Turbulence

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References

- G. K. Batchelor, *An introduction to fluid dynamics*, Cambridge University Press, Cambridge, 2002
- T. M. Boyd and J. J. Sanderson, *The physics of plasmas*, Cambridge University Press, Cambridge, 2003
- D. Biskamp, *Magnetohydrodynamic turbulence*, Cambridge University Press, Cambridge, 2003
- U. Frisch, *Turbulence. The legacy of A.N. Kolmogorov*, Cambridge University Press, Cambridge, 1995
- R. Bruno & V. Carbone, *The solar wind as a turbulence laboratory*, Living Rev. Sol. Phys. **10,** 2 (2013). <https://doi.org/10.12942/lrsp-2013-2>

Outline

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- New challenges from Parker Solar Probe
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Introduction

Why turbulence?

- ➢ It is the process by which a fluid (or gas) attempts to self-organize its energy.
- ➢ In the space context:
	- \triangleright It can heat the background plasma.
	- ➢ It can scatter cosmic rays
	- \triangleright It is a major factor for star formation

Turbulence is a fundamental and ubiquitous phenomenon on Earth.

Turbulence is a major factor in the car and aviation industry, by affecting drag

Enhances mixing: affects diffusion e.g., of contaminants and transport in general

Affects weather patterns, including hurricanes

Some canonical features of turbulent flows

https://www.youtube.com/watch?v=_UoTTq651dE&t=21s

- o Not steady and unpredictable
- o Multiscale
- o Enhanced mixing
- o Intermittent behavior

Simulation by C. Gonzalez showing the development of turbulence in a magnetized plasma.

Some canonical features of turbulent flows

Open problems in the solar wind

Laminar wind from Parker's theory

• First theory that predicts a solar wind, 1958

$$
\frac{d}{dr}(\rho U r^2) = 0, \quad p = c^2 \rho
$$

$$
U\frac{dU}{dr}=-\frac{1}{\rho}\frac{dp}{dr}-\frac{g}{r^2}
$$

Exploring the heliosphere out of the ecliptic: the Ulysses mission

Typical configuration (at solar minimum) of magnetic field structure and slow/fast wind

The solar wind measured by Ulysses

The solar wind is hotter than predicted

 $T \sim R^{-0.74}$

Fitted radial trend of temperature

 \boldsymbol{U} . dT $d\mathbf{R}$ $=-T$ 2 3 $\nabla \cdot \bm{U} \Rightarrow T \sim R^{-4/3}$

Predicted by adiabatic spherical expansion

Evidence for ongoing proton heating!

[Hellinger et al 2011]

Modeling "Wave-Driven" Winds

Steady state equations describing the conservation of mass and momentum fluxes of the average solar wind, and a simplified equation for the temperature:

$$
\nabla \cdot (\rho \, \vec{U}) = - \nabla \cdot < \delta \rho \, \delta \vec{U} >
$$

$$
\nabla \cdot (\rho \vec{U} \vec{U}) = -\nabla p^T + \frac{\vec{B} \cdot \nabla \vec{B}}{4\pi} - \rho \frac{GM_{\odot} \vec{R}}{R^3} - \nabla \leq
$$

$$
-\nabla \cdot \langle (\rho \delta \vec{U} \delta \vec{U} - \frac{\delta \vec{B} \delta \vec{B}}{4\pi}) + 2 \vec{U} \delta \rho \delta \vec{U}
$$

Perturbations in density, velocity and magnetic field contribute to the average solar wind via wave pressure and Reynolds stresses. They can also heat the plasma via dissipation (or other kinetic processes.)

 $\delta \vec{B} \delta \vec{B} >$ 8π $+\delta\rho\delta\vec{U}\delta\vec{U}$

$$
\vec{U} \cdot \nabla T = -U \frac{4 T}{3 R} + m_p \varepsilon
$$

Observed solar wind fluctuations

[Belcher & Davis1971]

- What heats and accelerates the solar wind?
	- Evidence points to the fact that fluctuations/turbulence play a crucial role
- If it's "waves", what's the problem then?
	- We need to understand how the energy stored in the field can be conveyed to the particles
	- To answer to this question, we need to go beyond laminar flow models and understand nonlinear evolution of fluctuations

Open questions

Turbulence in Fluids

From laminar to turbulent flows

laminar flow

turbulent flow

$Re < 2100$

$Re > 4000$

Reynolds number $Re = UL/\nu$

From laminar to turbulent flows

What is a turbulent flow?

- The two experiments in the same conditions report different signals.
- The velocity appears chaotic. The system is highly sensitive to small perturbations in the initial conditions (Lorenz 1963)
- However, the signals vary cyclically and display the same statistical properties (e.g, the mean and rms of the signal distribution). The signal also contains multiple scales.

Energy cascade (configuration space)

Schematic diagram of an eddy which undergoes an instability that fragments it into smaller structures

Energy cascade (Fourier space)

- Simplest model to analyze the combined effect of nonlinearity and diffusion
- Assume a periodic function: $u(x, t) = \sum_k A_k(t) e^{ikx}$
- Substitute into eq. (1)

(1)

$$
\dot{A_k} + \sum_{m} i m A_{k-m} A_m = -\nu k^2 A_k
$$
\nNonlinearity

 (2)

A simple example: Burgers equation

- Nonlinearity couples a given mode with higher wave-number modes (i.e. smaller scales)
- Even if initially all of the energy was contained in a single mode (say, $k=1$), the nonlinear term acts as a source (like a forcing term) for all of the other accessible modes to the system
- This leads to transfer of energy at smaller scales, aka, energy cascade
- Does energy cascade proceeds ad-infinitum? No!

$$
e^{-\nu k^2 t}
$$

$$
\dot{A_k} + \sum_{m} i \, m A_{k-m} A_m = -\nu k^2 A_k
$$
\nNonlinearity\ndissipation

• Dissipation acts on single components leading inexorably to their decrease

$$
\dot{A}_k = -\nu k^2 A_k, \Longrightarrow A_k(t) = A_k(0)
$$

A simple example: Burgers equation

A simple example: Burgers equation

Navier-Stokes equation for incompressible fluids

 ∂u

 $\frac{\partial}{\partial t} + (\boldsymbol{u} \cdot \boldsymbol{\nabla}) \boldsymbol{u} = -\boldsymbol{\nabla} p +$ 1 R_e $\nabla^2\boldsymbol{u}$ $\nabla \cdot \mathbf{u} = 0$

[[]Banerjee, Supratik. (2014)]

Energy conservation

Kinetic energy/mass K ≡ 1 2 $\overline{ }$ \boldsymbol{V} $\bm{u}\cdot \bm{u}dV=$

1 2 $\overline{ }$ \boldsymbol{V} u^2 dV

• From incompressible Navier-Stokes, energy conservation law:

$\nu\,\bm{u}\cdot(\nabla^2\bm{u})dV$ (3)

$$
\frac{\partial K}{\partial t} + \int_{V} \{ \boldsymbol{u} \cdot \nabla p + \boldsymbol{u} \cdot [(\boldsymbol{u} \cdot \nabla)] \boldsymbol{u} \} dV = - \int_{V} dV
$$

 $(1) = \int [\nabla \cdot (u p) - p(\nabla \cdot u)]dV = 0$ (assuming periodicity and incompressibility) $(2) = \int$ **u** · 1 2 $\nabla \cdot u^2 - u \times (\nabla \times u) \, dv = \int \left[\frac{1}{2} \right]$ 2 $\nabla \cdot (\mathbf{u}u^2) - u^2 (\nabla \cdot \mathbf{u}) - \mathbf{0} \, dv = 0$ $(3) = \int_V v \, u \cdot (\nabla^2 u) dV = \int_V v [\nabla \cdot (u \cdot \nabla u) - |\nabla u|^2] = \int_V v |\nabla u|^2 dV$

Energy conservation

• From incompressible Navier-Stokes, energy conservation law:

• In the presence of energy 'pump' (source) **f**

$$
\frac{\partial K}{\partial t} = -\int_{V} \nu (\nabla u)^2 dV
$$

$$
\frac{\partial K}{\partial t} = \int_{V} f \cdot u dV - \int_{V} \sqrt{v(u)^{2} d}
$$

- Nonlinear terms and pressure transfer energy in a conservative way (inertial range)
- In steady-state energy is dissipated at the same rate at which it is let into the system
- The dissipation rate ε does not depend on viscosity!

 \mathcal{E}

- How small are the dimensions at which viscous effects prevail?
- What happens between the scales where energy is injected into the flow and those where it is dissipated?
- Theory of Kolmogorov provides answers to those questions. It starts from hypothesis motivated by empirical observations:
- (1) At very large but finite Reynolds number, the turbulent flow is statistically homogeneous and isotropic
- (2) At very large but finite Reynolds number, the turbulent flow is self-similar at scales smaller than the injection scale and larger than dissipative.
- (3) At very large but finite Reynolds number, the turbulent flow has a finite, nonvanishing mean dissipation rate per unit mass

Kolmogorov phenomenology (1941)

The main elements of phenomenology are:

- \circ λ , the scale under consideration (between injection scale L and dissipative scale η)
- α *u*_λ, the typical value of the velocity associated to scales ~ λ : *u*_λ ~ $\sqrt{$ < $\delta u^2(\lambda)$ >
- σ τ_{NL} the "eddy turnover time" associated with scale λ

- o Eddies of nearly the same size interact through their gradients and distort one another. τ_{NL} represents the timescale of this interaction through which energy flows across scales.
- \circ The rate of energy transfer from scales ~ λ to smaller ones is: Γ

Kolmogorov phenomenology (1941)

$$
I_{\lambda} \sim \frac{u_{\lambda}^2}{\tau_{NL}} \sim \frac{u_{\lambda}^3}{\lambda}
$$

- At the top and bottom of the inertial range we find:
- $\delta \epsilon \sim U^3/L$ (energy injection rate at macroscopic scale)
- \circ Kolmogorov dissipation scale η , where dissipation must balance inertia:

Kolmogorov phenomenology (1941)

• Inertial range there is neither direct energy input nor direct dissipation

$$
\Pi_{\lambda} \sim \frac{u_{\lambda}^2}{\tau_{NL}} \sim \frac{u_{\lambda}^3}{\lambda} \sim \varepsilon
$$

$$
u_{\lambda} \sim \varepsilon^{\frac{1}{3}} \lambda^{1/3}
$$

• Thus:

$$
\varepsilon \sim \frac{\nu}{\eta^2} u_\eta^2 \Rightarrow \eta \sim \varepsilon^{\frac{1}{4}} v^{\frac{3}{4}} \sim L R_e^{-\frac{3}{4}}
$$

Energy spectrum in fluids

$E = \int u^2(k)dk = \int E(k)dk,$ 1 \boldsymbol{k} $\mathcal{E}_{\mathcal{C}}$ 2 $\overline{3}k$ − 2 $\overline{3}$) = C_k ε 2 $\frac{1}{3}k^{-5/3}$

 $C_k = 1.6$

Typical magnetic field energy spectrum in the solar wind at 1AU (Kiyani et al. Phil. Trans. R. Soc. A. **373** 2015)

Solar wind turbulent spectrum

Turbulence in the Earth's magnetosphere

Magnetosheath

Magnetotail

Interstellar turbulence

MHD turbulence

- MHD contains 2 timescales: \bigcirc
	- Fluid timescale of overturning eddies
	- Wave propagation timescale Θ
- MHD has also the magnetic field that is coupled to the velocity field \bigcirc
- In the presence of an average background magnetic field, the system is \bigcirc not isotropic

From hydro to MHD

Alfvén waves

• Transverse to both B_0 and k

• Oscillations are driven by magnetic field tension and inertia

- restoring force
-

ransverse/incompressible)

• Magnetic tension only provides the

$$
\boldsymbol{B}_0 \cdot \delta \boldsymbol{u} = 0 \qquad \qquad \boldsymbol{k} \cdot \delta \boldsymbol{u} = 0 \qquad (t
$$

- Incompressible means that there is not perturbed density and pressure
- Relation between B and u fluctuations:

Incompressible MHD

Introduce Elsasser variables

$\nu \nabla^2$ u

$$
\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P + \mathbf{B} \cdot \nabla \mathbf{B} +
$$

$$
\frac{\partial \mathbf{B}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{u} + \eta \nabla^2 \mathbf{E}
$$

$$
\nabla \cdot \mathbf{u} = 0
$$

$$
\nabla \cdot \mathbf{B} = 0
$$

 $\mathbf{z}^{\pm} = \mathbf{u} \pm \frac{\mathbf{b}}{\sqrt{4\pi\rho}}$

Incompressible MHD - reloaded

Sum and subtract **u** and **B** equations and use new variables:

dissipation

- z^{\pm} represent "counterpropagating" Alfvénic fluctuations
- Two equations displaying a similar symmetry as Navier-Stokes are recovered, however with important differences: the nonlinearity couples different fields, and there is a linear propagation term

- o Nonlinear interactions occur between 'counter-propagating' Alfvénic fluctuations
- o $k_{\parallel}V_a \gg k_{\perp}z^{\pm}$ turbulence in weak

$$
\text{or } k_{\parallel}V_a \ll k_{\perp}z^{\pm} \text{ turbulence is strong}
$$

- o Iroshnikov (1964) and Kraichnan (1965) argue that in MHD, wave propagation (τ_A) is more important that turbulent "eddy" interaction (τ_e) .
- \circ The hypothesis is that interactions are weak, $\tau_A =$ λ_{\parallel} V_a $\ll \tau_e \equiv$ that turbulence is isotropic, $\lambda_{\parallel} \sim \lambda_{\perp}$.

λ u_{λ} . The other hypothesis is

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- The timescale for the two wave packets to cross each other is of the order of the Alfvén time $\Delta t \sim$ λ_{\parallel} V_a ∼ λ V_a
- o One (weak) eddy interaction will lead to a variation $\Delta u_1 \ll u_2$ in time Δt

$$
\Delta u \sim \Delta t \frac{u_{\lambda}^2}{\lambda} \sim \frac{\lambda}{V_a} \frac{u_{\lambda}^2}{\lambda}
$$

 \circ The non-linear decay of the wave packets in such weak interactions can only occur after several interactions, $\tau_{NL} \sim N \Delta t$

o After N random collisions there is a cumulative change $\left(\frac{\Delta u}{\Delta u}\right)$ $u \tJ_N$

$$
\frac{\Delta u_{\lambda,N}}{u_{\lambda}} \sim \sqrt{N} \frac{\Delta u_{\lambda}}{u_{\lambda}} \sim \sqrt{N} \frac{\lambda}{V_{\alpha}} \frac{u_{\lambda}}{\lambda}
$$

- \circ Thus, a number of collision $N \sim$ V_a u_{λ} 2 velocity $\frac{\Delta u}{\Delta t}$ $\overline{\mathcal{U}}$ $~\sim 1.$
- The nonlinear time, is thus longer than Kolmogorov's:

is required to have a relative cumulative variation of

$$
\tau_{NL} \sim N\Delta t \sim \left(\frac{v_a}{u_{\lambda}}\right)^2 \frac{\lambda}{v_a}
$$

This longer nonlinear time leads to a different prediction for the energy:

$$
E(k) \sim (\varepsilon v_A)^{\frac{1}{2}} k^{-3/2}
$$

Goldreich-Sridhar (1995)

- The model of Goldreich-Sridhar was introduced to include anisotropy of the turbulent cascade. It is based on critical balance, $\frac{\lambda_{\perp}}{\lambda_{\perp}}$ u_{λ} ∼ λ_{\parallel} v_A so that $\tau_{NL}^{\pm} \sim \lambda_{\perp}/z^{\mp}$
- o Under these assumptions one recovers the Kolmogorov scaling:

, and on the strong turbulence condition

- o Other anisotropic models by (Grappin et al. 2014; Boldyrev 2006, Ng & Bhattacharjee 1997, Galtier et al. 2000)
- o To what extent can we apply this phenomenology to solar wind turbulence?

$$
E(k_{\perp}) \sim k_{\perp}^{-5/3}
$$
 with $k_{\parallel} \sim k_{\perp}^{3}$, $E(k_{\parallel}) \sim k_{\parallel}^{-2}$

Turbulence in the inner heliosphere with Parker Solar Probe

(Chen et al. ApJS 2020, Shi et al, 2021)

0.9 0.8 0.7 0.6 $0.5 \space \widehat{\overline{\overline{a}}}$ 0.4 0.3 0.2 0.1

 Ω

Magnetic field anisotropy

The turbulent cascade is

• However, shallower $(E \sim k_{\perp}^{-})$ −3/2) spectra are also observed [Sioulas

• Observations seem to favor critical balance far from the sun and k_\perp^- −5/3

V & B spectra are different

[Podesta et al 2006 & C. Smith slides]

Two types of wind/two types of turbulence

[Helios data, Grappin et al. JGR 1990]

Two types of wind/two types of turbulence

• There is a "standard type of turbulence", balanced $\sigma_c \approx 0$

$$
\sigma_c = \frac{|z^+|^2 - |z^-|^2}{|z^+|^2 + |z^-|^2}
$$

• And a highly imbalanced

Turbulence in the inner heliosphere with Parker Solar Probe

- $|\delta$ **B**|/B₀~1
- \cdot δ **B** $|/B_0 << 1$
- δ**B**/√ρμ0~∓δ**V**
- mainly propagating outward
- Developed spectrum
- δρ/ρ<<1

Alfvénic turbulence properties

[Matteini et al 2024]

- $|\delta$ **B** $|/B_0$ ~1
- $δ$ B $/B_0 << 1$
- δ**B**/√ρμ0~∓δ**V**
- mainly propagating outward
- Developed spectrum
- δρ/ρ<<1

Waves and Turbulence in the Solar Wind: Why is it So Complicated?

- Both nonlinearities and expansion effects (inhomogeneity) are crucial
- Also large scale shears in the wind can play a role

Challenges from PSP

Switchbacks

• Switchbacks are large amplitude Alfvénic fluctuations sometimes leading to a kink backwards of the magnetic field

Switchbacks

(Rouafi et al. Sp Sci Rev. 2023)

• The corresponding signature in velocity is a

• In switchbacks, $V \& B$ are correlated like in Alfvén waves and $|B|^2 \simeq const \Rightarrow$ spherical

- radial jet.
- polarization
- fluctuations

• They are embedded in the flux of turbulent

What is their origin and impacts in interplanetary turbulence?

- Related to coronal jets?
- Formed nonlinearly during evolution in heliosphere?
- Reconnection in the corona?
- What is their role in the turbulent cascade?

(MacNeil et al. MNRAS 2020)

A product of reconnection (Drake et al. 2021; Zank et al 2021.)

> Induced by velocity shears (Landi et al. GRL 2006; Ruffolo et al ApJS 2020; Schwadron et al 2020, Toth et al 2023)

Induced by expansion (Squire et al. 2020; Mallet et al. 2021; Shoda et al. 2021)

Summary

- The solar wind is an excellent laboratory to study turbulence in magnetized plasmas
- Turbulence can provide a mechanism to heat and accelerate the solar wind, as well as to heat the solar corona (wave-driven winds)
- Turbulence is indeed observed in-situ everywhere in the heliosphere
- While similar concept from hydrodynamic turbulence can be applied to the solar wind, the details still need to be framed in a coherent theory.
- Parker Solar Probe is giving us a glimpse of what the solar wind looks like closer to the sun, a lot of work is still needed to fully understand evolving waves and turbulence in the interplanetary space!

Thanks!