar west.

How is that related to the predominant source regions of solar energetic particles?

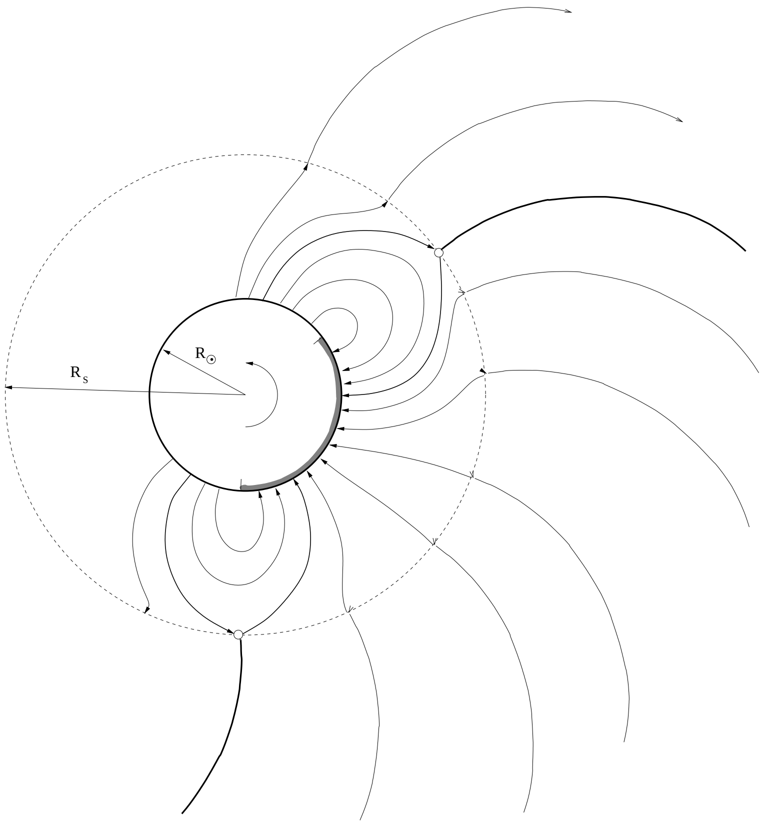


Fig. 2.5. A schematic depiction of a potential-field source-surface (PFSS) model as viewed from above the Sun's North pole (the sense of rotation is indicated by a semi-circular arrow). . . . [Fig. H-I:4.7]

2.6 Assumptions and fidelity of MHD models

Activity F: **List and discuss factors that limit the fidelity of models of Sun-planet couplings via the solar wind** *(1) in 'quiescent' background state and (2) for (I)CMEs? Include both numerical and observational factors. Consider these factors from the perspectives of (a) fundamental science and (b) the use of models in forecasts on time scales of, say, hours (for flare irradiance and prompt energetic particles for spaceflight and communication), days to a week (for impact of wind streams and ICMEs for geomagnetic activity), and years (solar dynamo impact on satellite orbits and interplanetary travel). Start with the modeling based on what is discussed in Chapters 4, 5, 6, and 8.*

Groups 2, 6, 3, 8

1: 'quiescent' background state

- Limited coverage of the solar surface
- Inadequate measurement of forces on the solar near-surface field (current systems)
- Lack of understanding of 'reconnection' [definition?: "a change of connectivity of plasma elements" (Pontin and Priest, 2022, LRSP); or my favorite: failure of the frozen-in (ideal-MHD) condition]
- Inadequate understanding of energy conversion in the corona
- Miserable coverage of the heliosphere, including the Sun-Earth line
- Interaction of streams and shocks
- The Earth is a very small target viewed from the Sun: $d_\oplus \sim 200 R_\odot$.
- Poor knowledge of magnetospheric field coupling and internal dynamics
- Miserable coverage of the magnetosphere
- Insufficient modeling capabilities for impact assessment of geomagnetic activity
- Intractable scale ranges
- 'Secretiveness' about SWx impacts
- ...

2: CMEs:
All of the above, plus:
Hours: N/A
Days:

- Poor understanding of destabilization of the solar magnetic field
- Unknown field structure leaving the Sun
- Inadequately known field state in the magnetosphere
- Insufficient lead time to model geomagnetic response for forecasting
- ...

Years:

- No validated predictive dynamo model
- ...

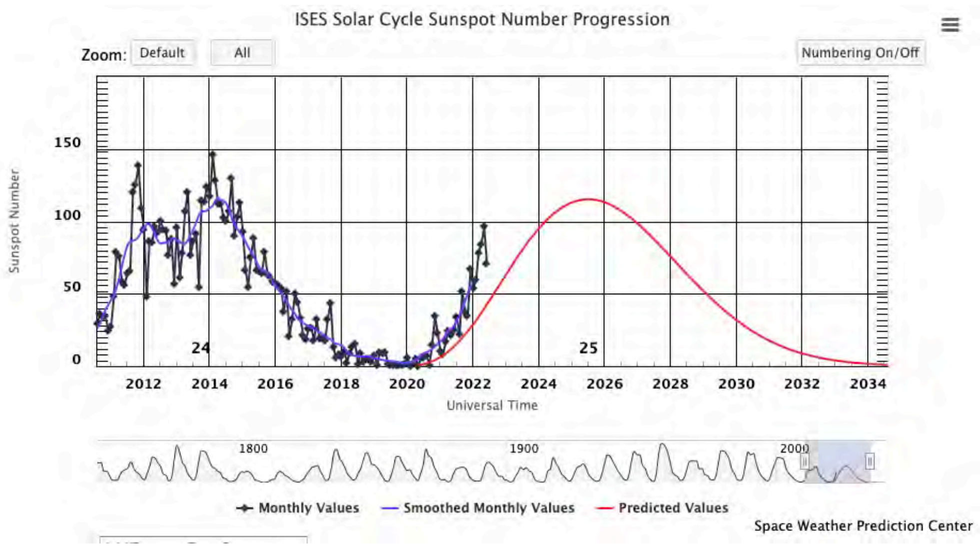


Fig. 2.6. SWPC solar cycle prediction, downloaded 2022/08/01 from https://www.swpc.noaa.gov/products/solar-cycle-progression

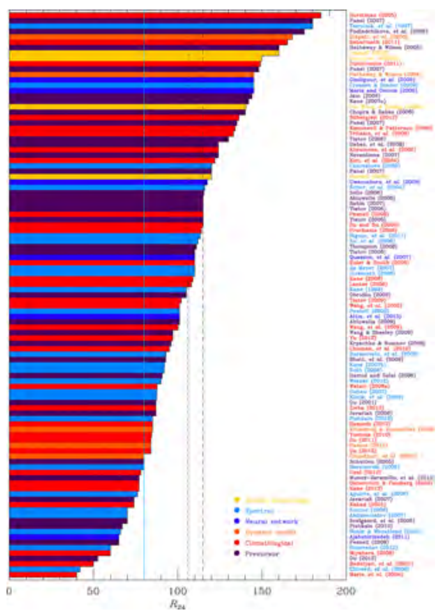


Fig. 2.7. Predictions of solar cycle 24 from Pesnell (2016, Space Weather 14, 10).

3

Universal processes and settings

Focus: other planetary systems / other times
Learning objective(s): 'the exception(al) proves [tests?] the rule'

3.1 Universal processess

Activity G: **Compile a list of physical processes common to solar/stellar and (exo-)planetary space physics** *and consider what parameters might cause these same processes to manifest differently (for example: effect of rotation on convection in slow-moving, fast-spinning planets vs. fast-moving, slow-spinning stars).*

Groups 1–8

Processes (in order of 'popularity' in the homework):

1,2,3,4,6,7 Flows and waves: (M)HD/kinetic, helio-/geo-seismology, (atmospheric, heliospheric, ISM) shocks, (scale-coupling, heating, diffusive dissipation) turbulence, (couplings/mixing through) (neutral) atmospheres, stellar (binary) winds, ...

2,3,4,5,7,8 (Time dependent) Dynamo [definition?]; power source (cooling, fusion, fission, gravitational settling/solidification, tides), power loss: planetary crust/oceans/atmosphere

2,4,5,6,7 Reconnection

1,2 MHD: current sheets, flux ropes, collisional-collisionless; neutral component (ISM neutrals, ionospheres, coolest stars); dust

3,8 Tides and torques: binary stars, satellites/moons, irradiation

4,7 Radiative transfer: in vs. out, clouds, greenhouse gases / line blanketing, radiation pressure (hot-star winds, comets), emission/absorption lines

1,6 Convection: neutral gas, (magnetized) plasma, magma; rotation

4,6 Chemical processes

2,7 'Phase' change: (coronal, atmospheric) rain or evaporation [be careful with terminology!], ionization/recombination, charge exchange

?2 Magnetic instabilities *(2:flares,CMEs)*: field evolution, emergence/intrusion, external driving including perturbations

3 Exosphere

$\sim$ 7 Energy conversion

?1 Gravity

- Particle acceleration
- ...

Parameters involved in shaping the process

1,2,4,5,6 Magnetic geometry

1,2,3,5,8 (Differential) Rotation (up to magnetars!)

1,2,3,4,8 (Chemical, Ion, Isotopic) Species present / composition / gradients

2,4,6 Plasma (and magma, ocean) conditions

3,8 Orbital properties

1,2 Energy ?source?, transport (convect, conduct, radiate)

1,3 Evolutionary phase

3 Tides (i.e., satellites, ...)

4 Bulk plasma velocity

1 Size

1$\sim$ Dimensionless numbers

2 Seed field [?]

3.2 Revisiting the Parker solar wind: other stars

Quick note:

Value of the critical radius (Eq. 2.11) in Sun and Sun-like MS stars. With T_6 in MK:

$$\frac{r_{\mathrm{c},\odot}}{R_\odot} \approx \frac{6}{T_6}. \tag{xii}$$

So, as expected, the critical point moves *outward* with *de*creasing temperature.

For other cool main-sequence stars (approximating data in Eker et al. (2018) Fig. 6 (top) below 1.2 solar masses):

$$\frac{R_*}{R_\odot} \approx \frac{M_*}{M_\odot}, \tag{xiii}$$

so that:

$$\frac{r_{\mathrm{c},*}}{R_*} \approx \frac{6}{T_6}, \tag{xiv}$$

i.e., essentially independent of mass when expressed in terms of stellar radii.

Insight: scaling relationships offer quick insights

3.3 Revisiting the Parker solar wind: centrifugal forces

Activity 14 (or H): Show: The momentum balance in Eq. (2.7) describes a radially-flowing wind over a non-rotating Sun. In reality, the Sun is rotating, and the magnetic field reaching into the heliosphere enforces the wind to co-rotate with the Sun, out to a distance where it becomes too weak to enforce such co-rotation (somewhere between 10 to 20 solar radii, or 0.05 to 0.1 AU). **Show that for a sufficiently slowly rotating Sun, ignoring the centrifugal force is warranted.** *At what rotation period of a star like the Sun does the centrifugal force at, say, $2R_\odot$ counteract gravity by more than 10%? The centrifugal force in the wind would have been important for the very young Sun, see Sec. 10.2.1. Moreover, in the early phases of star-disk systems, centrifugal forces may be important in driving a cold wind; see Sect. 7.2.4.*

Groups 7, 5

Answer: The simplest Parker solar wind model as discussed in Section 2.2 ignores the magnetic field, and therefore also centrifugal forces and angular momentum (why?), but with a magnetic field, that is not necessarily warranted. Comparing the centrifugal force to gravity, as per Activity 14, yields:

$$\rho\omega^2 r < \frac{1}{10}\frac{\rho G M_\odot}{r^2}, \tag{3.1}$$

or $P = 2\pi/\omega > 1$ day at $2R_\odot$ for a solar-mass star, and longer at larger distances: at $20R\odot$, for example, the threshold period would have to exceed about 30 days.

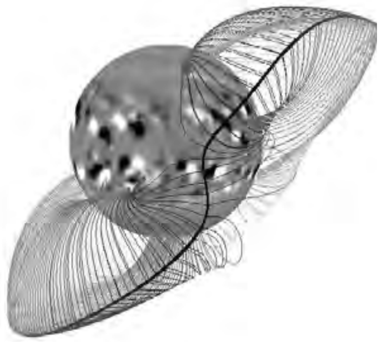
Insight: centrifugal forces are important for young stars; some very rapid rotators have phenomena called 'slingshot prominences."

3.4 Terrestrial dynamo to surface field

Activity I: Models for the Earth's magnetic field show a high degree of structure in the dynamo region in the core. Similarly, state-of-the-art models of the solar dynamo also reveal a high degree of structure. In contrast, the Earth's surface field and the Sun's heliospheric field are dominated by dipolar patterns. **Discuss how these difference arise.**

Groups 3, 4

Answer/Insight: multipolar fields decay quickly between solar surface ($1R_\odot$) and the source surface where the field becomes radial (at about $2.5R_\odot$), and between the Earth's dynamo (in the outer core, which reaches up to $\approx 0.5R_\oplus$) and Earth's surface (up to $1R_\oplus$). Another factor is the range of scales of convective and convectively-driven motions (such as the Sun's meridional circulation) compared to the object's radius.



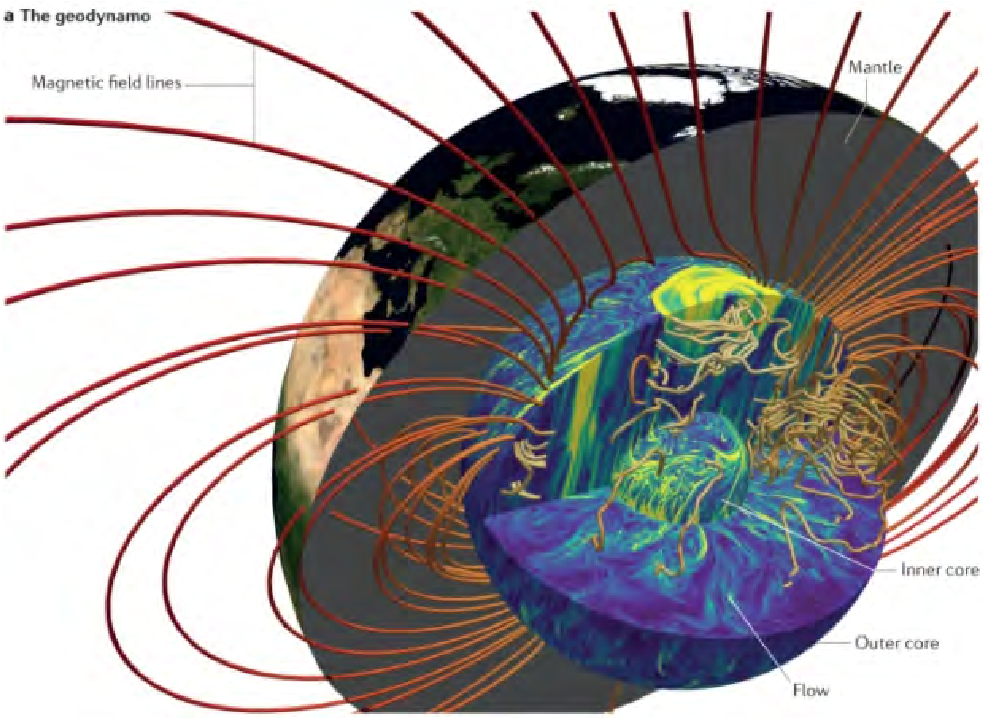


Fig. 3.1. A visualization of a geodynamo model. [Image source]

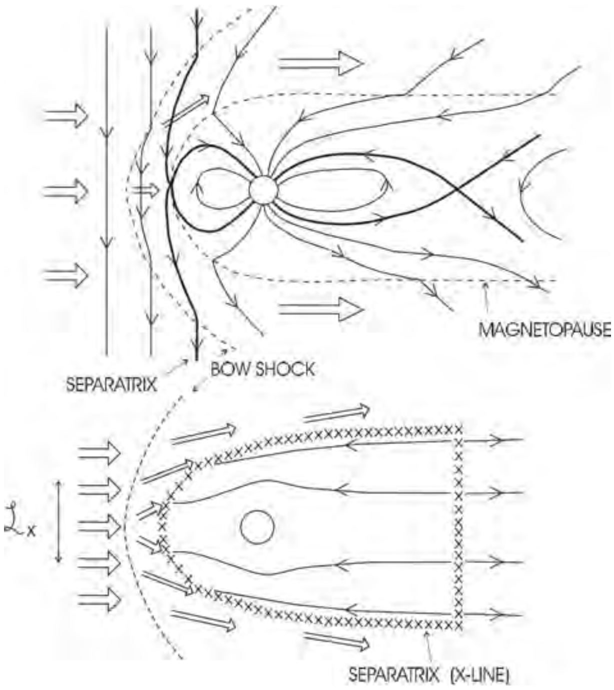
3.5 Different settings: planets

The magnetospheric magnetic field cycle starts for the field with (1) day-side reconnection to the field in the wind, is then (2) followed by being dragged towards night-side, from there (3) moving into the magnetotail, and after (4) reconnection in the current sheet the field (5) moves back towards the day-side to replenish (at least on average over longer periods) the flux lost from there in the reconnection process. That loop, called the Dungey cycle, can be visualized from Fig. 5.14 if the succession of drawn field lines is interpreted as a sequence of events for a single field line (and realizing that step (5) has to occur over lower magnetic latitudes to avoid the field that is at the same time involved in step (2)).

That entire process occurs within the magnetosphere. On the day-side, the boundary of the magnetosphere can be defined as the magnetopause. For a comparison:

Activity J: For Earth and the giant planets, **estimate for each of the planets the model-based magnetopause distance** $R_{\rm CF}$ *[Eq. (5.22), setting* $\xi = 2$*]. Assume the following: that the solar wind speed averages to roughly the same value at all the planets (say* $v_{\rm sw} = 400$ *km/s). Use info in Tables 5.2 and 5.3.*

Groups 6, 1



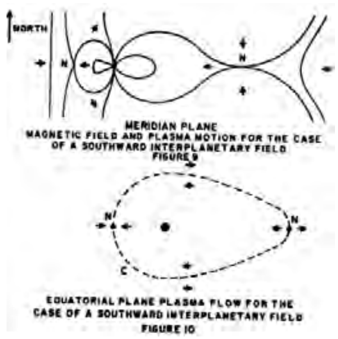


Fig. 3.2. **Left:** *Schematic representation of a magnetically open magnetosphere.* Top*: cut in noon-midnight meridian plane; [open arrows: solar wind bulk ow. Solid lines within magnetosphere: magnetic field lines (direction appropriate for Earth); thick lines are magnetic field lines within the 'separatrix surfaces' that separate open from closed or open from interplanetary field lines [. . .].* Bottom*: cut in equatorial plane; a line of × symbols represents intersection with the two branches of the separatrix; solid lines are streamlines of magnetospheric plasma flow, and $\mathcal{L}_\times$ represents the projection of the dayside magnetic reconnection region along streamlines into the solar wind. [Fig. 5.14, from H-I:10.3]* **Right:** *from "The interplanetary magnetic field and the auroral zones", J.W. Dungey, 1962, AFCRL 62-423. The concept of magnetic reconnection (and the involvement of neutral points) goes back to Ron Giovannelli in 1948.*

How much mass is carried by the solar wind?

Mass per unit area over a time interval Δt, with ion density $n_{\rm sw} \approx 10\,\text{cm}^{-3}$ and $v_{\rm sw} \approx 500\,\text{km/s}$ at Earth (and Moon), ignoring long-term change:

$$\dot{m} = n_{\rm sw} v_{\rm sw} (1.7 m_{\rm p}) \Delta t \approx 45\,{\rm mg/cm^2}\,\text{over a million years}$$

For the Sun as a whole, that amounts to

$$\dot{M}_\odot / M_\odot = n_{\rm sw} v_{\rm sw} (1.7 m_{\rm p}) \Delta t 4\pi d_\odot^2 / M_\odot \approx 6\,10^{-5}\,\text{over a billion years}$$

Answer: The magnetopause distance is determined primarily by the requirement that the total pressure (plasma plus magnetic) must have the same value on both sides of the discontinuity. At the Chapman-Ferraro distance of Eq. (5.22), the linear momentum flux density (or dynamic pressure) in the undisturbed solar wind, $\rho_{\rm sw}{v_{\rm sw}}^2$ at the sub-solar region equals the interior magnetic pressure of the dipole field, $B(r) = (1/8\pi)(\mu_{\rm p}/{\rm r}^3)^2$ with $\mu_{\rm p} = B_{\rm p}R_{\rm p}^3$ the magnetic dipole moment of the planet with equatorial field $B_{\rm p}$ and radius $R_{\rm p}$, with an extra factor $\xi \simeq 2$ to roughly correct for the added field from magnetopause currents:

$$R_{\rm CF} \approx R_{\rm p}\left(\frac{B_{\rm p}^2}{2\pi\rho_{\rm sw}v_{\rm sw}^2}\right)^{1/6} = 1.7\,10^{10}\,\frac{R_{\rm p}}{R_\oplus}\left(\frac{B_{\rm p}^2}{n_{\rm sw}}\right)^{1/6}, \qquad \text{(rewritten 5.22)}$$

where the solar wind speed has been set to 400 km/s in the righthand expression.
With data from Tables 5.2 and 5.3:

Planet	$B_{\rm p}$ (G)	$n_{\rm sw}$ (cm^{-3})	$R_{\rm p}/R_\oplus$	$R_{\rm CF}/R_{\rm p}$	$R_{\rm CF}$ (cm)
Earth	0.3	7.	1	12	$8.2\,10^9$
Jupiter	4.3	0.2	11.2	56	$4.0\,10^{11}$
Saturn	0.21	0.07	9.4	25	$1.5\,10^{11}$
Uranus	0.23	0.02	4.0	31	$8.0\,10^{10}$
Neptune	0.14	0.006	3.9	32	$8.1\,10^{10}$

Note the weak dependence on $n_{\rm sw}$: $1000^{1/6} \approx 3$.

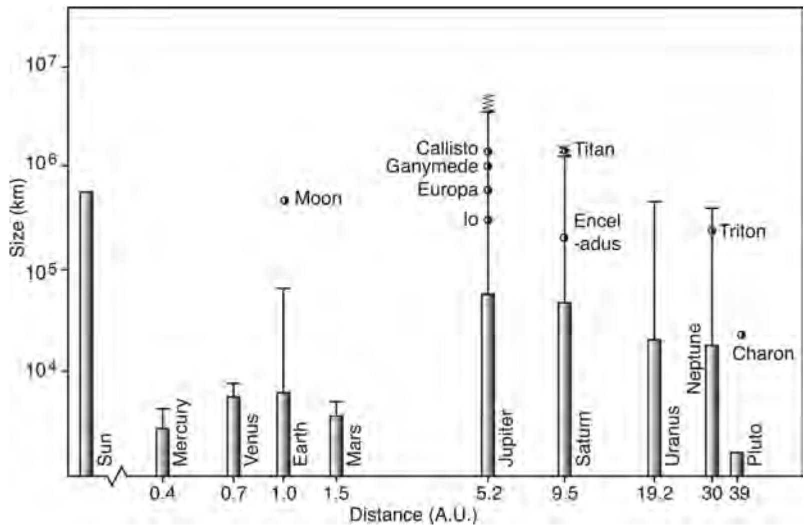


Fig. 3.3. A logarithmic plot of size of object vs. distance from the Sun for the planets (solid bars), their magnetospheres (thin bars) and the orbital radii of their primary moons. The range in sizes of the magnetospheres of Jupiter and Saturn are shown by the zig-zag lines. [H-1:13.1]

N.B. Magnetospheric scientists often contemplate processes in terms of electrical currents (solar and heliospheric physicists rarely do). In that light:

Another lesson from Parker (2014 Res. Astron. Astrophys. 14 1): "Then there is the notion that $\mathbf{j}$ causes $\mathbf{B}$ and is, therefore, the more fundamental field variable. But Maxwell's equations cannot be written in useful form in terms of $\mathbf{j}$ rather than $\mathbf{B}$. One has to use the Biot-Savart integral to eliminate $\mathbf{B}$, thereby converting the partial differential equations into intractable global integro-differential equations. So, lacking a tractable field equation, it is declared that $\mathbf{E}$, induced by $\mathbf{v}$, drives $\mathbf{j}$, which in turn produces $\mathbf{B}$. But it is the electric field ... in the moving frame of the plasma that drives the current $\mathbf{j}$, so in the absence of resistivity, [that electric field] is zero and there is no significant driving of $\mathbf{j}$. The magnetic field varies because it is mechanically deformed as it is carried along with the bulk fluid motion ... *The energy of the magnetic field changes because of the work done on it by* $\mathbf{v}$." [see also Sect. 5.5.7.1]

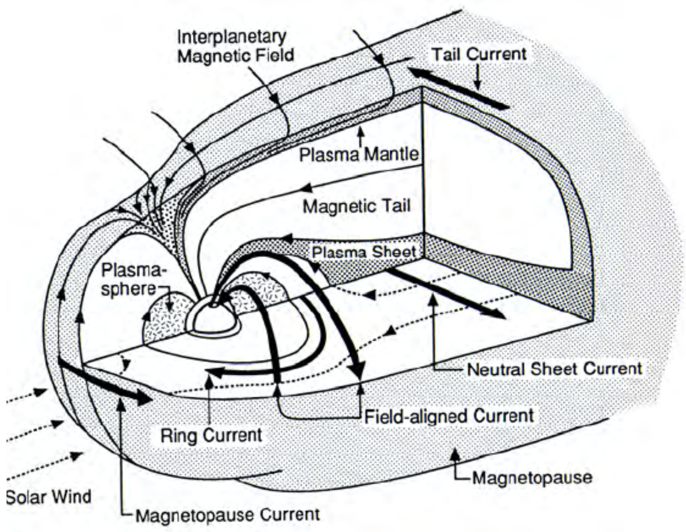


Fig. 3.4. The Earths magnetosphere and large-scale current systems [Kivelson and Russell, 1995]

3.6 Different settings: beyond the solar system

Activity K: **List, discuss, and compare/differentiate the factors that influence the interactions** *between (1) between the solar/stellar wind and orbiting (exo)planets, (2) between moons and (exo)planetary magnetospheres, and (3) a star and the interstellar medium that it moves through.*

Groups 8, 2

Discussion outcome for Activity K:
To consider (in no particular order):

2,8 Is the stellar wind magnetized? E.g., radiatively/acoustically-driven hot-star wind vs. magnetically (and conductively) driven cool-star wind
2,8 Is the relative motion supersonic, super-Alfvénic? E.g., binary star system, for close-in planet or planetary satellite, for ISM
2,8 Conductivity. E.g., ('non-'conducting) Earth's Moon vs. (internal dynamo in core) Jupiter's Ganymede vs. (induced field in ocean) Jupiter's Europa
2 ISM density and magnetic field, and relative motion to planetary system
8 Field orientation
8 Evolutionary stage
8 Orbital properties (binary, planet, satellite)
8 ISM properties

- Tidal coupling for (stellar, planetary) dynamo, (space) weather. E.g., in close binary, 'eyeball planet'; tidal forcing (warming interior)
- Climate/Weather: Irradiation (rotating star with evolving (large?) starspots on (rotating, orbiting) planet; plate tectonics (water and weathering)
- Planetary migration, asteroid/'KBO' impacts/star or 'rogue planet' passage
- . . .

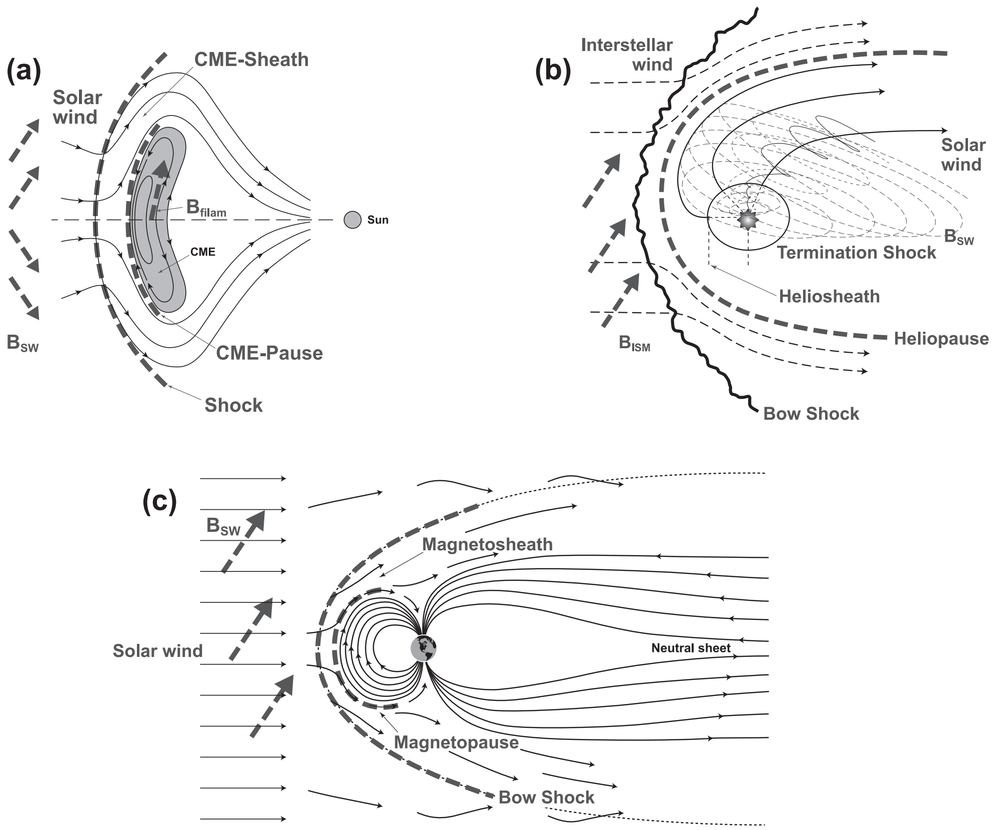


Fig. 3.5. Schematic comparison of shocks around CMEs, the heliosphere, and the magnetosphere. [Fig. H-II:7.1]

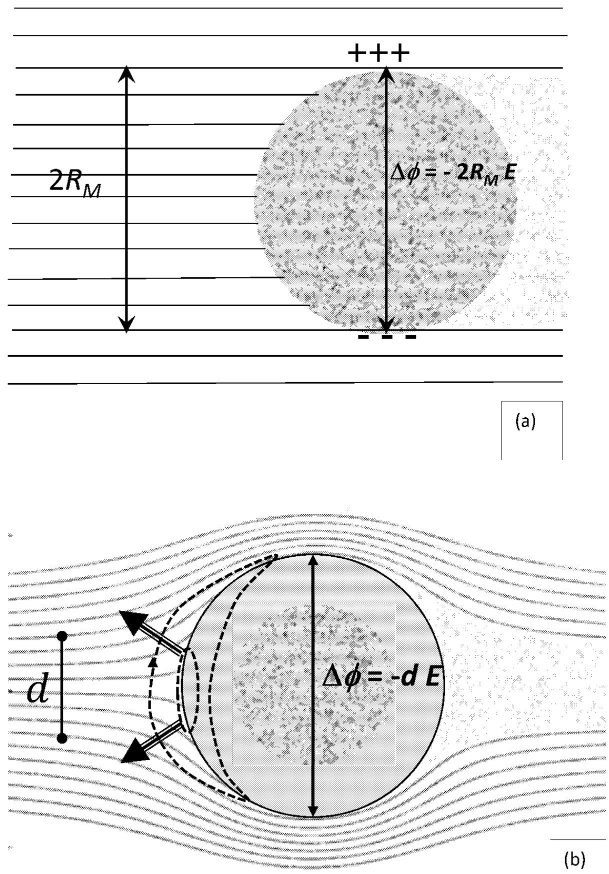


Fig. 3.6. Schematics of plasma flow (shown by lines of flow) at velocity $\mathbf{u}$ from the left onto (a) a non-conducting body and (b) a conducting body. In the plasma, $\mathbf{B}$ is into the paper, $\mathbf{E}$ is $-\mathbf{u} \times \mathbf{B}$ in both cases. Diagram (a) shows that a non-conducting body builds up surface charge that imposes a potential drop $\Delta\phi = -2R_M E$ across the diameter, producing an electric field that opposes the solar wind electric field. Diagram (b) shows the response of a conducting body that does not build up surface charge. Conducting paths allow current (shown schematically as a dashed line) to flow through the body and close in the incident flow. Heavy banded arrows identify the orientation of the resultant $\mathbf{j} \times \mathbf{B}$ force that diverts part of the incident flow. Because much of the incident flow has been diverted, the potential drop across the body is only $\Delta\phi = dE$, where $d < R_M$ is the distance in the incident flow between the flow lines that just graze the body. The electric field that penetrates the body is a fraction of the upstream field determined by the fraction of the upstream flow that impacts the surface. In the wake region, gray in both diagrams, the plasma pressure is reduced and the magnetic pressure is increased relative to the upstream values. [Fig. H-IV:10.2]